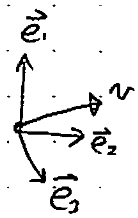
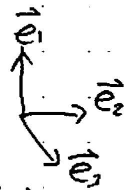
201110876 丹羽 良之

ベクトル

God

$$\vec{v} = \vec{e}_1 v_1 + \vec{e}_2 v_2 + \vec{e}_3 v_3$$
$$= (\vec{e}_1, \vec{e}_2, \vec{e}_3)\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$
$$= \underbrace{(\vec{e}_1, \vec{e}_2, \vec{e}_3)}_{\text{人間が勝手に決めたもの}} v$$





基底, 座標系

$(\vec{e}_1, \vec{e}_2, \vec{e}_3)$: 不定

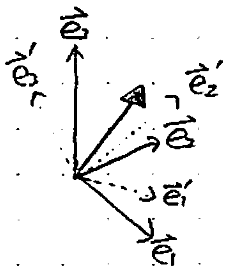
v : 不定

座標変換

$(\vec{e}_1, \vec{e}_2, \vec{e}_3)$を変化させたときどうみえるか

$\downarrow$

$(\vec{e}_1{}', \vec{e}_2{}', \vec{e}_3{}') \rightarrow v'$ 　　 $v \rightarrow v'$



規格直交をとる

↳ $e_i \cdot e_j = \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & \text{それ以外} \end{cases}$

$\vec{e}_1 = (\vec{e}_1, \vec{e}_2, \vec{e}_3)e_1$ 　　 $e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\vec{e}_i = (\vec{e}_1, \vec{e}_2, \vec{e}_3)e_i$

$$\begin{cases} \vec{v} = \vec{e}_i v_i \\ \vec{u} = \vec{e}_j u_j \end{cases}$$

$$\vec{v} \cdot \vec{u} = \vec{e}_i v_i \cdot \vec{e}_j u_j$$
$$= (\vec{e}_i \vec{e}_j) v_i u_j$$
$$= \delta_{ij} v_i u_j$$
$$= v_i u_j$$
$$= v \cdot u$$

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$(\vec{e}_1, \vec{e}_2, \vec{e}_3)$

$$\vec{e}'_i = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\, e'_i \qquad (e'_1)\,(e'_2)\,(e'_3)$$

$$(\vec{e}'_1, \vec{e}'_2, \vec{e}'_3) = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\underbrace{(e'_1, e'_2, e'_3)}_{T:\ 3\times 3\text{行列}}$$

$$\vec{e}'_i \cdot \vec{e}'_j = \delta_{ij} \qquad \tilde{T}T = \begin{pmatrix} \tilde{e}'_1 \\ \tilde{e}'_2 \\ \tilde{e}'_3 \end{pmatrix}(e'_1, e'_2, e'_3)$$

$$\begin{array}{c} \vec{e}'_i \cdot \vec{e}'_j \\ \| \\ e'_i \cdot e'_j \\ \| \\ \tilde{e}'_i \cdot e'_j \end{array}$$

$$= \begin{pmatrix} \tilde{e}'_1 e'_1 & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

規格直交系

$= E_3$: 3x3の単位行列

$$\tilde{T}T = E_3,\quad \tilde{T} = T^{-1},\quad TT^{-1} = E_3$$

直交行列 $T\tilde{T}$

$$\begin{aligned} \vec{v} &= (\vec{e}_1, \vec{e}_2, \vec{e}_3)\boxed{v} \\ &= (\vec{e}'_1, \vec{e}'_2, \vec{e}'_3)\, v' \\ &= (\vec{e}_1, \vec{e}_2, \vec{e}_3)\boxed{Tv'} \end{aligned}$$

$$v = Tv'$$

$$\vec{v} = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\, v$$

すべてのベクトル量の成分は $v = Tv'$ という1つの T で変換する。

Newton eq. $\vec{F} = m\vec{a}$ $m = m'$ 座標変換で不変(スカラー)

$$(\vec{e}_1, \vec{e}_2, \vec{e}_3)\boxed{\mathbb{F}} = \boxed{m}\,\boxed{(\vec{e}_1, \vec{e}_2, \vec{e}_3)\mathbb{a}}$$

$\mathbb{F} = m\mathbb{a}$ ある座標でのNewton e.g.

$$\begin{cases} \mathbb{F} = T\mathbb{F}' \\ \mathbb{a} = T\mathbb{a}' \end{cases}$$ ベクトルの成分の変換

$$T\mathbb{F}' = T(m\mathbb{a}')$$

$\mathbb{F}' = m\mathbb{a}'$ 座標変換後も成立

ベクトルの成分で表現, Newton e.g. は座標系によらず成立

座標変換とは?

位置ベクトル

$$\mathbb{r}\ (= \vec{e}_x x + \vec{e}_y y + \vec{e}_z z) = \vec{e}_1 x_1 + \vec{e}_2 x_2 + \vec{e}_3 x_3$$

$$\vec{r} = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\mathbb{r} = (\vec{e}_1{}', \vec{e}_2{}', \vec{e}_3{}')\mathbb{r}' \qquad \mathbb{r} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\mathbb{r} \to \mathbb{r}' \qquad \mathbb{r} = T\mathbb{r}'$$

$$|\mathbb{r}|^2 = \mathbb{r}\cdot\mathbb{r} = \tilde{\mathbb{r}}\cdot\mathbb{r} = (\widetilde{T\mathbb{r}'})T\mathbb{r}'$$
$$= \tilde{\mathbb{r}}'\cdot\tilde{T}T\cdot\mathbb{r}' = \tilde{\mathbb{r}}'\mathbb{r}' = |\mathbb{r}'|^2$$ 長さは変化しない

↳ 回転(反転, 向きさ変える)

$\mathbb{r} = T\mathbb{r}'$: 回転, 直交座標

$\mathbb{r} = T\mathbb{r}'$

$(\mathbb{r})_i = x_i = (T\mathbb{r}')_i = T_{ij}x_j'$ 行列の積

$\frac{\partial x_i}{\partial x_j} = T_{ij} \quad x_i = \frac{\partial x_i}{\partial x_j'}x_j'$

一般に $\mathbb{v} = T\mathbb{v}'$

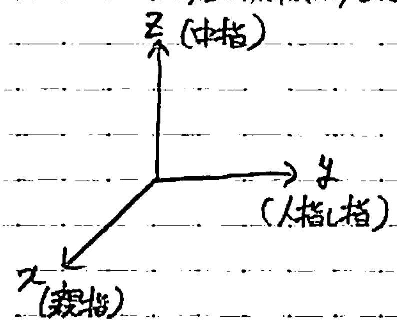
$(\mathbb{v})_i = v_i = (T\mathbb{v}')_i = T_{ij}v_j'$

$v_i = \frac{\partial x_i}{\partial x_j'}v_j'$ ベクトルの変換則

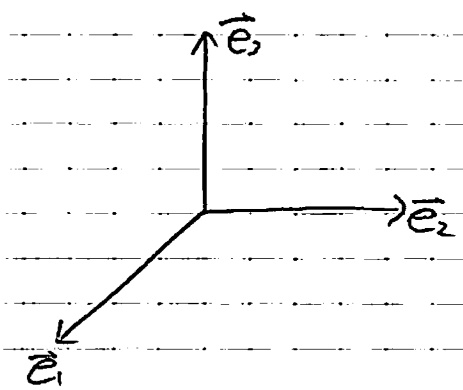
$x_i \to x_i'$ 座標変換

座標系の向き：右手系と左手系とって議論する

座標系が右手系とは？



$(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ が右手系とは？



外積 ×

$$\begin{cases} \vec{e}_1\times\vec{e}_2=\vec{e}_3 & \vec{e}_2\times\vec{e}_1=-\vec{e}_3 \\ \vec{e}_2\times\vec{e}_3=\vec{e}_1 & \vec{e}_3\times\vec{e}_2=-\vec{e}_1 \\ \vec{e}_3\times\vec{e}_1=\vec{e}_2 & \vec{e}_1\times\vec{e}_3=-\vec{e}_2 \end{cases} \quad \vec{e}_1\times\vec{e}_1=\vec{e}_2\times\vec{e}_2=\vec{e}_3\times\vec{e}_3=0$$

$$\vec{e}_i\times\vec{e}_j=\varepsilon_{ijk}\vec{e}_k \quad \left(\varepsilon_{ijk}=\begin{cases} 1 & ijk=(1,2,3)(2,3,1)(3,1,2) \\ -1 & \quad\ \ (1,3,2)(3,2,1)(2,1,3) \\ 0 & \quad\ \ \text{それ以外} \end{cases}\right)$$

$$\begin{cases} \vec{A}=\vec{e}_1A_1+\vec{e}_2A_2+\vec{e}_3A_3=(\vec{e}_1,\vec{e}_2,\vec{e}_3)\mathbb{A} \\ \vec{B}=\vec{e}_1B_1+\vec{e}_2B_2+\vec{e}_3B_3=(\vec{e}_1,\vec{e}_2,\vec{e}_3)\mathbb{B} \end{cases}$$

$$\begin{aligned} \vec{A}\times\vec{B} &= (\vec{e}_1A_1+\vec{e}_2A_2+\vec{e}_3A_3)\times(\vec{e}_1B_1+\vec{e}_2B_2+\vec{e}_3B_3) \\ &= \vec{e}_3A_1B_2-\vec{e}_2A_1B_3-\vec{e}_3A_2B_1+\vec{e}_1A_2B_3+\vec{e}_2A_3B_1-\vec{e}_1A_3B_2 \end{aligned}$$

$$\vec{A}\times\vec{B}=\vec{e}_1(A_2B_3-A_3B_2)+\vec{e}_2(A_3B_1-A_1B_3)+\vec{e}_3(A_1B_2-A_2B_1)$$

$$\vec{A}\times\vec{B}=(\vec{e}_1,\vec{e}_2,\vec{e}_3)\underbrace{\begin{pmatrix} A_2B_3-A_3B_2 \\ A_3B_1-A_1B_3 \\ A_1B_2-A_2B_1 \end{pmatrix}}_{=\mathbb{A}\times\mathbb{B}}$$

確認

$$\begin{aligned} \varepsilon_{12k}\vec{e}_k &= \varepsilon_{121}\vec{e}_1+\varepsilon_{122}\vec{e}_2+\varepsilon_{123}\vec{e}_3 \\ &= \vec{e}_3 \end{aligned}$$

$$\vec{A}=\vec{e}_iA_i \quad , B=\vec{e}_j\cdot B_j$$

$$\begin{aligned} \vec{A}\times\vec{B} &= (\vec{e}_iA_i)\times(\vec{e}_jB_j) \\ &= \underbrace{(\vec{e}_i\vec{e}_j)}_{=\varepsilon_{ijk}\vec{e}_k}A_iB_j \end{aligned}$$

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$$\vec{A}\times\vec{B} = \varepsilon_{ijk}\vec{e}_k A_i B_j$$
$$= \vec{e}_k \varepsilon_{ijk} A_i B_j$$

$k \to i \quad i \to j \quad j \to k$

$$\vec{A}\times\vec{B} = \vec{e}_i \underline{\varepsilon_{jki}} A_j B_k$$
$\quad \llcorner -\varepsilon_{jik} = (-)^2 \varepsilon_{ijk}$

$$= \vec{e}_i \varepsilon_{ijk} A_j B_k = \vec{e}_i (\mathbb{A}\times\mathbb{B})_i$$

$$(\mathbb{A}\times\mathbb{B})_i = \varepsilon_{ijk} A_j B_k$$

$$\begin{matrix} A_1 & A_2 & A_3 & A_1 \\ \times & \times & \times & \\ B_1 & B_2 & B_3 & B_1 \end{matrix} \quad \begin{pmatrix} A_2B_3 - A_3B_2 \\ A_3B_1 - A_1B_3 \\ A_1B_2 - A_2B_1 \end{pmatrix} \qquad \vec{A}\times\vec{B} = -\vec{B}\times\vec{A},\ \vec{A}\times\vec{A} = 0$$

角運動量 $\vec{L} = \vec{r}\times\vec{p}$, $\vec{p} = m\vec{v} = m\dot{\vec{r}}$: 運動量

Newton e.g. $\vec{F} = m\vec{a} = \dot{\vec{p}} \qquad m\vec{v} = m\dot{\vec{r}} = \vec{p}$

$$(\times\vec{r}) \quad \vec{r}\times\vec{F} = \vec{r}\cdot\dot{\vec{p}}$$

$$\dot{\vec{L}} = \frac{d}{dt}\vec{L} = \frac{d}{dt}\vec{r}\times\vec{p}$$

$$\underset{\llcorner \frac{\vec{p}}{m}}{\dot{\vec{r}}}\times\vec{p} + \vec{r}\times\dot{\vec{p}} = \frac{1}{m}\vec{p}\times\vec{p} + \vec{r}\times\dot{\vec{p}} = \vec{r}\times\vec{F}$$

$$\underline{\vec{r}\times\vec{F}} = \dot{\vec{L}}$$
$\llcorner \vec{N}$: 力のモーメント

$\vec{N} = \dot{\vec{L}}$ 角運動量の運動方程式
$(\vec{F} = \dot{\vec{p}})$