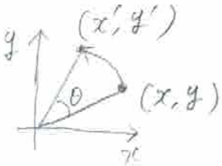
量子力学 II　第3回　[redacted]

回転操作　$U=R$



$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$R: \vec{r} \mapsto \vec{r}\,'$

$|\vec{r}| = |\vec{r}\,'|$　：長さが不変な連続変換

$\vec{r} \mapsto \vec{r}\,' = R\vec{r}$　　R：3×3の実行列

$|\vec{r}|^2 = (\vec{r}, \vec{r})$　内積　　　$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$= (x, y, z)\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

（$\tilde{\vec{r}} = {}^t\vec{r} = (x, y, z)$　と書くと，）

$= \tilde{\vec{r}}\,\vec{r}$

$|\vec{r}\,'|^2 = \tilde{\vec{r}}\,'\,\vec{r}\,'$

$= \widetilde{(R\vec{r})}\,R\vec{r}$

（一般の行列 A，B について　$\widetilde{AB} = \tilde{B}\tilde{A}$　…☆）

$= \tilde{\vec{r}}\,\tilde{R}R\vec{r}$

$= \tilde{\vec{r}}\,\vec{r}$

$\Rightarrow \tilde{R}R = E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$　である。

$\Rightarrow \tilde{R} = R^{-1}$　：直交行列

☆　$(\widetilde{AB})_{ij} = (AB)_{ji} = A_{jk}B_{ki} = (\tilde{B})_{ik}(\tilde{A})_{kj} = (\tilde{B}\tilde{A})_{ij}$

$\therefore\ \widetilde{AB} = \tilde{B}\tilde{A}$

無限小変換 δR について考える。

$R = 1 + \delta R = E_3 + \delta R$

$\tilde{R}R = (\widetilde{E_3 + \delta R})(E_3 + \delta R)$

$= E_3 + \widetilde{\delta R} + \delta R + \cancel{\delta R^2} = E_3$

$\Rightarrow \widetilde{\delta R} + \delta R = 0$　　$\Rightarrow \widetilde{\delta R} = -\delta R$　：反対称行列

$(\delta R)_{ij} = -(\delta R)_{ji}$　なので，$(\delta R)_{ij} = -\varepsilon_{ijk}\,\delta\omega_k$　と書く。

$R: \psi \mapsto \psi' = U\psi$

$\psi(\vec{r}) = \psi'(\vec{r}') = \psi'(R\vec{r})$ より.

$R = 1 + \delta R$
$R^{-1} = 1 - \delta R$

$$\psi'(\vec{r}) = \psi(R^{-1}\vec{r})$$
$$= \psi((1-\delta R)\vec{r})$$
$$= \psi(\vec{r}) \underbrace{- (\delta R\vec{r})\cdot\vec{\nabla}}_{*}\psi$$

$$* \quad -\delta R\vec{r}\cdot\vec{\nabla} = -(\delta R r)_i \partial_i$$
$$= -(\delta R)_{ia} r_a \partial_i$$
$$= \varepsilon_{iak}\delta\omega_k r_a \partial_i$$
$$= -\varepsilon_{kai} r_a \partial_i \delta\omega_k$$
$$= -(\vec{r}\times\vec{\nabla})_k \delta\omega_k$$
$$= -\delta\vec{\omega}\cdot(\vec{r}\times\vec{\nabla})$$

$$\therefore \psi'(\vec{r}) = \psi(\vec{r}) - \delta\vec{\omega}\cdot(\vec{r}\times\vec{\nabla})\psi$$
$$= \psi(\vec{r}) - \frac{i\delta\vec{\omega}(\vec{r}\times\vec{p})}{\hbar}\psi$$
$$= e^{-i\delta\vec{\omega}\cdot\vec{L}/\hbar}\psi$$

$\vec{r}\times\vec{p} = \vec{L}$
角運動量

$\psi \mapsto \psi' = U\psi$ より、 $U = e^{-i\vec{\omega}\cdot\vec{L}/\hbar}$

$|\psi\rangle \to |\psi'\rangle = e^{-i\vec{\omega}\cdot\vec{L}/\hbar}|\psi\rangle = U|\psi\rangle$

$U = e^{-i\vec{\omega}\cdot\vec{L}/\hbar}$

回転の母関数は角運動量

Hが回転操作で不変 → 角運動量が保存

$$[r_i, L_j] = [r_i, \varepsilon_{jkl} r_k p_l] = \varepsilon_{jkl}[r_i, r_k p_l] = \varepsilon_{jkl}([r_i, r_k]p_k + r_k[r_i, p_l])$$
$$= \varepsilon_{jkl}(0 + r_k \cdot i\hbar r_k \delta_{il}) = i\hbar\varepsilon_{jil} r_l$$
$$= i\hbar\varepsilon_{ijk} r_k$$

$L_j = \varepsilon_{jkl} r_k p_l$

$$[p_i, L_j] = [p_i, \varepsilon_{jkl} r_k p_l] = \varepsilon_{jkl}[p_i, r_k p_l] = \varepsilon_{jkl}([p_i, r_k]p_l + r_k[p_i, p_l])$$
$$= \varepsilon_{jkl}(-i\hbar\delta_{ik}p_l + 0) = -i\hbar\varepsilon_{jil}p_l$$
$$= -i\hbar\varepsilon_{ijk}p_k$$

変換則で物理量を分類
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