

Zoom & Univ. Tsukuba, Feb.11-13 (2022)



Welcome !

International workshop

*“Variety and universality of **Bulk-Edge** **C**orrespondence in topological phases:
From solid state physics to transdisciplinary concepts”
BE/BC2022 (Bulk-Edge/Boundary Correspondence)
(**BE/BC2022**)*

Yasuhiro Hatsugai

*Department of Physics
University of Tsukuba*

Zoom & Univ. Tsukuba, Feb.11-13 (2022)



Welcome !

Bulk-edge correspondence :
From Quantum Hall effects

Yasuhiro Hatsugai

Department of Physics
University of Tsukuba

Our project **BEC** Zoom & Univ. Tsukuba, Feb.11-13 (2022)

Bulk-Edge Correspondence

With edges

- ★ Observables
- ★ *In gap states*
- ★ *Low energy modes*

Solid states

As a bulk

- ★ Topology is hidden
- ★ *Gapped*
- ★ *Unconventional*

bulk-edge correspondence

Photonics & Newton's

"Math"

Cold atoms

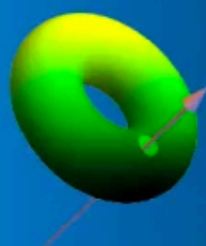


Variety and universality of bulk-edge correspondence in topological phases:

BE/BC2022

Feb.11-13 (2022)

From solid state physics to transdisciplinary concepts



Our project

BEC

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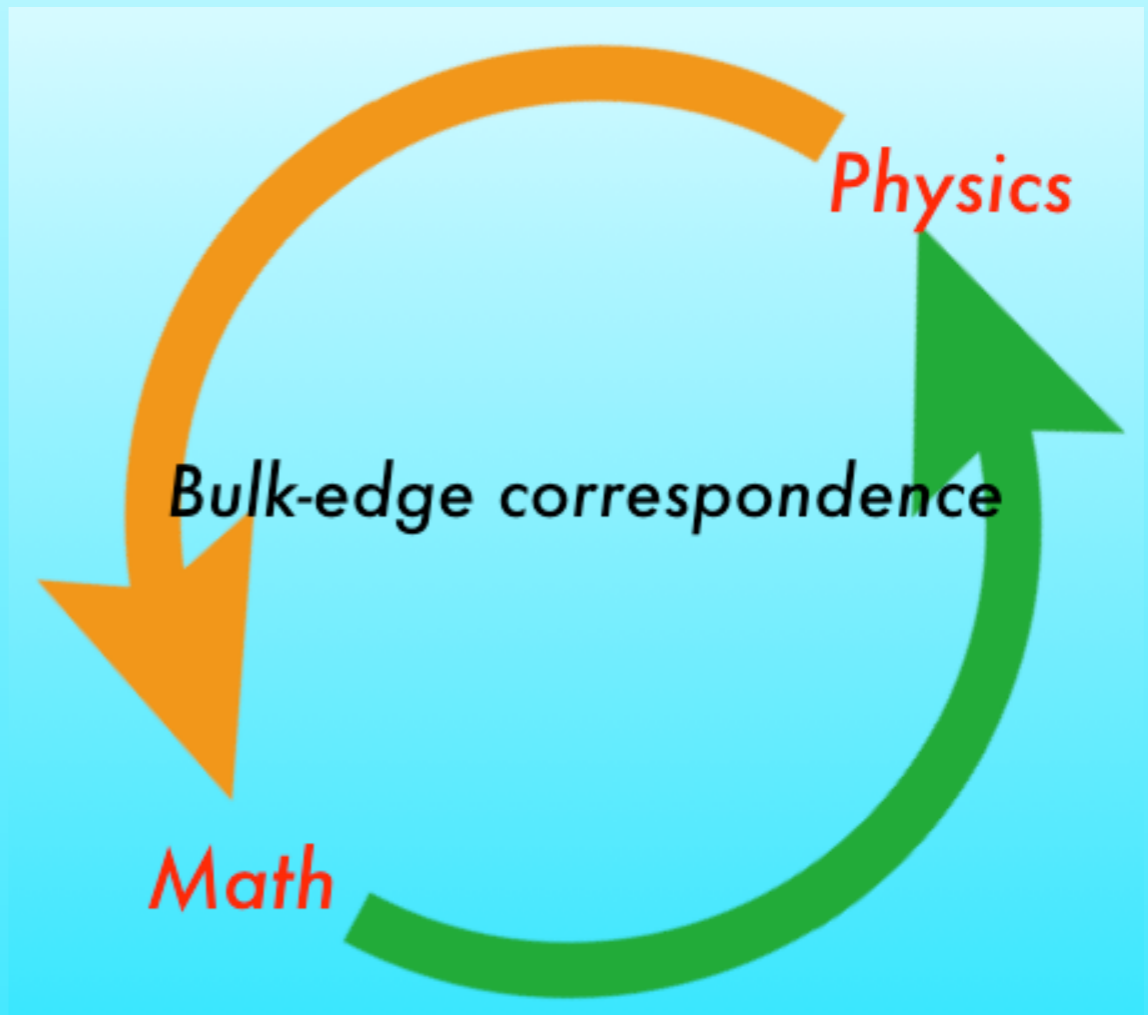
Bulk-Edge Correspondence

Solid states

bulk-edge correspondence

Photonics & Newton's

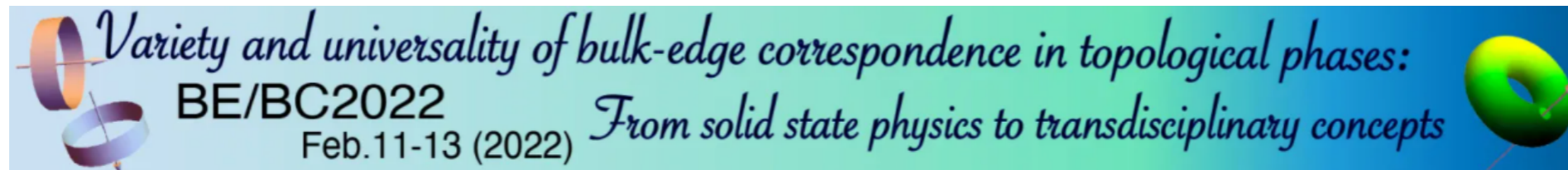
Cold atoms



History (international workshops)

We are now 7-th !

★ **BE/BC2022** Zoom, Feb.11-13 (2022)



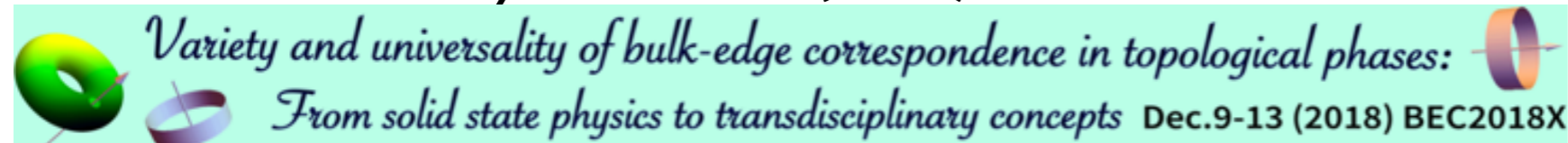
★ **BE/BC2020F** Tsukuba (virtualized due to COVID19), Feb.28-29 (2020)



★ **NTTI2019 & BEC2019** Hiroshima, July.14-19 (2019)



★ **BEC2018X** Tokyo, Dec.9-13 (2018)



★ **BEC2018 Jan** Tsukuba, Jan. 6-8 (2018)



★ **BEC2016** Kyoto, Sep. 27-30 (2016)



★ **BEC2015** Tokyo, Sep. 27-29 (2015)



Project 2

2017-2022.3

Supports by



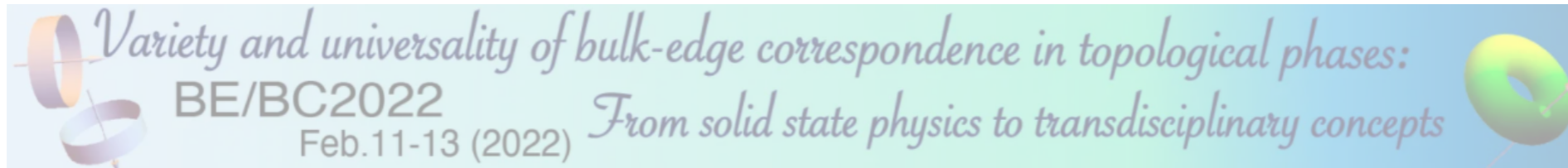
Project 1

2014-2017.3

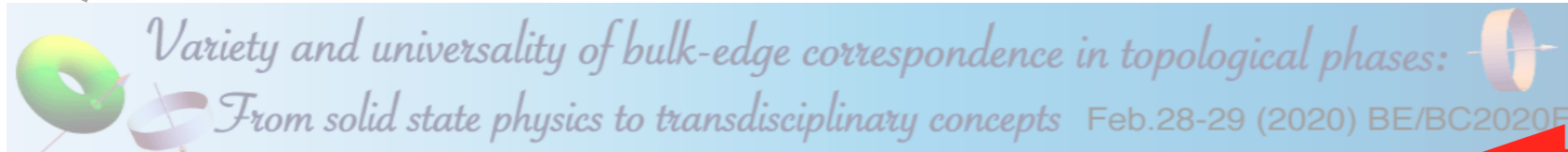
Surprises/Big events !

We are now 7-th !

★ **BE/BC2022** Zoom, Feb.11-13 (2022)



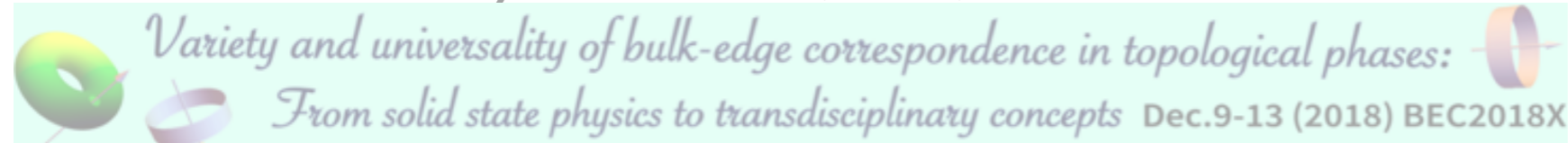
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Coronavirus disease (COVID-19) pandemic



Photo: A. Mahmoud
David J. Thouless
Prize share: 1/2



Photo: A. Mahmoud
F. Duncan M. Haldane
Prize share: 1/4



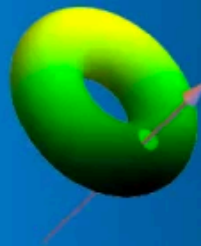
Photo: A. Mahmoud
J. Michael Kosterlitz
Prize share: 1/4

THE NOBEL PRIZE

The Nobel Prize in Physics 2016

2017-2022.3

2014-2017.3



Started from here

Quantum Hall effects

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electrons in a magnetic field

VOLUME 45, NUMBER 6

PHYSICAL REVIEW LETTERS

11 AUGUST 1980

**New Method for High-Accuracy Determination of the Fine-Structure Constant
 Based on Quantized Hall Resistance**

K. v. Klitzing

*Physikalisches Institut der Universität Würzburg, D-8700 Würzburg, Federal Republic of Germany, and
 Hochfeld-Magnetlabor des Max-Planck-Instituts für Festkörperforschung, F-38042 Grenoble, France*

and

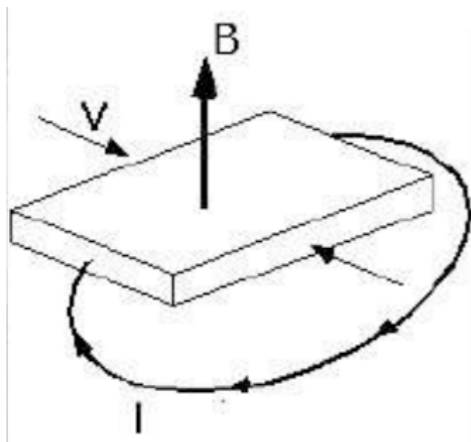
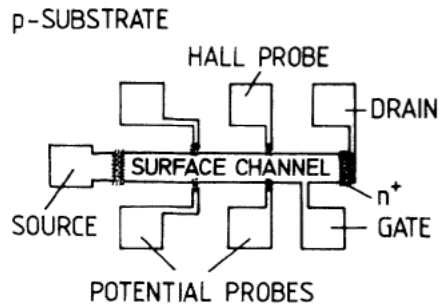
G. Dorda

Forschungslaboratorien der Siemens AG, D-8000 München, Federal Republic of Germany

and

M. Pepper

Cavendish Laboratory, Cambridge CB3 0HE, United Kingdom



$$I_x = \sigma_{xy} V_y$$

$$-\sigma_{xy} = e^2 i / h$$

$i = 1, 2, 3 \dots$ Integer QHE

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Quantum Hall effects

VOLUME 49, NUMBER 6

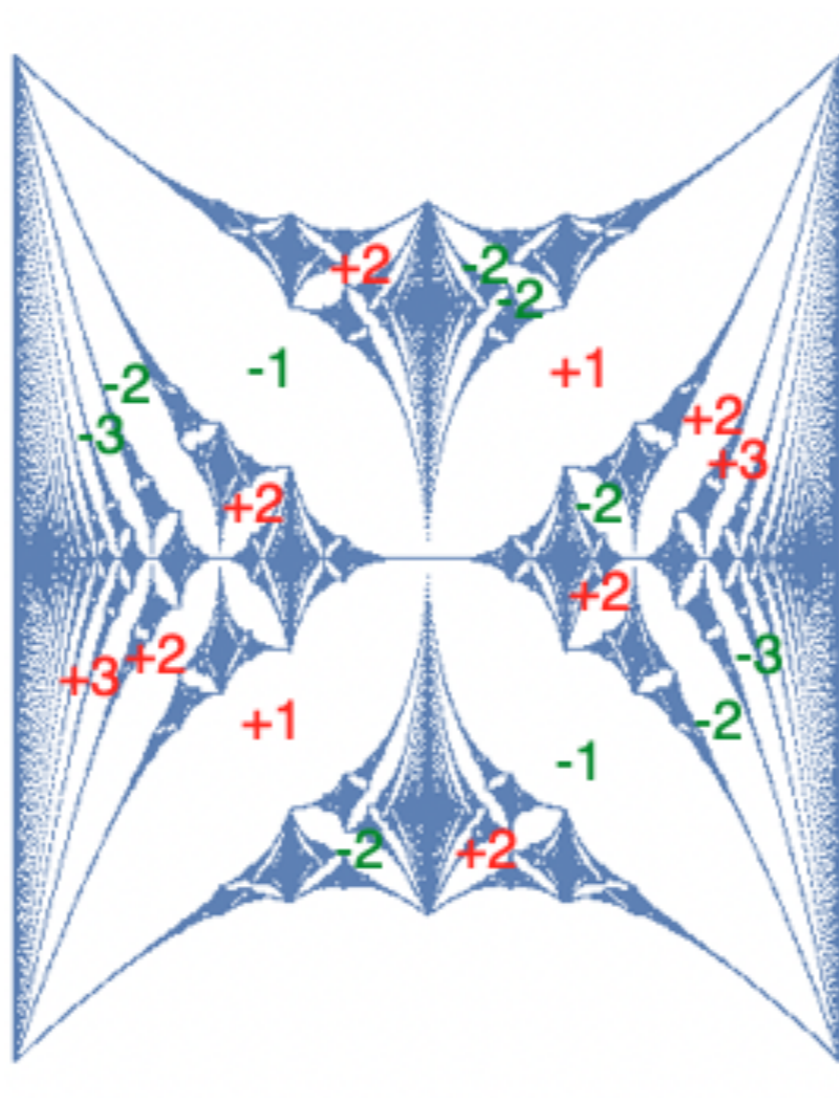
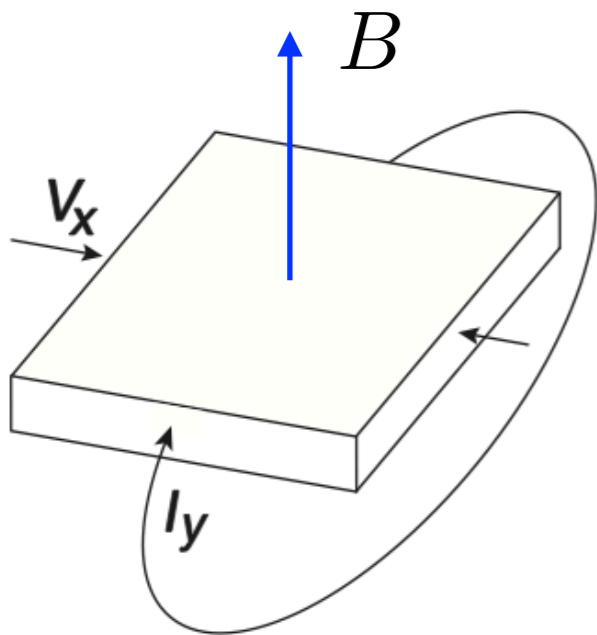
PHYSICAL REVIEW LETTERS

9 AUGUST 1982

Quantized Hall Conductance in a Two-Dimensional Periodic Potential

D. J. Thouless, M. Kohmoto,^(a) M. P. Nightingale, and M. den Nijs
 Department of Physics, University of Washington, Seattle, Washington 98195
 (Received 30 April 1982)

TKNN



$$I_y = \sigma_{yx} V_x$$

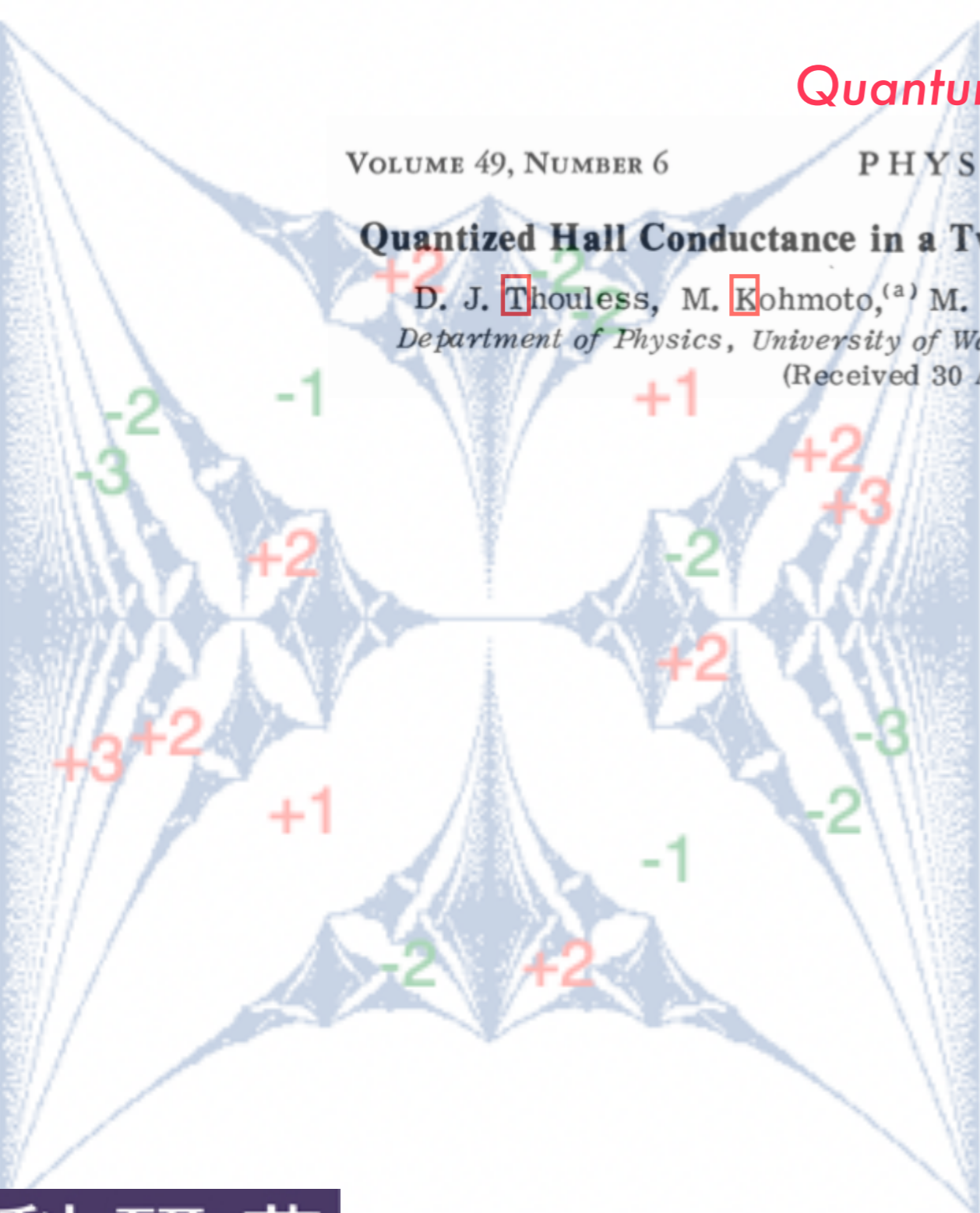
$$\sigma_{yx} = \frac{e^2}{h} \times n$$

$$n = 1, 2, 3, \dots$$

TKNN integers

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Quantum Hall effects



TKNN

Topological numbers for condensed matter physics

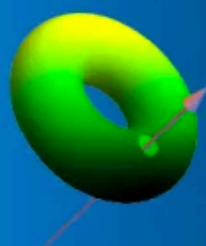
Cool!

(exceptional ?)

I was a grad. student in the late 80'.

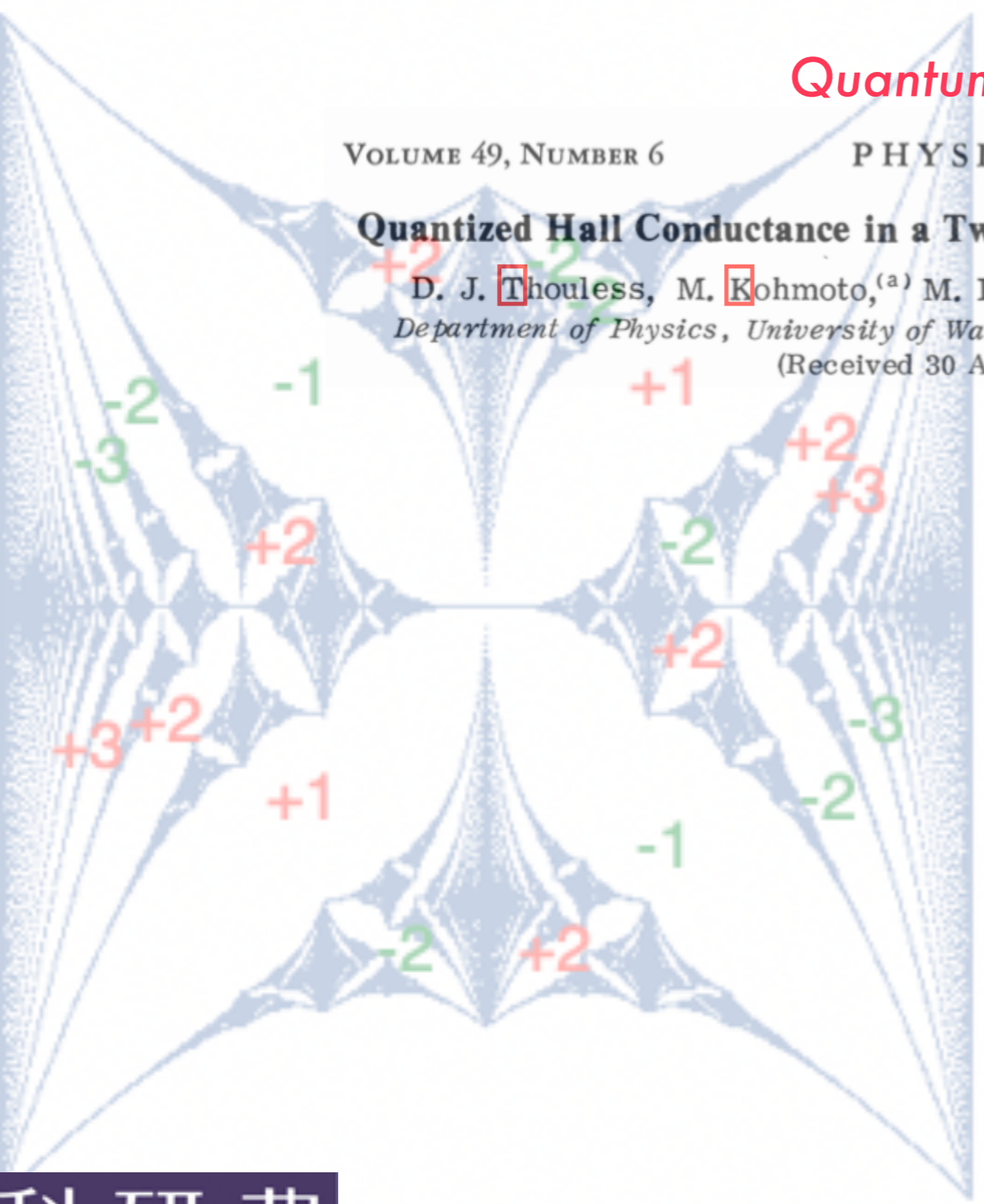


Variety and universality of bulk-edge correspondence in topological phases: BE/BC2022 Feb.11-13 (2022) *From solid state physics to transdisciplinary concepts*



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Quantum Hall effects



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Topological numbers for condensed matter physics

Cool !

THE
NOBEL
PRIZE

Universal !



Photo: A. Mahmoud
David J. Thouless
Prize share: 1/2



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F. Duncan M. Haldane
Prize share: 1/4



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Prize share: 1/4

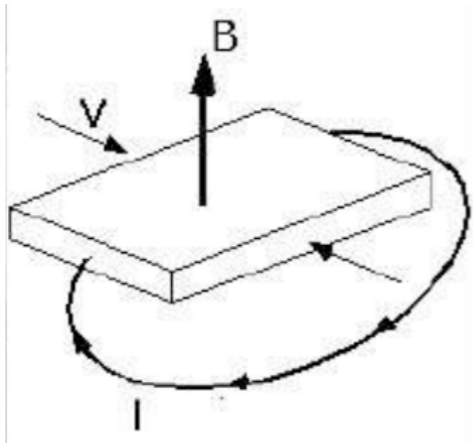
科研費
KAKENHI

Grant-in-Aid for Scientific Research (S) Project No.17H06138

筑波大学
University of Tsukuba

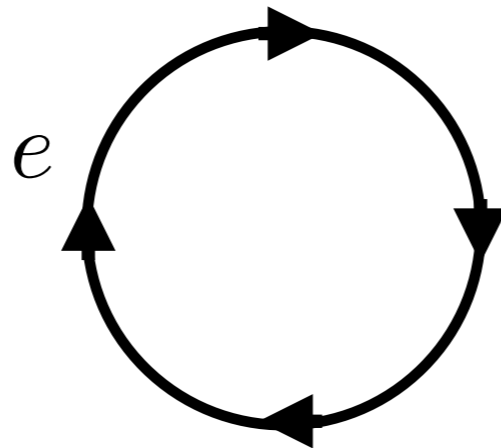
Quantum Hall effects

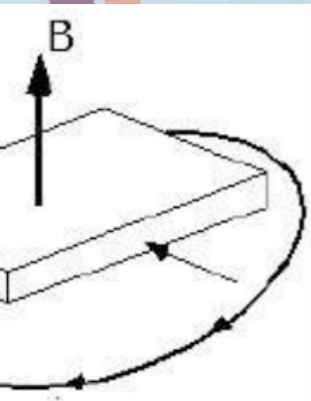
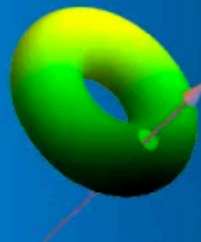
electrons in a magnetic field



$$F = ev \times B$$

Circular (cyclotron) motion
due to Lorentz force





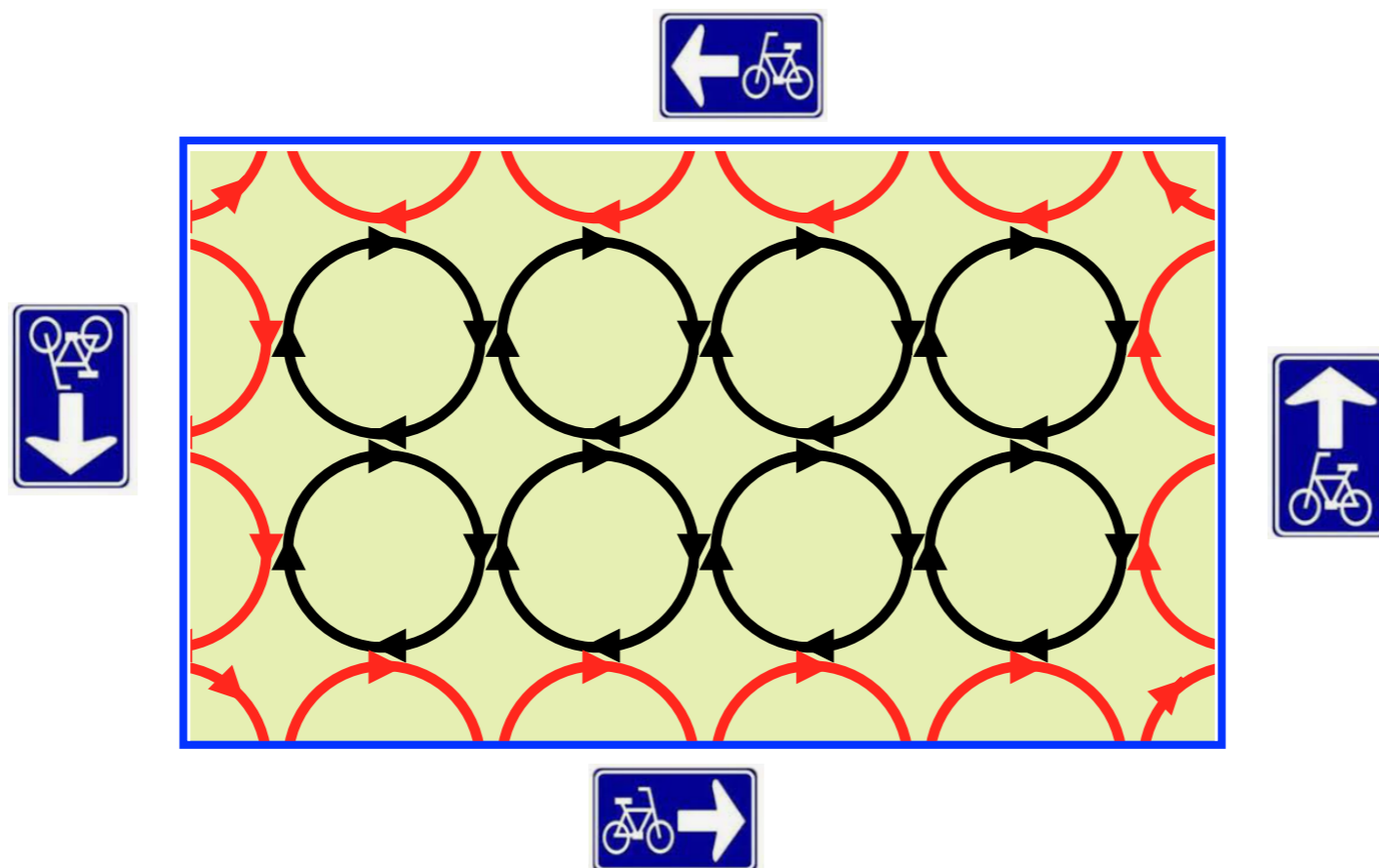
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Quantum Hall effects

With boundaries

One way going !!

Cannot stop !



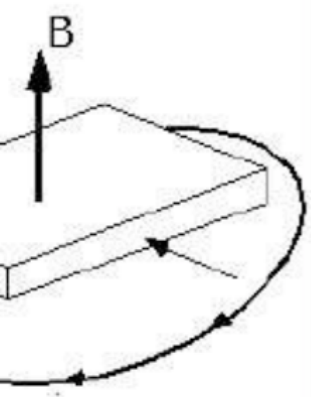
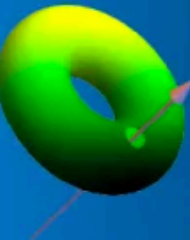
"skipping orbit"

Variety and universality of bulk-edge correspondence in topological phases:

BE/BC2022

Feb.11-13 (2022)

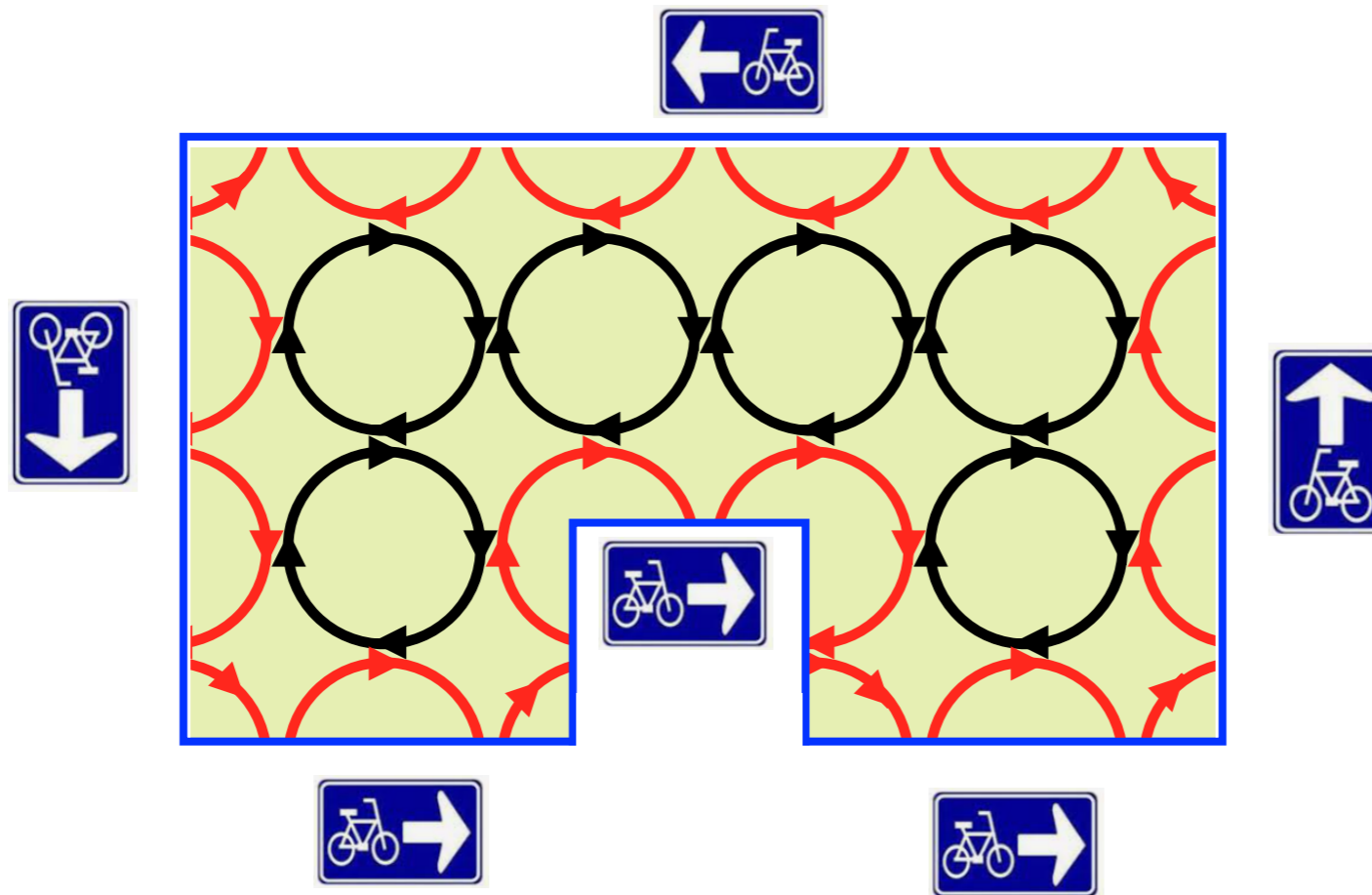
From solid state physics to transdisciplinary concepts



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Quantum Hall effects

With boundaries



One way going !!

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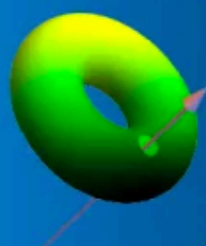
No back scattering

Stable for impurities !!

"Topological" stability

"skipping orbit"

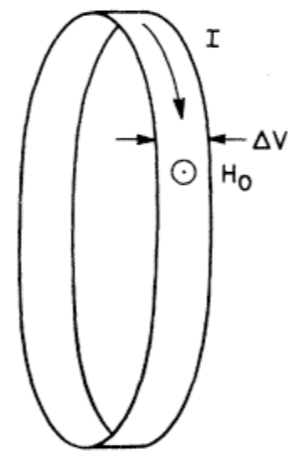
Edge states are essential ! Halperin '82



Quantum Hall effects

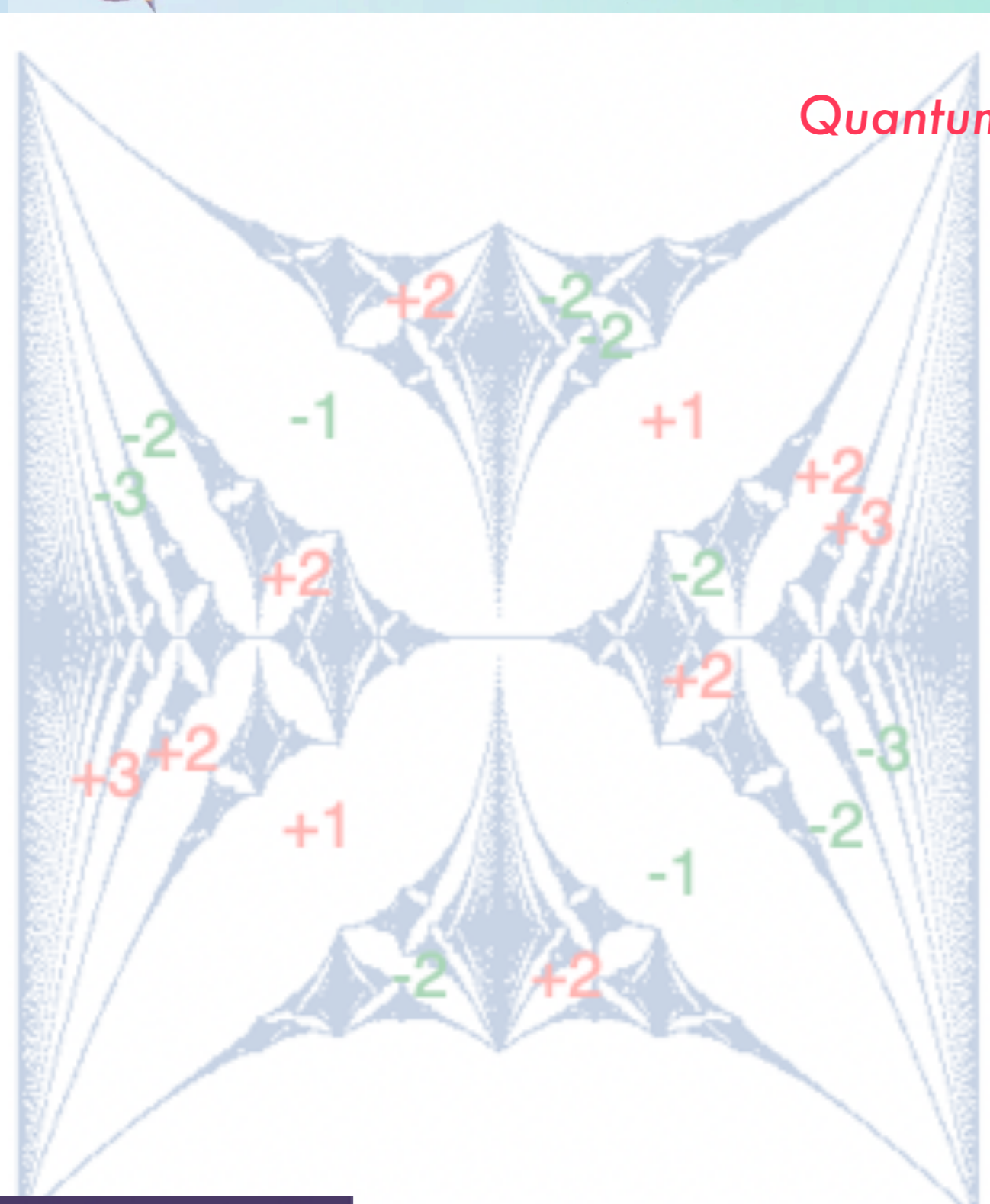
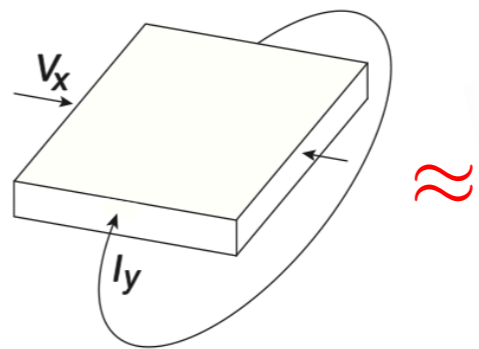
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QHE due to edge states



Laughlin's argument '81

"cylinder"



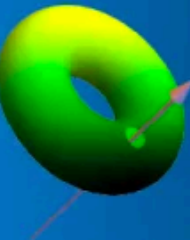


Variety and universality of bulk-edge correspondence in topological phases:

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Zoom & Univ. Tsukuba, Feb.11-13 (2022)

Quantum Hall effects

QHE due to edge states

$$C_j = I_j - I_{j-1}$$

TKNN integer

=

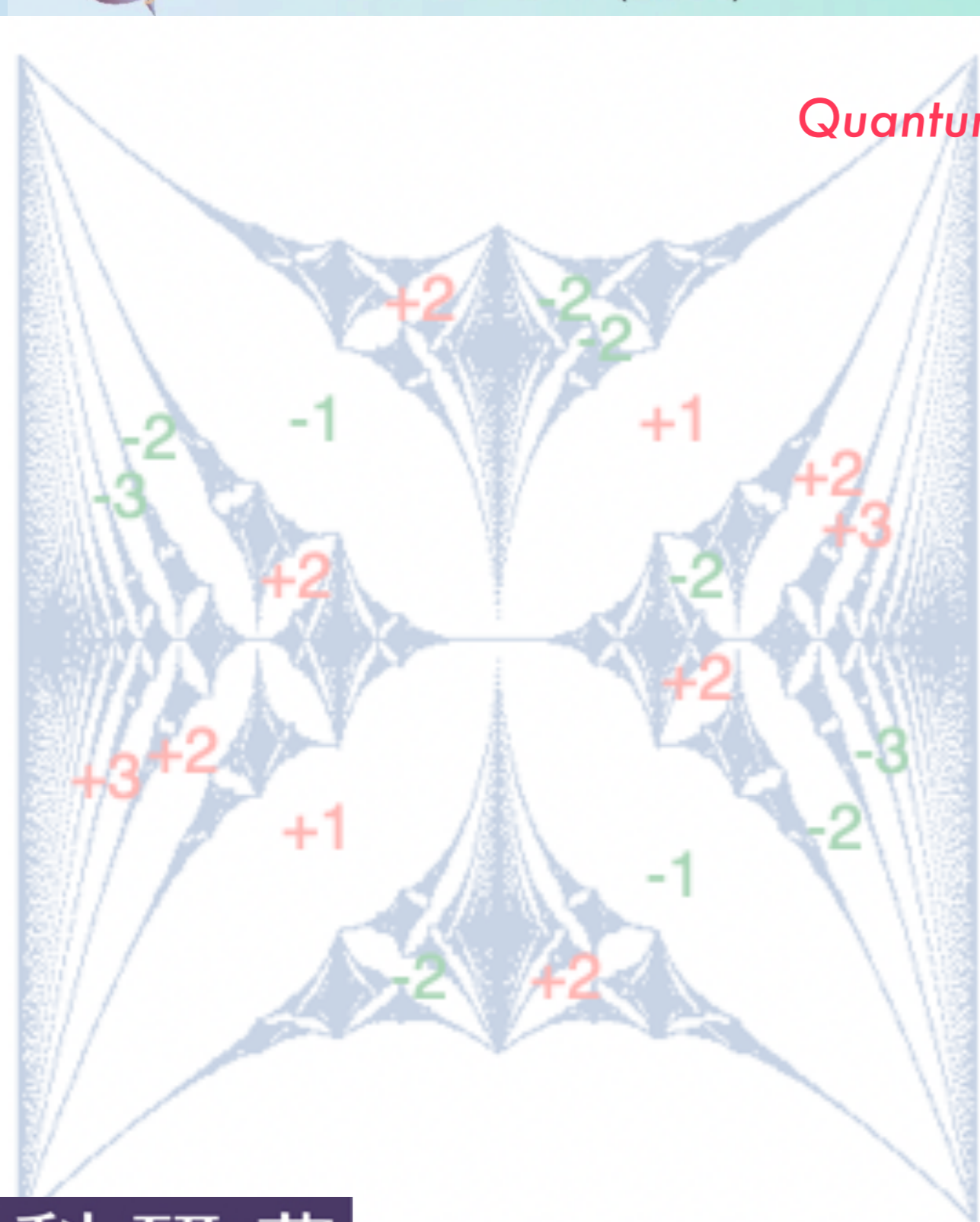
Number of edge states

$\sigma_{xy}^{\text{bulk}}$

=

$\sigma_{xy}^{\text{edge}}$

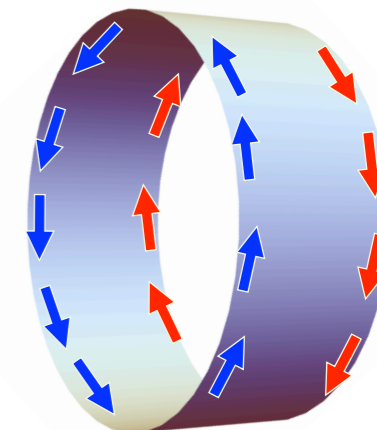
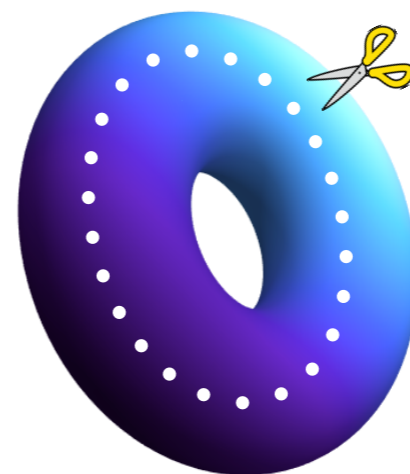
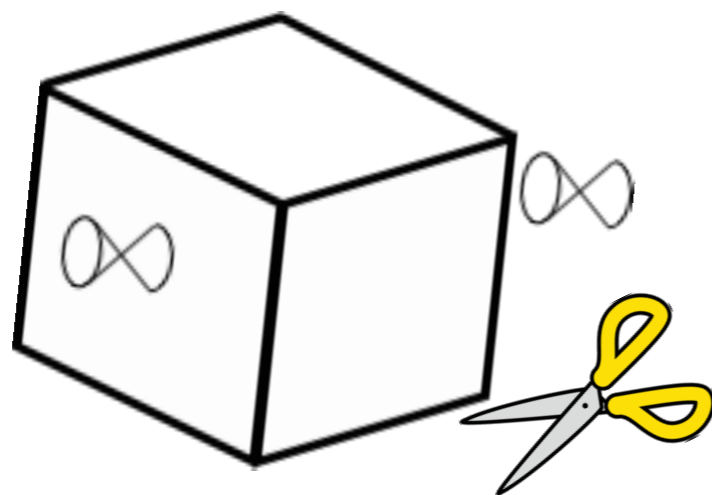
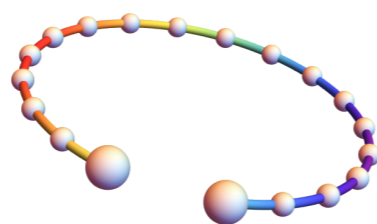
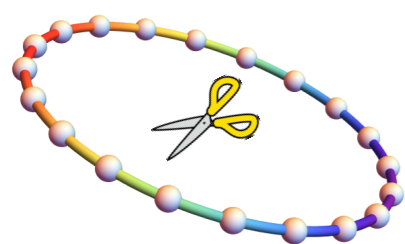
YH '93



BEC

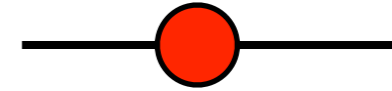
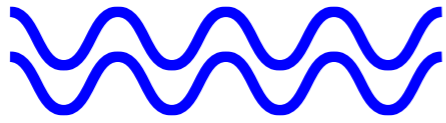
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Bulk-Edge Correspondence



Consider bulk with edges
Topological number : mostly hidden

Bulk-edge correspondence



Bulk state
(scattering state)
Bulk Gap
Non trivial Vacuum

Control
with
each other



Edge state
(Bound state)
Particles in the gap

QHE

They can not be independent

History...

Laughlin '81 : Gauge invariance on **cylinder**

Halperin '82: Role of edge states in QHE

X.G.Wen '90 : Gauge invariance with edges

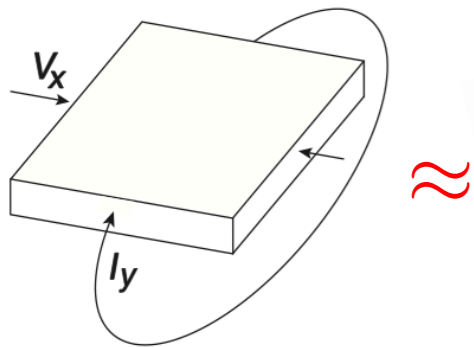
YH '93 : BEC of IQHE (Hofstadter)

Kitaev '01 Majorana boundary states of superconductors

Ryu-YH '02 BEC of graphene, d-wave superconductivity

Qi-Wu-Zhang '06, BEC : general theorem

... more & more...



≈



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History...

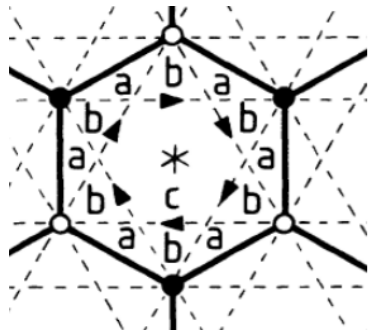
Haldane model

Haldane '88

Quantum Spin Hall effects

Kane-Mele '05

Bernevig-Hughes-Zhang '06



Topological number is **hidden** !

Observe edge states !

predicted by the non-trivial bulk

Topological phases & Bulk-edge correspondence

Quantum Hall effects
Quantum spin Hall effects
Topological insulators

'Quantum'
Schrödinger eq.

Before

2005 (Millennium)

After

Surprise in this Millennium

Do NOT need quantum mechanics

Topological photonics

Haldane-Raghu '05

Wang-Chong-Joannopoulos-Soljačić '08

Edge states of photons

Hafezi-Demler-Lukin-Taylor '11

Topological mechanics

Prodan-Prodan '09

Kane-Lubensky '14

V. Vitteli et al. '14

Topological circuit

Albert et al. '15

Topological phases & Bulk-edge correspondence

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Topological circuit

Albert et al. '15

Try to explore more in our project from 2017...

Project 2

2017-2022.3

Project 1

2014-2017.3

New wave!

2005 (Millennium)

After

Surprise in this Millennium

Do NOT need quantum mechanics

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Topological mechanics

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Topological circuit

Albert et al. '15

Further surprises

Talks in the workshop

...

Quantum to classical

Newton



$$x \cdot p = p \cdot x$$

“Quantization”



Schrödinger



$$x \cdot p \neq p \cdot x$$

Classical mechanics/EM fields

Newton eq.
Maxwell eq.

Quantum mechanics

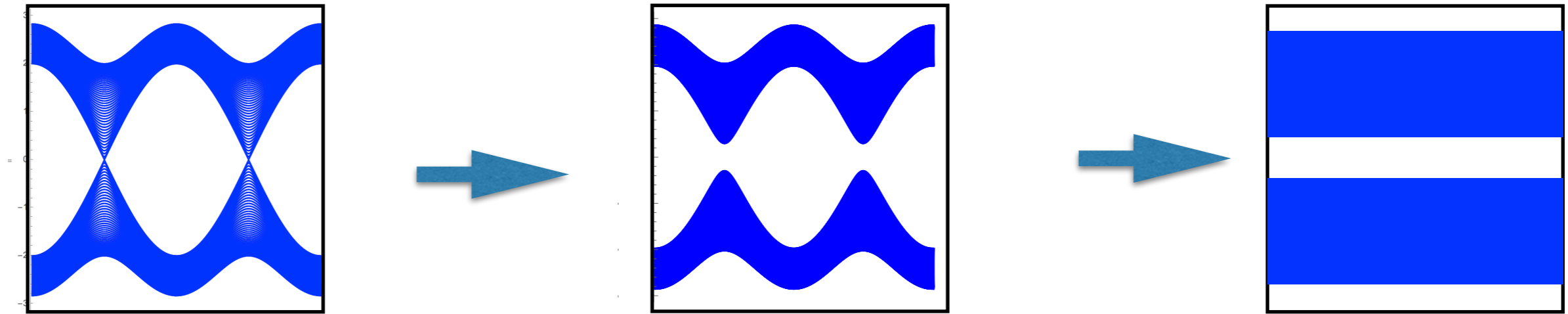
Schrödinger eq.



“Bulk-edge correspondence”

Massive Dirac fermions (level crossing) is essential

Key ingredients

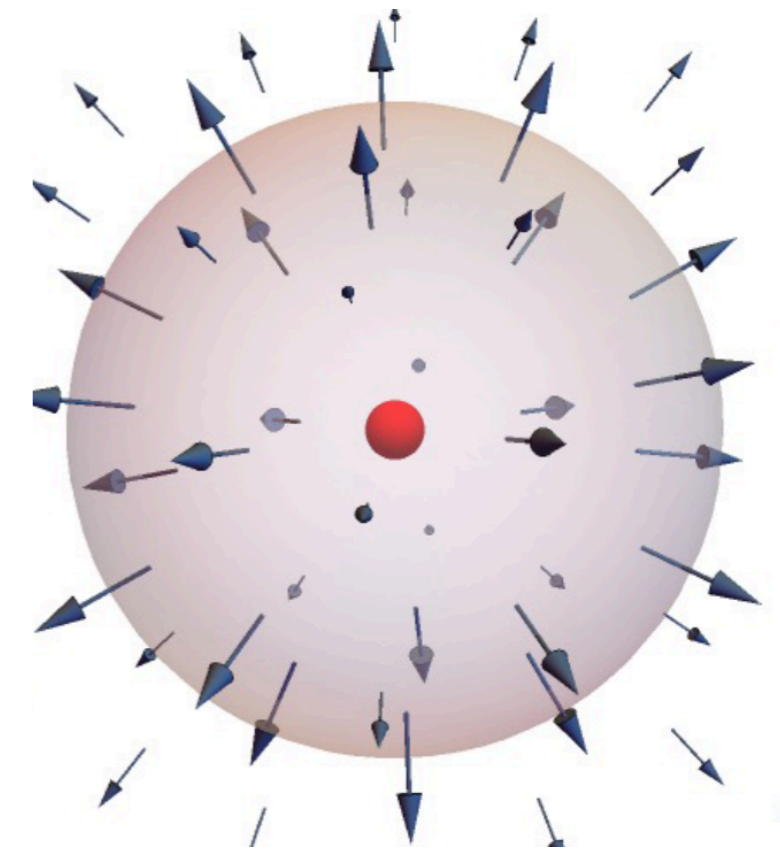


Bulk

*Dirac fermions/gap closing
in parameter space*

If gapped, they are intrinsically hidden !

Massless Dirac fermions = gap closing



2x2 eigen value problem $\mathbf{R} = (x, y, z)$

$$H = \begin{pmatrix} z & x - iy \\ x + iy & -z \end{pmatrix},$$

$$H\psi = \epsilon\psi, \quad \epsilon = \pm R, \quad R = |\mathbf{R}|$$

$$\mathbf{A} = \psi^\dagger \nabla \psi \quad \text{Berry connection}$$

$$\mathbf{B} = \nabla \times \mathbf{A} = \pm \frac{i}{2} \frac{\mathbf{R}}{R^3}$$

$$\nabla \cdot \mathbf{B} = \pm 2\pi i \delta(\mathbf{R})$$

(magnetic) Dirac mono pole

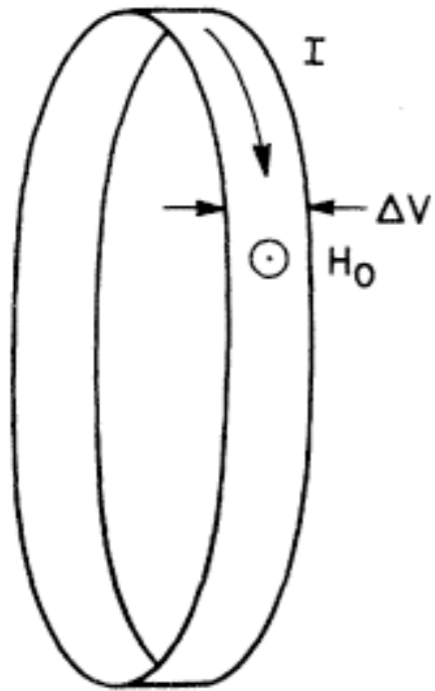
Maxwell eq.

$$\mathbf{E}_{EM} = \frac{e}{4\pi\epsilon_0} \frac{\mathbf{R}}{R^3}$$

$$\nabla \cdot \mathbf{E}_{EM} = \frac{e}{\epsilon_0} \delta(\mathbf{R})$$

point charge

Non trivial Berry connection



Gauge fields (synthetic in general)

+

Laughlin argument

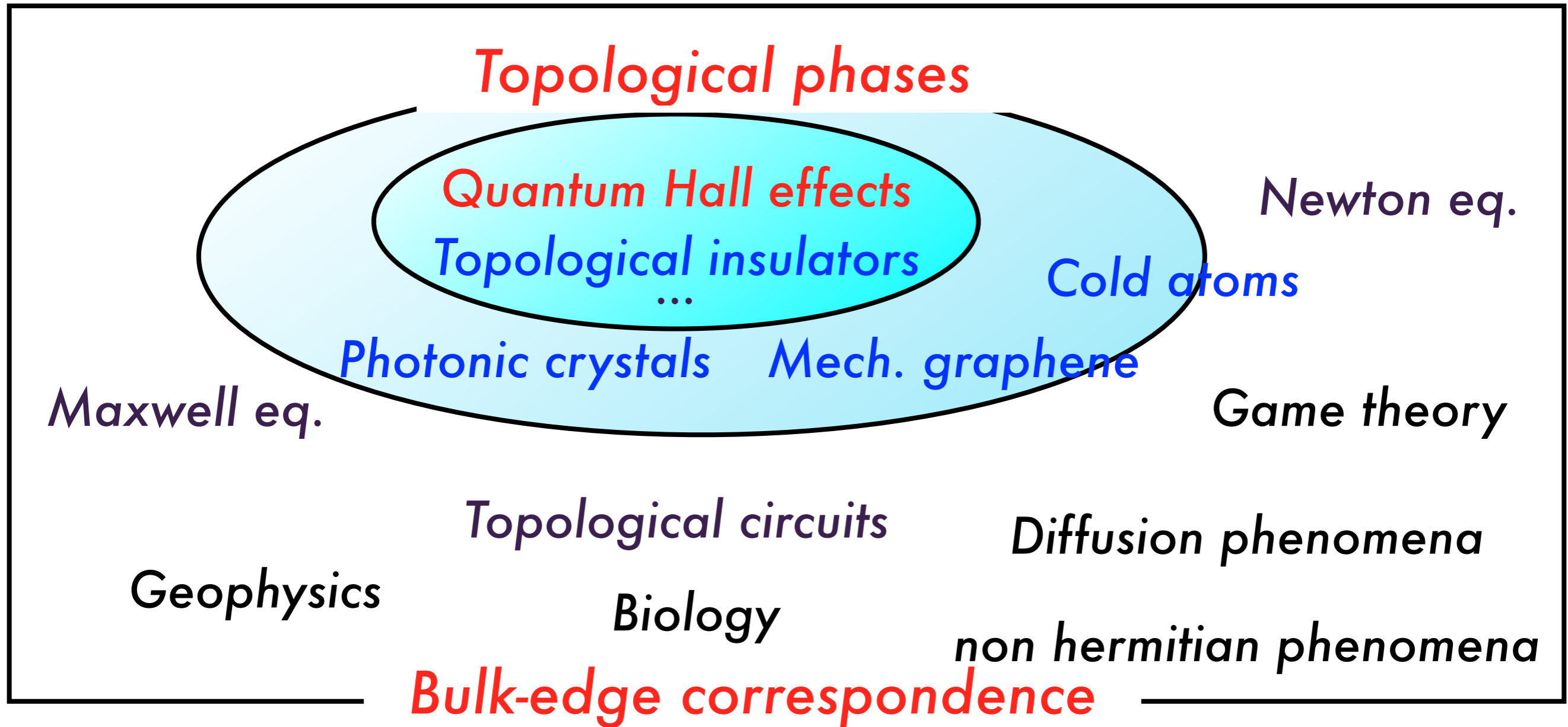
||

Bulk-edge correspondence



Topology & BE/BC are everywhere

Edge states are everywhere : topological origin





Variety and universality of bulk-edge correspondence in topological phases:

From solid s

Two years ago

Feb.28-29 (2020) BE/BC2020F

Univ. Tsukuba, Feb.28-29 (2020)

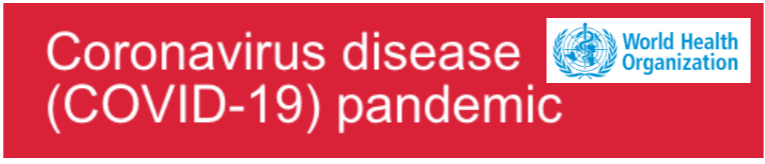
(Virtual)

Exchange idea between **theorists** & **experimentalists**

BE/BC2020F among **different fields**

Enjoy this virtual workshop !

Feb. 28-29 (2020)



+ Lots of new results of the participants today

Zoom & Univ. Tsukuba, Feb.11-13 (2022)

Exchange idea between **theorists** & **experimentalists**
BE/BC2022 among **different fields**

Enjoy this workshop !

By zoom (new wisdom)

“Science of Bulk-edge correspondence”

Beyond topological phases