(15日) 復習

Laughlin の議論 → $\sigma_{xy} = \frac{e^2}{h} C$ ： C：未定の整数

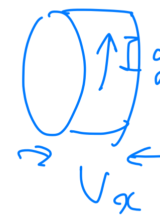
ホール伝導度

∥

gauge 不変性 の議論

☆ TKNN / NTW の議論によると

Niu-Thouless-Wu



$I_y = \underbrace{\sigma_{yx}} V_x$

C：Chern 数 (今日の呼び方では)

∥

TKNN 数

$$C = \frac{1}{2\pi i}\int_0^{2\pi} d\theta_x \int_0^{2\pi} d\theta_y \left[\langle \partial_x \psi | \partial_y \psi \rangle - \langle \partial_y \psi | \partial_x \psi \rangle \right]$$

⇔ $\theta_{x,y}$ ： <u>twisted b.c.</u>

$$H(\theta_x, \theta_y)|\psi(\theta_x \theta_y)\rangle = |\psi\rangle E$$

$|\partial_x \psi\rangle$?　$|\psi\rangle$ をパラメター微分する

$H(x)|\psi\rangle = |\psi\rangle E$　位相が不定

$$|\partial_x \psi\rangle = \lim_{\Delta x \to 0} \frac{|\psi(x+\Delta x)\rangle - |\psi(x)\rangle}{\Delta x}$$

✓ $|\psi(x+\delta x)\rangle \xrightarrow[\delta x \to 0]{} |\psi(x)\rangle$ とは限らない

$H(x+\delta x)|\psi(x+\delta x)\rangle = |\psi(x+\delta x)\rangle E(x+\delta x)$

$H(x)|\psi(x)\rangle = |\psi(x)\rangle E(x)$

独立(自由)にそれぞれ位相がついているかもしれない

一般には微分はできない

何か位相を決める必要：gauge 固定

"Berry 接続"

$$\langle\psi|d\psi\rangle = dx_i \underbrace{\langle\psi|\partial_i\psi\rangle} = dx_i A_i$$

$$|\psi\rangle \to |\psi\rangle g =: |\psi_g\rangle \qquad A_i := \langle\psi|\partial_i\psi\rangle$$

$$, \quad g = e^{i\theta}$$

$$A = \langle\psi|d\psi\rangle$$

$$A_g = \langle\psi_g|d\psi_g\rangle = g^{-1}\langle\psi|d(|\psi\rangle g)$$

$$= g^{-1}\underbrace{\langle\psi|d\psi\rangle}_{A} g + g^{-1}\overbrace{\langle\psi|\psi\rangle}^{1} dg$$

$$= g^{-1}Ag + g^{-1}dg$$

gauge 変換

$$= A + i\,d\theta$$

$$(A_g)_i = A_i + i\partial_i\theta$$ ← 電磁場 (Maxwell) の gauge 変換と同型

$$C = \frac{1}{2\pi i}\int d^2x \left[\langle \partial_x \psi | \partial_y \psi\rangle - \langle \partial_y \psi | \partial_x \psi \rangle \right]$$

$$\partial_x\big(\underbrace{\langle \psi | \partial_y \psi\rangle}_{A_y}\big) - \partial_y\big(\underbrace{\langle \psi | \partial_x \psi\rangle}_{A_x}\big)$$

$$= \langle \partial_x \psi | \partial_y \psi\rangle - \langle \partial_y \psi | \partial_x \psi\rangle + \cancel{\langle \psi | \partial_x \partial_y \psi\rangle} - \cancel{\langle \psi | \partial_y \partial_x \psi\rangle}$$

$$\partial_x \partial_y = +\partial_y \partial_x$$

$$= \frac{1}{2\pi i}\int d^2x \left(\partial_x A_y - \partial_y A_x\right)$$

$$= \frac{1}{2\pi i}\int d^2x \left(\mathrm{rot}\,\vec{A}\right)_z$$

$$= \frac{1}{2\pi i}\int d^2x \; B_z$$

$\vec{A} = \vec{A}_g + i\nabla\theta$ と変換して

$\mathrm{rot}\,A = \mathrm{rot}\,\vec{A}_g = \vec{B}$

$(\because \mathrm{rot}\,\nabla = 0)$

gauge 不変な表式 (A or $|\psi\rangle$ が必要) gauge固定が必要

gauge 不変な Chern数の表式 (Avron-Seiler-Simons '83 ?)

記法を整理

$$C = \frac{1}{2\pi i}\int dx\, dy \left[\langle \partial_x \psi | \partial_y \psi \rangle - \langle \partial_y \psi | \partial_x \psi \rangle \right]$$

$x = x_1,\quad y = x_2$

$dx_i \wedge dx_j = -dx_j \wedge dx_i$

$$= \frac{1}{2\pi i}\int dx_i\, dx_j \langle \partial_i \psi | \partial_j \psi \rangle$$

$dx_1 dx_2 = -dx_2 dx_1$

$dx_1 \wedge dx_2$

dx_2

dx_1

$dx_2 \wedge dx_1$

$\because\ dx_1 dx_2 \langle \partial_1 \psi | \partial_2 \psi \rangle + dx_2 dx_1 \langle \partial_2 \psi | \partial_1 \psi \rangle$, $\quad dx_2 dx_1 = -dx_1 dx_2$

$$= dx_1 dx_2 \left[\langle \partial_1 \psi | \partial_2 \psi \rangle - \langle \partial_2 \psi | \partial_1 \psi \rangle \right]$$

$$= \frac{1}{2\pi i}\int \langle \underbrace{dx_i\, \partial_i \psi}_{d\psi} | \underbrace{dx_j\, \partial_j \psi}_{d\psi} \rangle$$

$$= \frac{1}{2\pi i}\int \langle d\psi | d\psi \rangle$$

$$C = \frac{1}{2\pi i}\int \langle d\psi | d\psi \rangle$$

gauge 不変な表式 (Projection で書く)

$$C = \frac{1}{2\pi i}\int \mathrm{Tr}\, P\,dP\,dP = \frac{1}{2\pi i}\int \mathrm{Tr}\, P\,(dP)^2 \quad //$$

⇒ "Dirac monopole" 3D // $\boxed{2\times 2}$

$$C = \frac{1}{2\pi i}\int \langle d\psi | d\psi \rangle$$

: 独立変数がふえてない (不変)

$$|\psi(\theta_x, \theta_y)\rangle$$

$$H(\theta_x \theta_y)\,|\psi(\theta_x \theta_y)\rangle$$

$$H(\theta_x, \theta_y) = \tilde{H}(\vec{R})$$

$$\vec{R} = \begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix}.$$

具体的には $H = \vec{R}\cdot\vec{\sigma}$ 2×2の時 //

$$H(\theta_x,\theta_y) = \vec{R}(\theta_x,\theta_y)\cdot\vec{\sigma}$$

R_x, R_y, R_z : 実数

$$= \begin{pmatrix} R_z & R_x - iR_y \\ R_x + iR_y & -R_z \end{pmatrix}.$$

$$H(\theta_x,\theta_y) = \tilde{H}(\vec{R})$$

$H|\psi\rangle = |\psi\rangle E$

$\tilde{H}|\tilde{\psi}\rangle = |\tilde{\psi}\rangle E$

$$C = \frac{1}{2\pi i}\int \langle d\tilde{\psi}|d\tilde{\psi}\rangle$$

$|\psi(\theta_x\theta_y)\rangle = |\tilde{\psi}(\vec{R})\rangle$

$$= \frac{1}{2\pi i}\int dR_i\, dR_j \langle\partial_i\tilde{\psi}|\partial_j\tilde{\psi}\rangle$$

$\tilde{B}_z$ 〃 $(\mathrm{rot}\,\tilde{A})_z$ 〃

$$= \frac{1}{2\pi i}\int d\vec{S}\cdot\vec{\tilde{B}}$$

▨ 積分

$$dR_x dR_y\left(\langle\partial_x\tilde{\psi}|\partial_y\tilde{\psi}\rangle - \langle\partial_y\tilde{\psi}|\partial_x\tilde{\psi}\rangle\right)$$
$$+\, dR_y dR_z\,\tilde{B}_x$$
$$+\, dR_z dR_x\,\tilde{B}_y$$

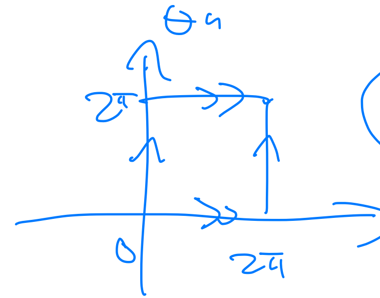
$$C = \frac{1}{2\pi i}\int_{T^2} d\theta_x d\theta_y \, \underbrace{\langle \nabla\psi | \times | \nabla\psi\rangle}_{\langle \partial_x\psi | \partial_y\psi\rangle - \langle \partial_y\psi | \partial_x\psi\rangle}$$

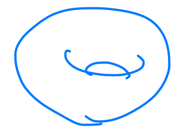
θ_x
θ_y

"外積的"に表した

$$= \frac{1}{2\pi i}\int_{\vec{R}(T^2)} d\vec{S}\cdot\tilde{\vec{B}}$$

// 面積分





境界の無い面

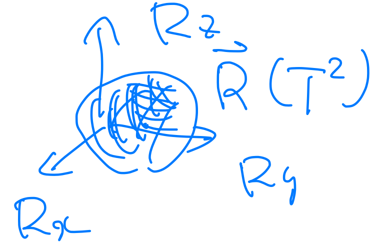
2×2 の時

$|\psi\rangle$ が必要

$$H = \vec{R}\cdot\vec{\sigma}$$

$$H(\vec{\theta}) = \tilde{H}(\vec{R})$$

$$\underbrace{\vec{R} = \vec{R}(\theta_x, \theta_y)}$$



$$\tilde{\vec{B}} = \langle \nabla\psi | \times | \nabla\psi\rangle$$

3成分

$$\tilde{\vec{B}} = \begin{pmatrix} \tilde{B}_x \\ \tilde{B}_y \\ \tilde{B}_z \end{pmatrix}$$

$P = |\psi\rangle\langle\psi|$ gauge 固定不用 (不要) $\quad P^2 = P$

$$\mathrm{Tr}\left(P\, \nabla P \times \nabla P\right)$$

ベクトル解析 (行列の)

$\langle\psi|\psi\rangle = 1$

$\Rightarrow \langle\nabla\psi|\psi\rangle = -\langle\psi|\nabla\psi\rangle$

$$= \mathrm{Tr}\, P^2 (\nabla P \times \nabla P)$$

$$= \mathrm{Tr}\, P (\nabla P) \times (\nabla P) P$$

$$= \mathrm{Tr}\, |\psi\rangle\langle\psi| \left(|\nabla\psi\rangle\langle\psi| + |\psi\rangle\langle\nabla\psi| \right) \times \left(|\nabla\psi\rangle\langle\psi| + |\psi\rangle\langle\nabla\psi| \right) |\psi\rangle\langle\psi|$$

$$= \mathrm{Tr}\left(\overset{①}{|\psi\rangle\langle\psi|\nabla\psi\rangle\langle\psi|} + \overset{②}{|\psi\rangle\langle\nabla\psi|} \right) \times \left(\overset{③}{|\nabla\psi\rangle\langle\psi|} + \overset{④}{|\psi\rangle\langle\nabla\psi|\psi\rangle\langle\psi|} \right)$$

$$= \mathrm{Tr}\Big[\overset{①\times③}{|\psi\rangle\langle\psi|\nabla\psi\rangle \times \langle\psi|\nabla\psi\rangle\langle\psi|} + \overset{①\times4}{|\psi\rangle\langle\psi|\nabla\psi\rangle \times \underbrace{\langle\nabla\psi|\psi\rangle}_{-\langle\psi|\nabla\psi\rangle}\langle\psi|} = 0$$

$$+ \overset{②\times③}{|\psi\rangle\langle\nabla\psi| \times |\nabla\psi\rangle\langle\psi|} + \overset{②\times④}{|\psi\rangle\langle\nabla\psi|\psi\rangle \times \langle\nabla\psi|\psi\rangle\langle\psi|} = \mathrm{Tr}\, |\psi\rangle\langle\nabla\psi| \times |\nabla\psi\rangle\langle\psi|$$

$$\mathrm{Tr}\, P\, \nabla P \times \nabla P = \mathrm{Tr}_{2\times 2}\, |\psi\rangle \left(\langle \nabla\psi| \times |\nabla\psi\rangle \right) \langle\psi|$$

$$= \mathrm{Tr}_{1\times 1} \langle \nabla\psi| \times |\nabla\psi\rangle = \langle \nabla\psi| \times |\nabla\psi\rangle = \vec{B}$$

$$C = \frac{1}{2\pi i} \int d\vec{S} \cdot \underbrace{\mathrm{Tr}\, P(\nabla P \times \nabla P)}_{=\vec{B}}$$ gauge固定なしで計算可能

2×2で $\vec{B} = \mathrm{Tr}\, P\, (\nabla P \times \nabla P)$?

$\rightarrow \nabla P \rightarrow dP$ 微分形式でも可能(成分)

一般にはもう少し変更が必要

$$\boxed{F = dA + A^2}$$ ← 非可換ベリー接続

多体問題で必要

Dirac Monopole

$$H = \vec{R}\cdot\vec{\sigma}$$

$$\mathrm{Tr}\,H = 0 \quad \because \mathrm{Tr}\,\vec{\sigma} = 0$$

エネルギーは $\pm E$

$$H^2 = E^2\cdot\sigma_0$$

$$H^2 = (\vec{R}\cdot\vec{\sigma})(\vec{R}\cdot\vec{\sigma})$$

$$= |R|^2\sigma_0 + i\vec{\sigma}\cdot\underbrace{(\vec{R}\times\vec{R})}_{0}$$

$$= |R|^2 = E^2$$

公式 $(\vec{A}\cdot\vec{\sigma})(\vec{B}\cdot\vec{\sigma})$

$$= \begin{pmatrix} A_z & A_x - iA_y \\ A_x + iA_y & -A_z \end{pmatrix}\begin{pmatrix} B_z & B_x - iB_y \\ B_x + iB_y & -B_z \end{pmatrix}$$

check

$$= \begin{pmatrix} * & * \\ * & * \end{pmatrix} = (\vec{A}\cdot\vec{B})\underbrace{\sigma_0}_{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} + i\vec{\sigma}\cdot(\vec{A}\times\vec{B})$$

エネルギー $\pm R$, $R = \sqrt{\vec{R}^2} = \sqrt{R_x^2 + R_y^2 + R_z^2}$

$\pm E$ の固有状態

$$H|\psi_\pm\rangle = |\psi_\pm\rangle R(\pm),$$

$$P_\pm = |\psi_\pm\rangle\langle\psi_\pm| \leftarrow ?$$

状態を求めずとも

$$\left\{\begin{aligned} P_+ &= \frac{1}{2}\left(1_2 + \frac{H}{R}\right) \\ P_- &= \frac{1}{2}\left(1_2 - \frac{H}{R}\right) \end{aligned}\right.$$

projection

$$P_+ + P_- = 1_2, \quad P_+^2 = P_+$$

$$P_+|\psi_+\rangle = \frac{1}{2}\left(1 + \frac{R}{R}\right)|\psi_+\rangle = |\psi_+\rangle$$

$$P_+|\psi_-\rangle = \frac{1}{2}\left(1 - \frac{R}{R}\right)|\psi_-\rangle = 0$$

$$P_-|\psi_+\rangle = 0$$

$$P_-|\psi_-\rangle = |\psi_-\rangle$$

ϕ $\oplus$ 固有值 $\oplus$

$$P = \frac{1}{2}\left(1 + \hat{R}\cdot\vec{\sigma}\right) \qquad \hat{R} = \frac{\vec{R}}{R}$$

$$\varepsilon_{abc} = \begin{cases} 1 & (123)(2.31)(312) \\ -1 & (132)(321)(213) \\ 0 & \text{それ以外} \end{cases}$$

$$\vec{B} = \mathrm{Tr}\, P\, \nabla P \times \nabla P$$

$$B_a = \varepsilon_{abc}\,\mathrm{Tr}\, P\, \partial_b P\, \partial_c P$$

$$= \varepsilon_{abc}\,\mathrm{Tr}\, \frac{1}{2^3}\left(1 + \hat{R}\cdot\vec{\sigma}\right)\left[\left(\partial_b \hat{R}\right)\cdot\vec{\sigma}\right]\left[\partial_c \hat{R}\cdot\vec{\sigma}\right]$$

$$= \frac{1}{8}\underset{\leftrightarrow a}{\varepsilon_{abc}}\,\mathrm{Tr}\left(1 + \hat{R}\cdot\vec{\sigma}\right)\left\{\left(\partial_b \hat{R}\cdot\partial_c \hat{R}\right)\sigma_0 + i\,\vec{\sigma}\cdot\left(\partial_b \hat{R}\times\partial_c \hat{R}\right)\right\}$$

0   $b \leftrightarrow c \rightarrow 0$ 0

$$B_a = \frac{1}{8}\varepsilon_{abc}\,\mathrm{Tr}\,(1+\hat{R}\cdot\vec{\sigma})\left\{(\partial_b\hat{R}\cdot\partial_c\hat{R})\,\sigma_0 + i\,\vec{\sigma}\cdot(\partial_b\hat{R}\times\partial_c\hat{R})\right\}$$

$$= \frac{i}{8}\varepsilon_{abc}\,\mathrm{Tr}\left[\hat{R}\cdot(\partial_b\hat{R}\times\partial_c\hat{R})\,\sigma_0 + i\,\vec{\sigma}\cdot\left[\hat{R}\times(\partial_b\hat{R}\times\partial_c\hat{R})\right]\right]$$

$$= \frac{i}{4}\varepsilon_{abc}\,\hat{R}\cdot(\partial_b\hat{R}\times\partial_c\hat{R})$$

$$= \frac{i}{4}\varepsilon_{abc}\,\hat{R}_i\,\varepsilon_{ijk}\,\partial_b\hat{R}_j\,\partial_c\hat{R}_k$$

$$= \frac{i}{4}\varepsilon_{abc}\,\varepsilon_{ijk}\,\hat{R}_i\,\partial_b\hat{R}_j\,\partial_c\hat{R}_k$$

$$B_a = \frac{i}{4}\,\varepsilon_{abc}\,\varepsilon_{ijk}\,\hat{R}_i\,\partial_b \hat{R}_j\,\partial_c \hat{R}_k$$

$$\partial_a \hat{R}_i = \partial_a \frac{R_i}{R} = \frac{\delta_{ai}}{R} + \frac{-1}{R^2}\,\partial_a R\; R_i \qquad R = \sqrt{\cdots}$$

$$= \delta_{ai}\frac{1}{R} - \frac{1}{R^2}\,\frac{2R_a}{2\sqrt{\ }}\,R_i$$

$$= \frac{\delta_{ai}}{R} - \frac{R_a R_i}{R^3}$$

$$= R^{-1}\left(\delta_{ai} - \hat{R}_a \hat{R}_i\right)$$

$$= \frac{1}{R^2}\frac{i}{4}\,\varepsilon_{abc}\,\varepsilon_{ijk}\,\hat{R}_i\left(\delta_{bj} - \hat{R}_b \hat{R}_j\right)\left(\delta_{ck} - \hat{R}_c \hat{R}_k\right)$$

対称 → 0

$$= \frac{i}{4R^2}\,\varepsilon_{abc}\,\varepsilon_{ijk}\,\hat{R}_i\,\delta_{bj}\,\delta_{ck}$$

$$= \frac{i}{4R^2}\,\underbrace{\varepsilon_{abc}\,\varepsilon_{ibc}}_{=2\delta_{ai}}\,\hat{R}_i = \frac{i}{2R^2}\hat{R}_a$$

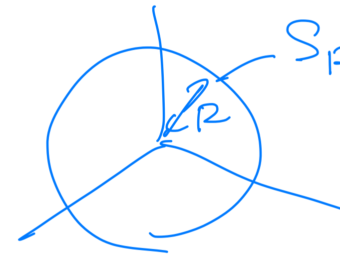
$$= i\,\frac{\hat{R}_a}{4R^2}$$

$$\vec{B} = i\,\frac{\hat{R}}{2R^2} = i\,\frac{1}{2}\,\frac{\vec{R}}{R^3}$$: 点電荷の時の電場

$$\phi = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \to \vec{E} = \frac{Q}{4\pi\varepsilon_0}\frac{\tilde{R}}{R^3}$$

磁場が点電荷の時の電場

$$\vec{B} = i\,\frac{1}{2}\,\frac{\vec{R}}{R^3} \qquad \vec{\nabla}\cdot\vec{B} = 0 \quad (R \neq 0) \quad \text{Maxwell}$$



$$\int_{\partial V_R = S_R} d\vec{S}\cdot\vec{B} = 4\pi R^2\, i\,\frac{1}{2}\,\frac{1}{R^2}$$

$$\| \qquad = i\pi 2$$

$$\int_{V_R} d^3R\ \nabla\cdot\vec{B} = i\pi 2$$

$$\nabla\cdot B = i\pi\,\delta^{(3)}(R)$$

磁場が単磁極

Dirac Monopole

$$C = \frac{1}{2\pi i}\cdot 2\pi i = 1$$

charge (磁荷/e) $= +1$

$2\times 2 \Rightarrow$ Berry $\Leftrightarrow$ Mono pole

☆ 非可換 Berry 接続

$$H(x)|\psi(x)\rangle = |\psi(x)\rangle E \qquad A = \langle\psi|d\psi\rangle \;:\; 1\times 1$$

$|\psi\rangle$: 縮退なし $\Rightarrow$ M 重に縮退

Wilczek-Zee '84

拡張

Hatsugai

$$\Psi = (|\psi_1\rangle, |\psi_2\rangle, \cdots, |\psi_M\rangle)$$

$A = \Psi^\dagger d\Psi$ 行列 $\quad M\times M$

$$= \begin{pmatrix} \langle\psi_1| \\ \vdots \\ \langle\psi_M| \end{pmatrix} (|d\psi_1\rangle, \cdots, |d\psi_M\rangle)$$

$$= \begin{pmatrix} \langle\psi_1|d\psi_1\rangle & \langle\psi_1|d\psi_2\rangle & \cdots & \langle\psi_1|d\psi_M\rangle \\ \vdots & & \ddots & \\ \langle\psi_M|d\psi_1\rangle & & \cdots & \langle\psi_M|d\psi_M\rangle \end{pmatrix}$$

gauge 対称 (位相の自由度)

$$H|\psi\rangle = |\psi\rangle E \;\Rightarrow\; H|\psi_g\rangle = |\psi_g\rangle E \;,\; |\psi_g\rangle = |\psi\rangle g \;//$$

$$\langle\psi|\psi\rangle = \langle\psi_g|\psi_g\rangle = 1 \qquad g = e^{i\theta} \in U(1)$$

$$H|\psi_1\rangle = |\psi_1\rangle E$$

$$\vdots$$

$$H|\psi_M\rangle = |\psi_M\rangle E$$

M 重縮退　線型結合もO.K.

$$\Psi_g = (|\psi_1'\rangle, \cdots |\psi_M'\rangle)$$

$$= \Psi g$$

$$\Psi_g^\dagger \Psi_g = E_M = \underbrace{\begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}}_{M} \Big\} M$$

$$\Psi^\dagger \Psi = E_M$$

$$\Psi_g^\dagger \Psi_g = E_M = g^\dagger \underbrace{\Psi^\dagger \Psi}\, g = g^\dagger g \;,\; g^\dagger = g^{-1}$$

unitary

$U(M)$

$$A_g = \Psi_g^\dagger d\Psi_g = g^{-1}\Psi^\dagger d(\Psi g) = g^{-1}\Psi^\dagger d\Psi + g^{-1}\underbrace{\Psi^\dagger \Psi}_{= E_M} dg$$

$$= g^{-1} A g + g^{-1} dg$$

gauge 变换 $\quad x = (x_1 \cdots\ )$

$$A = \underbrace{A_i dx_i}$$

$$A_i = \begin{pmatrix} \langle \psi_1 | \partial_i \psi_1 \rangle & & \langle \psi_1 | \partial_i \psi_M \rangle \\ \vdots & \ddots & \\ \langle \psi_M | \partial_i \psi_1 \rangle & & \langle \psi_M | \partial_i \psi_M \rangle \end{pmatrix}$$

$$A_g = g^{-1} A g + g^{-1} dg$$

多電子系の状態 (free fermion) 相互作用なし (格子上)

$x=(\phi_x,\phi_y)$ N粒子の電子系 free

$$H=\sum_{ij} t_{ij}\, c_i^\dagger c_j = \mathbf{c}^\dagger h\, \mathbf{c}$$

$$= (c_1^\dagger \cdots c_N^\dagger)\overbrace{\begin{pmatrix} \\ h \\ \\ \end{pmatrix}}^{N}\begin{pmatrix} c_1 \\ \vdots \\ c_N \end{pmatrix}$$

$h(\phi_x,\phi_y)$

$$H|G\rangle = |G\rangle E$$

$|G\rangle = ?$ フェルミ面までつめる.

一体問題 $h\phi_\ell = \varepsilon_\ell \phi_\ell$ $\quad \begin{bmatrix} & \\ & \end{bmatrix}\begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}\varepsilon_\ell$

ϕ_ℓ : N次元ベクトル

$$\phi_\ell^\dagger \phi_{\ell'} = \delta_{\ell\ell'} \qquad \varepsilon_1 \le \varepsilon_2 \le \cdots \le \varepsilon_M < \mu \le \varepsilon_{M+1} < \cdots$$

フェルミ面まで Mコつめる.

h 対角化

$$h\underbrace{(\phi_1 \cdots \phi_N)}_{\Phi} = (\phi_1 \cdots \phi_N)\underbrace{\begin{pmatrix} \varepsilon_1 & & \\ & \ddots & \\ & & \varepsilon_N \end{pmatrix}}_{\varepsilon}$$

$h\Phi = \Phi\varepsilon$, $h = \Phi\varepsilon\Phi^\dagger$ $\Phi^\dagger = \Phi^{-1}$

$$H = \underbrace{c^\dagger \Phi}_{d^\dagger \checkmark} \varepsilon \underbrace{\Phi^\dagger c}_{d} = d^\dagger \varepsilon d = \sum_l \varepsilon_l d_l^\dagger d_l$$

$$d^\dagger = c^\dagger \Phi$$
$$(d_1^\dagger \cdots d_N^\dagger) = c^\dagger (\phi_1, \phi_2, \cdots \phi_N)$$
$$d_l^\dagger = c^\dagger \phi_l$$

$$|G\rangle = \prod_{l=1}^{M} d_l^\dagger |0\rangle$$

$$= \prod_{l=1}^{M} (c^\dagger \phi_l)|0\rangle \Leftarrow |\Phi\rangle_M$$

$$\Phi = \underbrace{(\phi_1 \cdots \phi_M)}^{M}$$

$$= d_1^\dagger \cdots d_M^\dagger |0\rangle$$

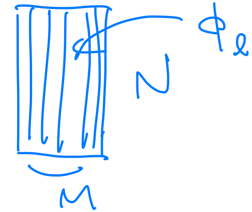
[illegible] $a_l = \begin{pmatrix} * \\ \vdots \\ * \end{pmatrix}$

一般化 $\underline{\underline{|A\rangle}} = \prod_{l=1}^{M} c^\dagger a_l |0\rangle$, $A = \overbrace{(a_1, \cdots, a_M)}^{M} \Big\} N = \underbrace{\boxed{A}}_{M \times N}$

$$\underbrace{\langle A|B\rangle}_{\text{公式}} = \det_M A^\dagger B = \det_M \boxed{}_{M \times N}\ \boxed{}_{N \times M} = \det \boxed{}_{M \times M}$$

公式 //

$$|G\rangle = |\phi\rangle \qquad \phi = (\phi_1 \cdots \phi_M) =$$



$$A = \langle G | dG\rangle$$ ⌋ 多電子系のベリー接続を求める

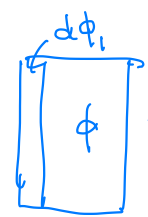
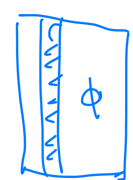
$$C = \frac{1}{2\pi i}\int d^2\theta\, dA \quad ,\ \text{など}$$

$$|dG\rangle = d\left(\prod_{\ell}^{M} c^\dagger \phi_\ell |0\rangle\right)$$

$$= (c^\dagger d\phi_1)(c^\dagger \phi_2)(c^\dagger \phi_3)\cdots(c^\dagger \phi_M)|0\rangle$$

$$+ (c^\dagger \phi_1)(c^\dagger d\phi_2)(c^\dagger \phi_3)\cdots(c^\dagger \phi)|0\rangle$$

$$+ \cdots$$

$$A = \langle G | dG\rangle = \det_M \phi^\dagger$$  $$+ \det \phi^\dagger$$  $$+ \cdots$$

$$
= \det\left(\underbrace{\begin{bmatrix} \phi_1^\dagger \\ \phi_2^\dagger \\ \vdots \\ \phi_M^\dagger \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} d\phi_1 & \phi \end{bmatrix}}_{M} \right) + \cdots\cdots + = \sum_{l=1}^{M} \phi_l^\dagger \, d\phi_l
$$

$$
= \mathrm{Tr}\, A
$$

$$
\det \begin{pmatrix} \phi_1^\dagger d\phi_1 & 0 & 0 & \cdots \\ \phi_2^\dagger d\phi_1 & 1 & 0 & \\ \vdots & 0 & 1 & \\ \phi_M^\dagger d\phi_1 & 0 & & \ddots \; 1 \end{pmatrix} + \phi_2^\dagger d\phi_2 + \cdots + \phi_M^\dagger d\phi_M
$$

(M-1)次元の単位行列

$$
\phi_1^\dagger d\phi_1
$$

$$
A = \phi^\dagger d\phi = \begin{bmatrix} \phi^\dagger \end{bmatrix} \underbrace{\begin{bmatrix} d\phi \end{bmatrix}}_{M} \Big\} N =
$$

$$
(A)_{ij} = \phi_i^\dagger \, d\phi_j
$$

$$|G\rangle = \prod_{l=1}^{M} c^\dagger \phi_l |0\rangle$$

$$\phi = (\phi_1 \cdots \phi_M)$$

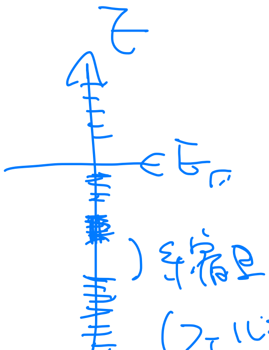
$N\times M$行列

$$A = \langle G | dG \rangle = \mathrm{Tr}\, A$$

$$A = \phi^\dagger d\phi$$

ϕのgauge変換 $\phi_g = \phi g$

$$= (\phi_1 \cdots \phi_M)\, g$$



(フェルミ面以下) $\longleftrightarrow$ フェルミ面以下の状態をまぜる

g不変なもの $\to$ フェルミ面以下での縮退に依存しない物を考える

Chern数はg不変

$$\boxed{B = \mathrm{rot}\, A}$$

$$C = \frac{1}{2\pi i}\int \mathcal{F}$$

$$\mathcal{F} = \langle dG | dG \rangle$$

$$= \underline{\underline{\mathrm{Tr}\, dA}}$$

$$= \mathrm{Tr}\, F$$

$$\mathrm{Tr}\, A^2 = 0$$

$$A = \phi^\dagger d\phi = \begin{array}{cc} M & \\ \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} & M \end{array}$$

$$F = dA + A^2$$

$$\left[F_g = dA_g + A_g^2 = g^{-1} F g \right]$$

↑ あしたやります.