Adiabatic heuristic principle on a torus and anyon pumping

Univ. of Tsukuba
Koji Kudo

Today's talk

-Introduction

- Adiabatic deformation

- Adiabatic deformation of QH states

Kudo-Hatsugai, PRB **102** 125108 (2020)

- Model
- Results
  - Adiabatic heuristic principle on a torus
  - Generalized Streda formula

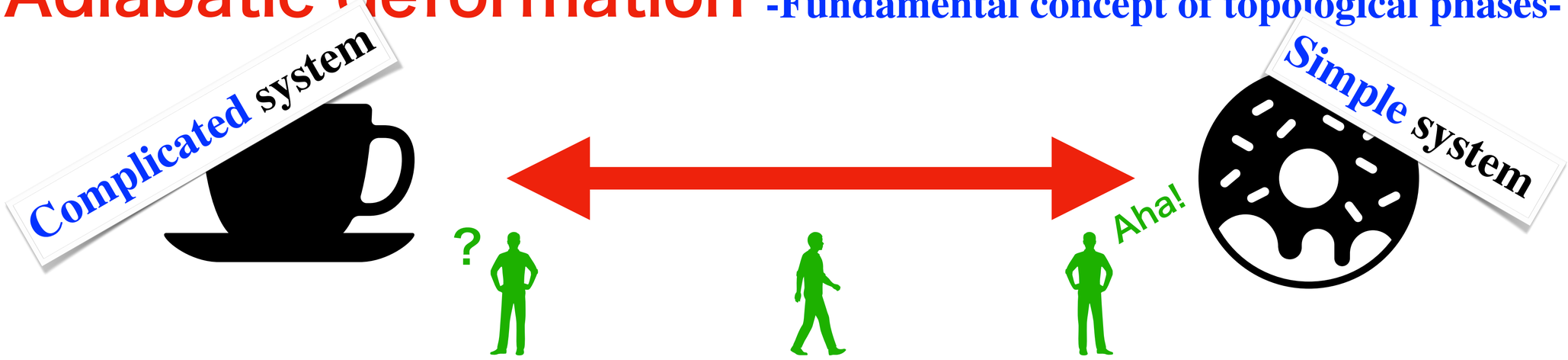
- Adiabatic deformation of charge pump

Kudo-Kuno-Hatsugai, in preparation

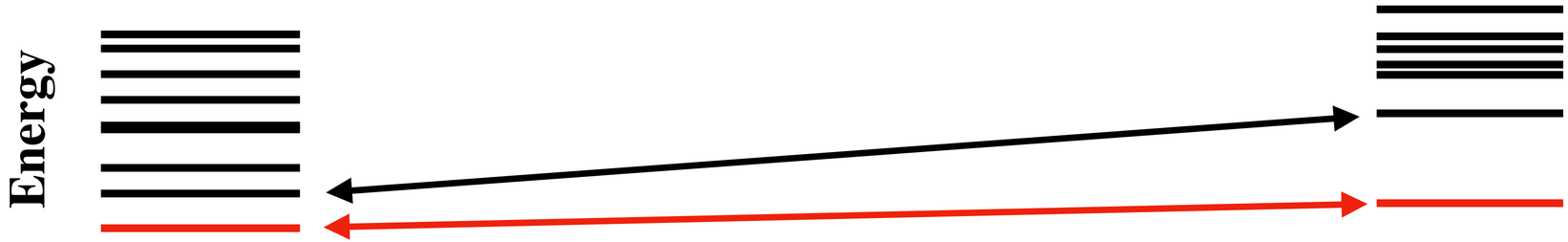
- Model
- Results
  - Noninteracting fermion pumping (warm-up)
  - Fractional anyon pumping

Topological phases

★ Adiabatic deformation -Fundamental concept of topological phases-



- **Gap remains open**



- **Topological invariants**

  e. g.) Chern number, Quantized Berry phase, …

Today's talk,

- QH states
- Charge pump

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✓- Adiabatic deformation of QH states

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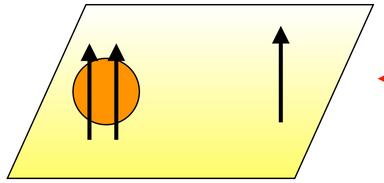
Flux-attachment

IQH states



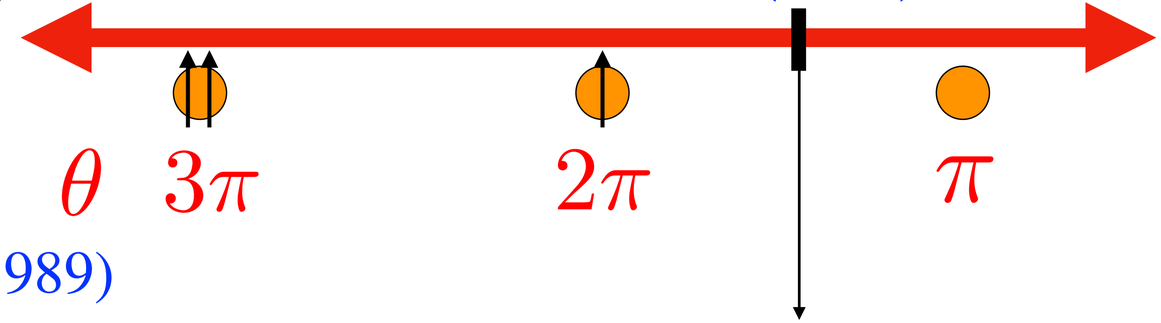
ν of  = 1

Jain (1989)

 is a fermion.

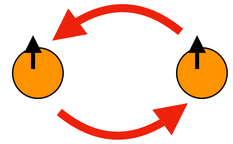
Adiabatic heuristic principle

Greiter-Wilczek (1990)



θ 3π 2π π

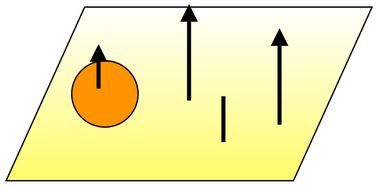
System of anyons



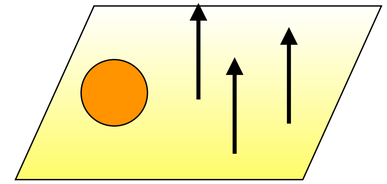
$\Psi \rightarrow e^{i\theta}\Psi$

Wilczek (1982)

Wu(1984)



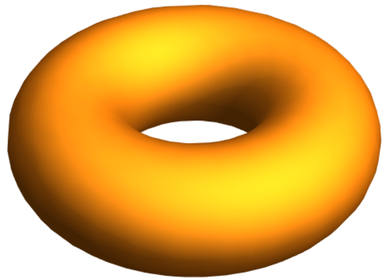
FQH states



$$\nu = \frac{\#\,\bullet}{\#\,\uparrow} = 1/3$$

● : electron

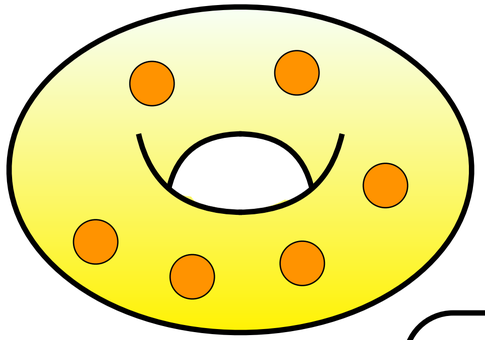
↑ : flux quantum



Systems on a torus

Anyons on a torus

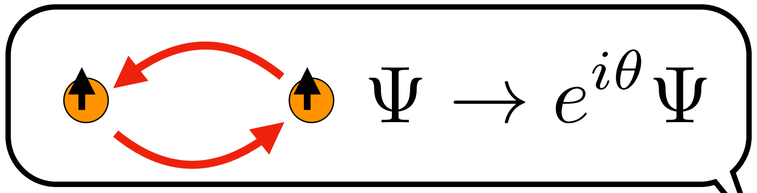
☆ Constraints due to the braid group

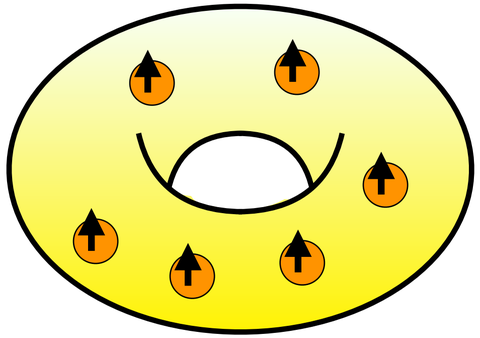


Fermions

Einarsson (1990)
Wen(1990)
Greiter-Wilczek (1992)

Basis: $|\boldsymbol{r}_1, \cdots, \boldsymbol{r}_N\rangle$





Anyon with $\theta = \dfrac{n}{m}\pi$

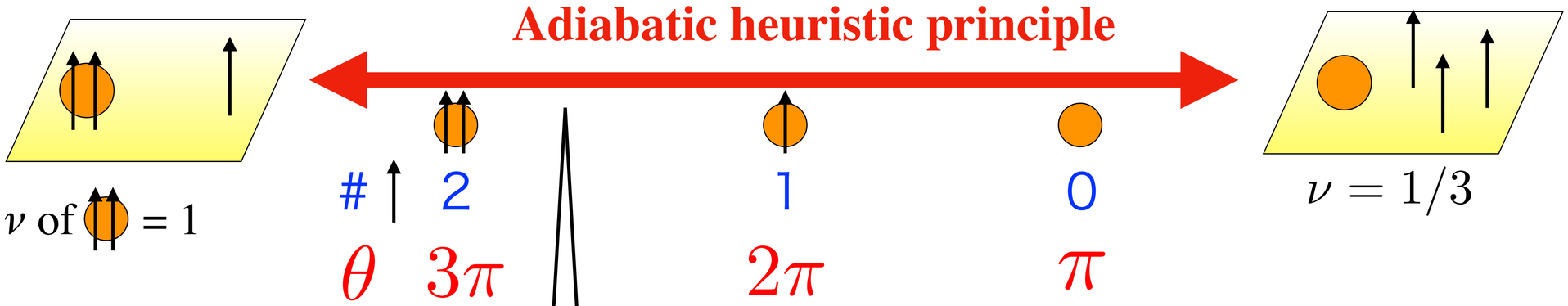
Basis: $|\boldsymbol{r}_1, \cdots, \boldsymbol{r}_N; w\rangle\,, \quad w = 1, \cdots, m$

$$\dim H_{\theta=(n/m)\pi} = m \times \text{const.}$$

Flux-attachment

★ IQH states

★ FQH states



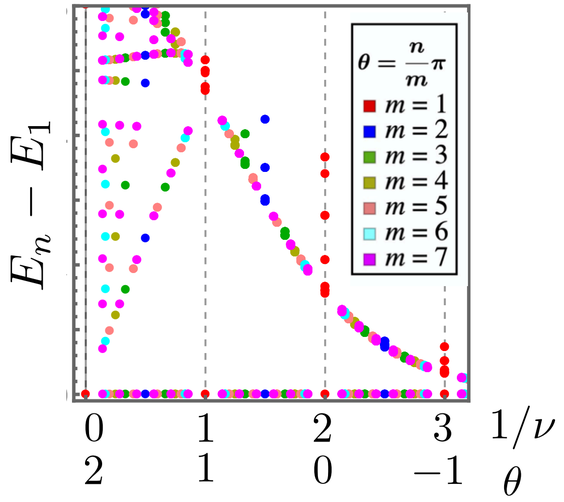
On a torus, $\dim H_{\theta=(n/m)\pi} = m \times \text{const.}$

Continuous change of the Hamiltonian H_θ

Adiabatic deformation ??

Overview of our results

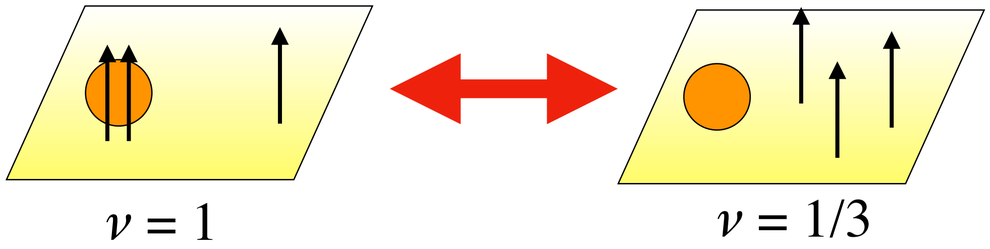
- **Numerical demonstration on a torus**



- **Discovery of the generalized Streda formula**

Topological degeneracy?

Unique ⟷ 3-fold



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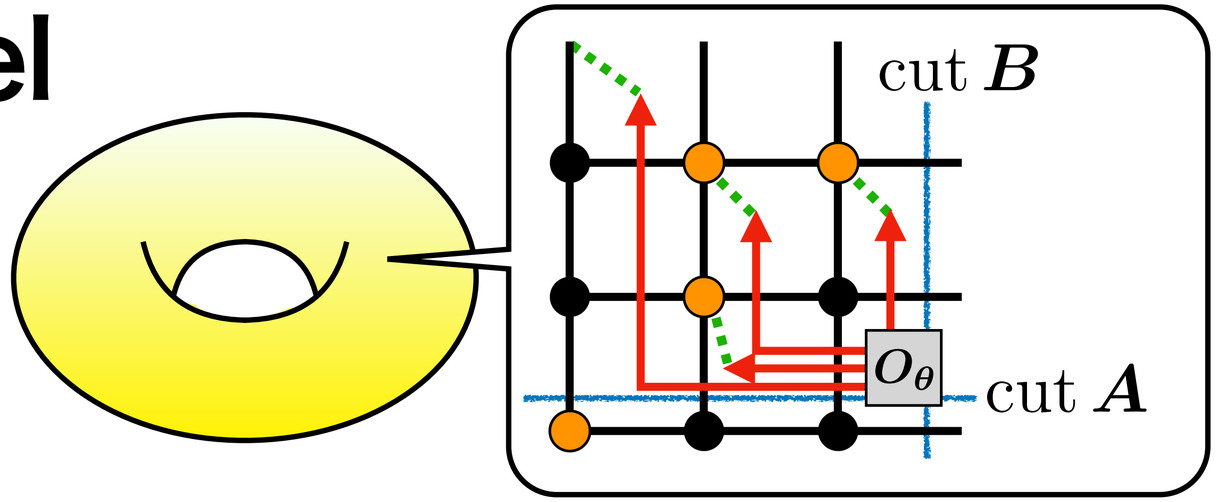
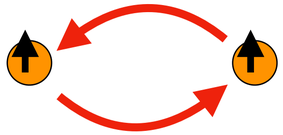
Model

Model



Anyons



$\Psi \to e^{i\theta}\Psi$

☆ Hamiltonian:

External fluxes

Statistical fluxes

$$H = -t\sum_{\langle ij\rangle} e^{i\phi_{ij}}\, e^{i\theta_{ij}}\, c_i^\dagger c_j + V\sum_{\langle ij\rangle} n_i n_j$$

Wen-Dagotto-Fradkin (1990)
Hatsugai-Kohmoto-Wu (1991)

☆ Dimension: $N_D \equiv \dim H_{\theta=(n/m)\pi} = m \times \text{cons}$

e.g.) (Fermion) ●×4 on 10×10 sites: $N_D = \binom{100}{4} = 3921225$

(Anyon) ⬆×4 on 10×10 sites: $N_D = 7\binom{100}{4} = 27448575$

with $\theta = (1/7)\pi$

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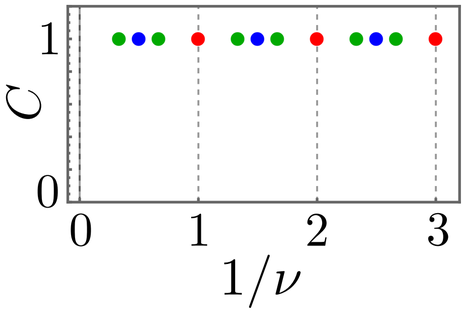
Adiabatic principle

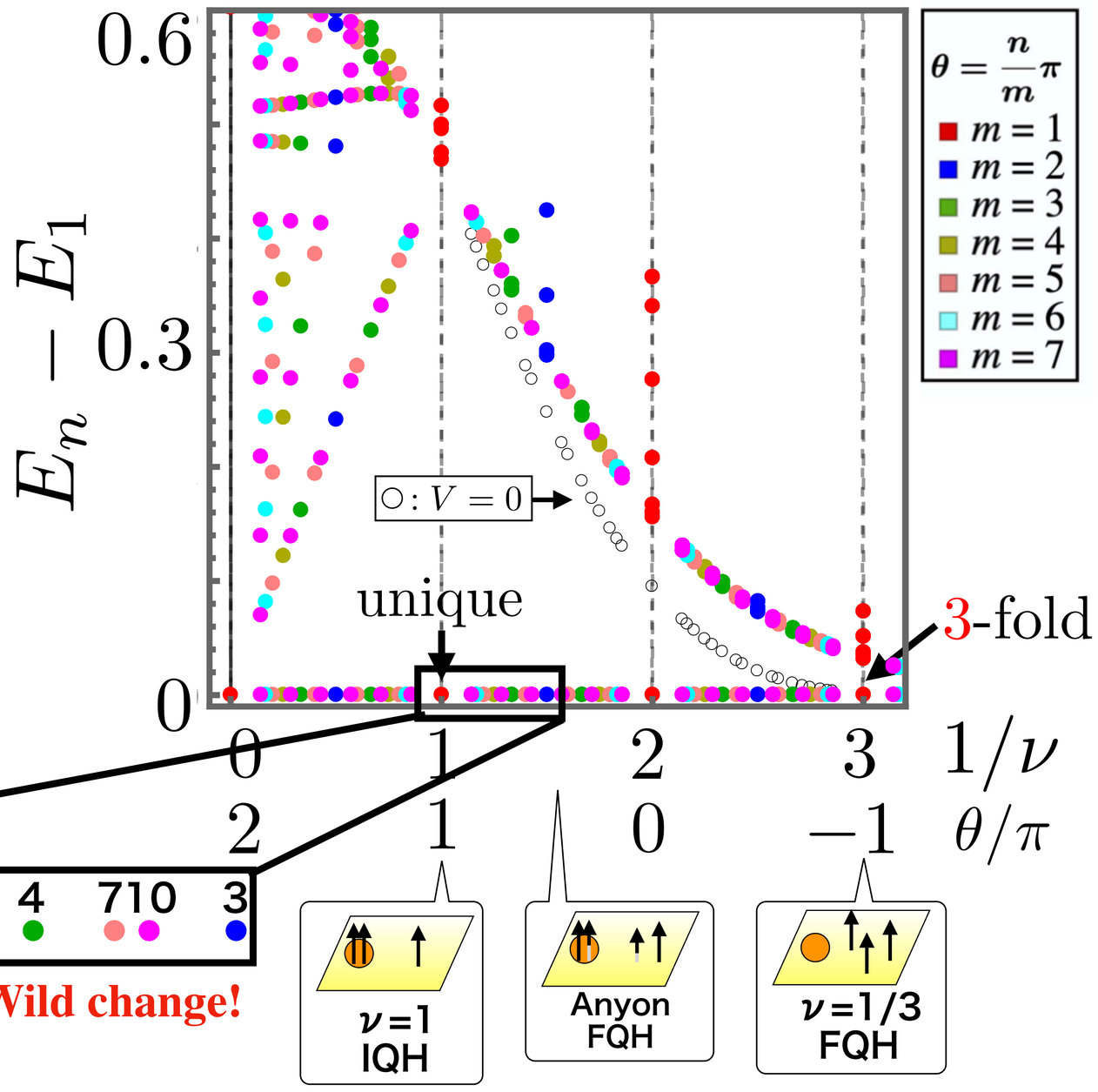
- Energy gap
  - $\dim H_{\theta=(n/m)\pi} = m \times \text{cons}$
  - **Gap remains open**
  - **Wild change of the degeneracies**
  - **Adiabatic invariant of the gap**

Many-body Chern number



Niu-Thouless-Wu (1985)

Results



Adiabatic principle

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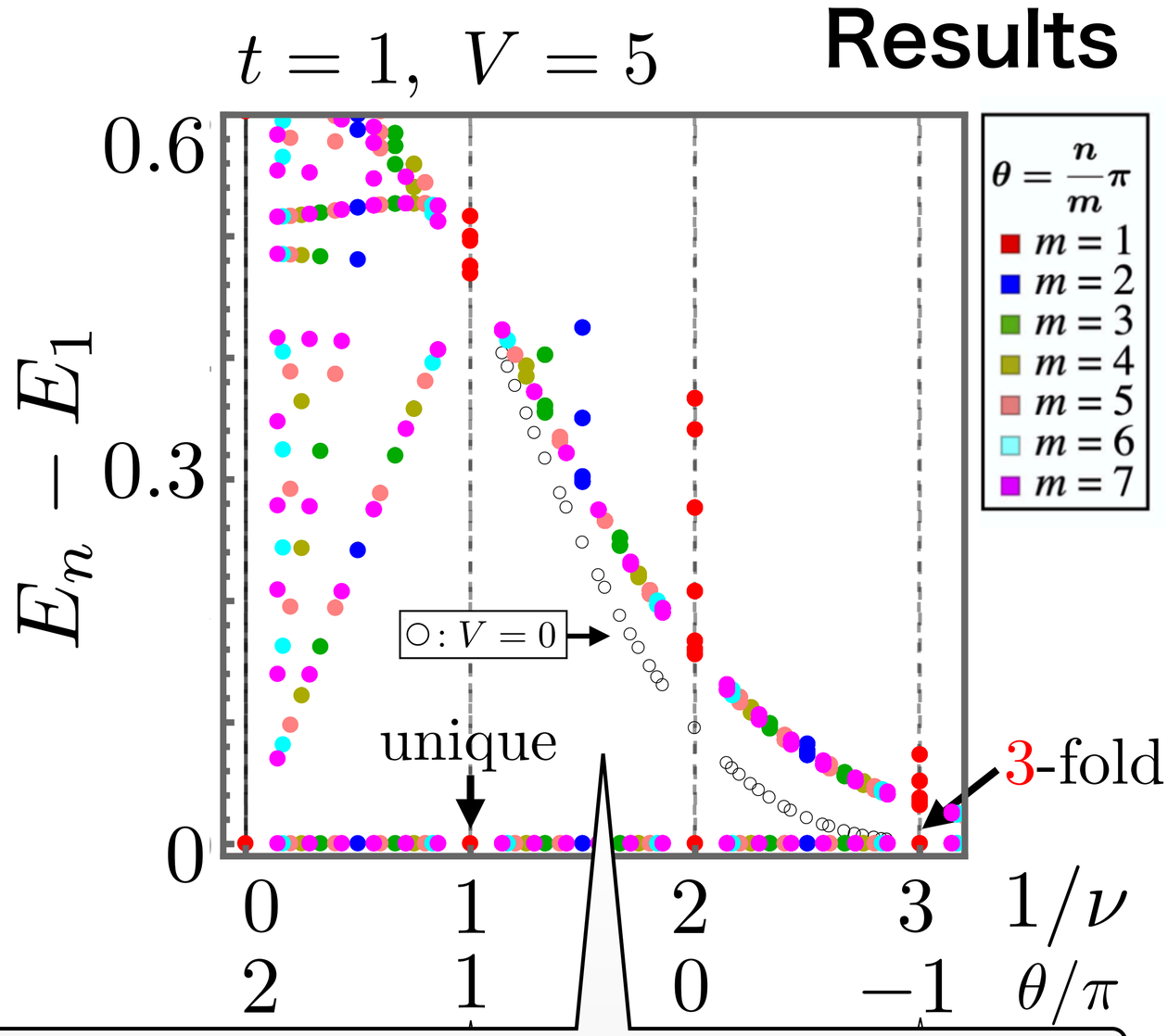

Adiabatic & Topological invariance

Continuity of the state $|\blacksquare\rangle \longleftrightarrow |\bullet\rangle$

Continuity of the gap

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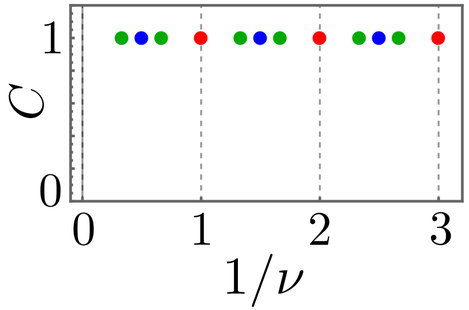
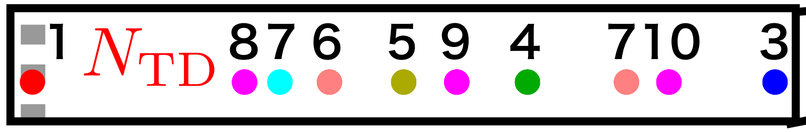
Generalized Streda formula

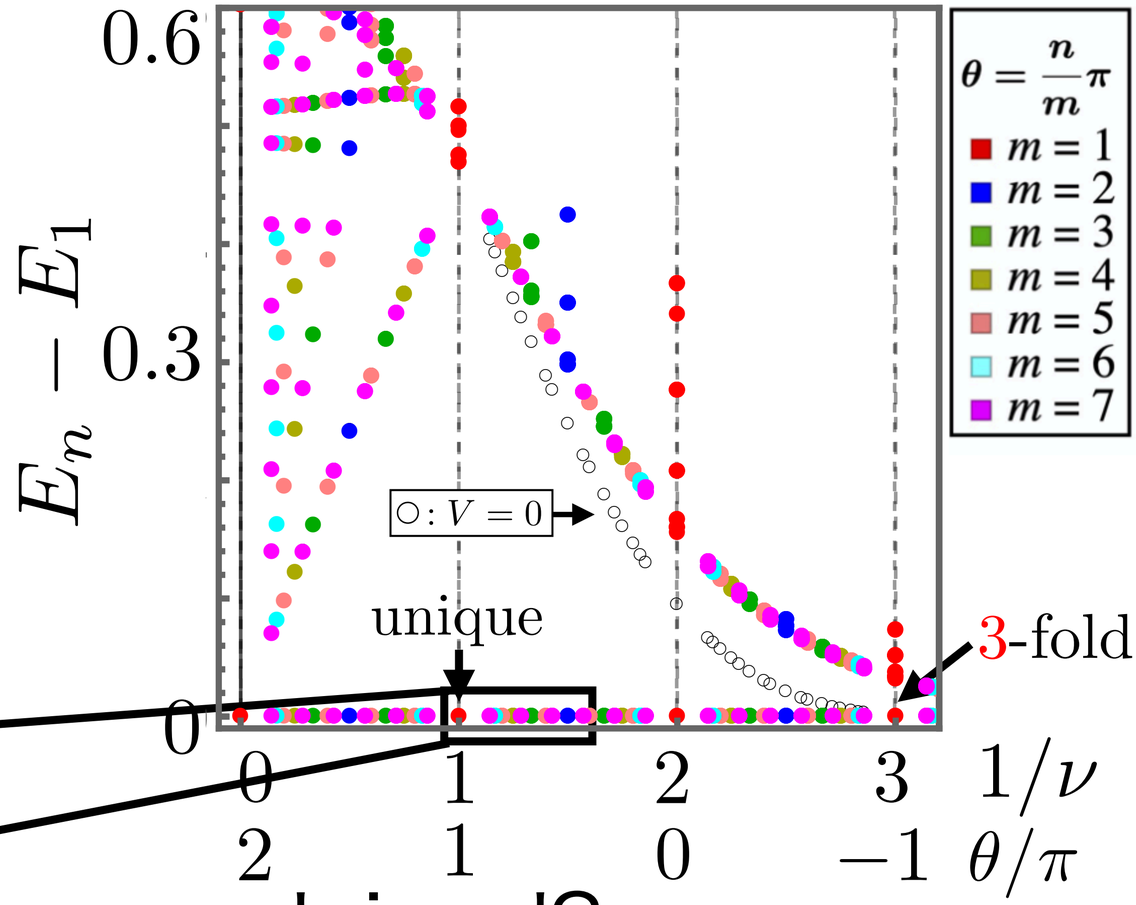
Results

- Many-body Chern number C
  - **Adiabatic invariant**



- Topological degeneracy N_{TD}
  - **Wild change**

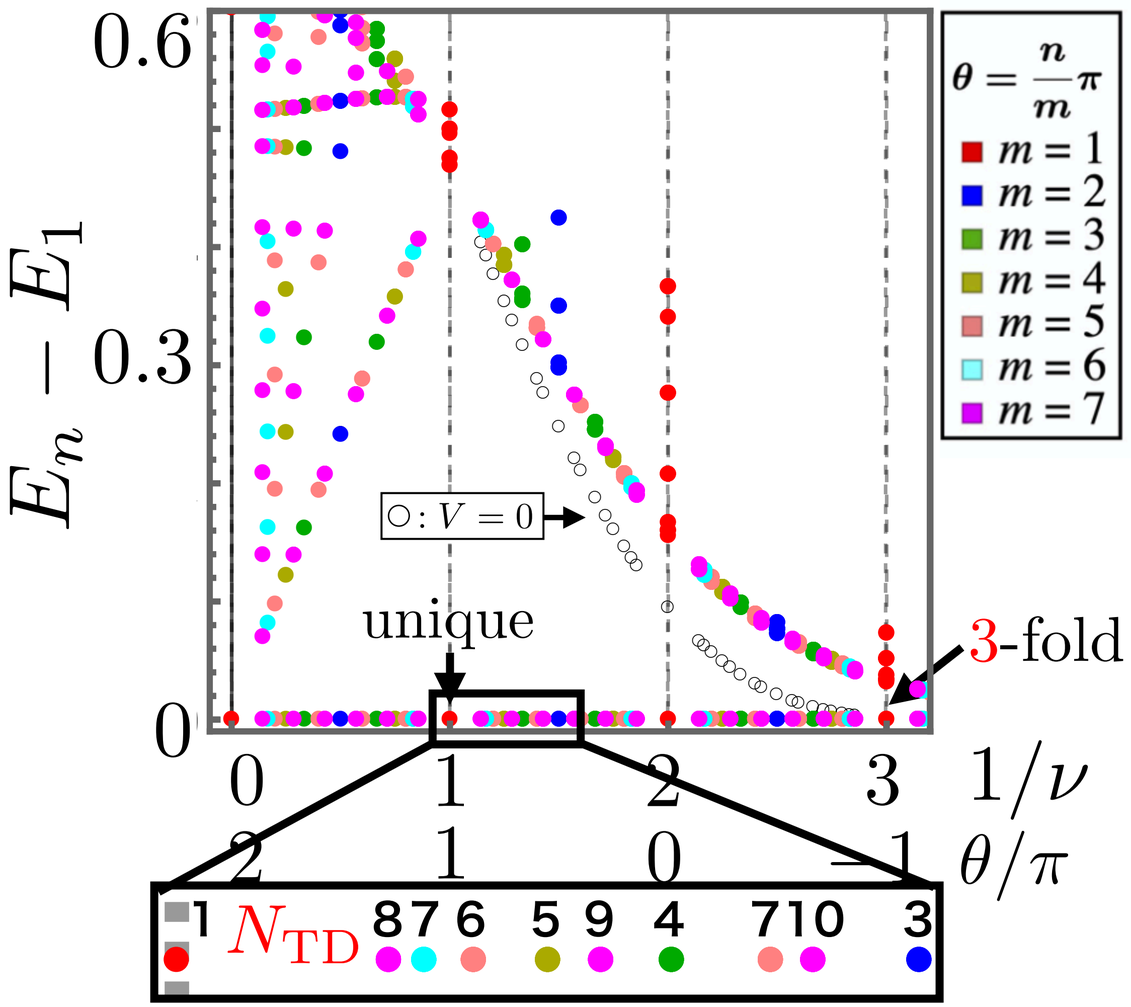


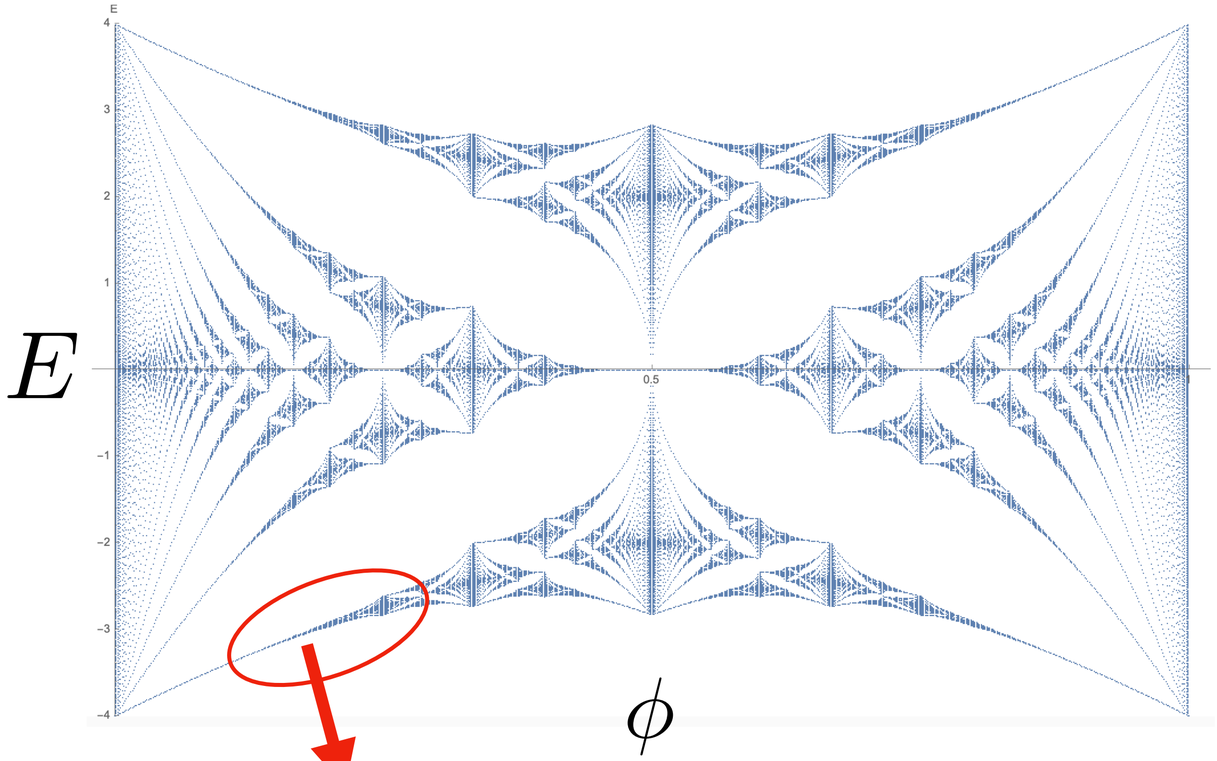


How is the wild change of N_{TD} explained?

→ We analytically show a **new** formula: $\left.\dfrac{\Delta N_{\mathrm{TD}}}{\Delta(m/\nu)}\right|_{\theta=(n/m)\pi} = C$

Generalized Streda formula





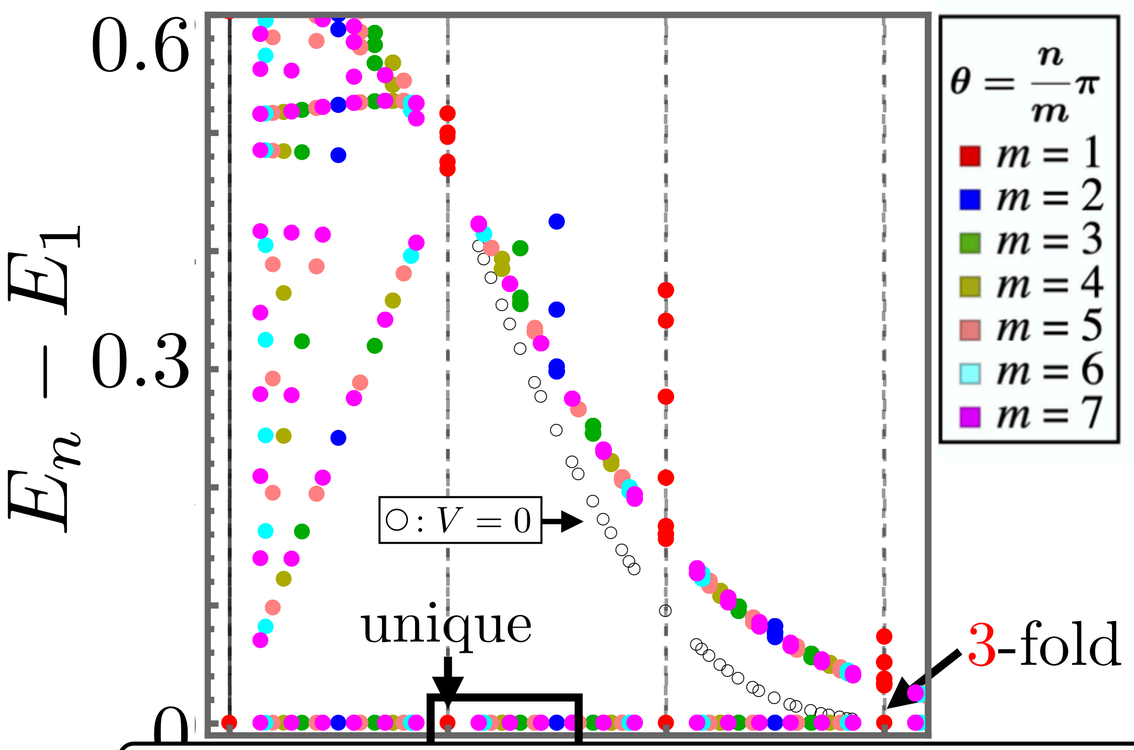
Band filling $\rightarrow \dfrac{\Delta N_{\mathrm{band}}}{\Delta\phi} = C$ Streda formula

new formula: $\dfrac{\Delta N_{\mathrm{TD}}}{\Delta(m/\nu)} = C$ Similar structure

$\theta = (n/m)\pi$

Hofstadter (1976), Streda (1982)
Thouless-Kohmoto-Nightingale-Nijs (1982)

Generalized Streda formula



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Continuity of the state $|\blacksquare\rangle \longleftrightarrow |\bullet\rangle$

Continuity of the gap

new formula: $\left.\dfrac{\Delta N_{\mathrm{TD}}}{\Delta(m/\nu)}\right|_{\theta=(n/m)\pi} = C$

Change in states = Topological invariant