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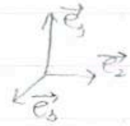
ベクトル

$$\vec{v} = \vec{e}_1 v_1 + \vec{e}_2 v_2 + \vec{e}_3 v_3 = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\,\mathbb{v}$$

← 見え方

God　　human



基底座標系

$(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ 不定　$\mathbb{v}$:不定

$(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ を変化させたときにどのように見えるか？ → 座標変換

$(\vec{e}_1{}', \vec{e}_2{}', \vec{e}_3{}') \to \mathbb{v}'$

規格座標系をとる　$\vec{e}_i \cdot \vec{e}_j = \delta_{ij} = \begin{cases} 1 & (i=j) \\ 0 & \text{それ以外} \end{cases}$

$\vec{e}_1 = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\mathbb{e}_1$　$\vec{e}_i = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\mathbb{e}_i$　※ $\mathbb{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\vec{v} = \vec{e}_i v_i$　$\vec{u} = \vec{e}_j u_j$

$\vec{v} \cdot \vec{u} = \vec{e}_i v_i \cdot \vec{e}_j u_j = (\vec{e}_i \cdot \vec{e}_j) v_i u_j = \delta_{ij} v_i u_j = v_i \cdot u_j = \mathbb{v} \cdot \mathbb{u}$

$(\vec{e}_1{}', \vec{e}_2{}', \vec{e}_3{}')$　$\vec{e}_i{}' = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\mathbb{e}_i{}'$　$(\mathbb{e}_1{}')(\mathbb{e}_2{}')(\mathbb{e}_3{}')$

$(\vec{e}_1{}', \vec{e}_2{}', \vec{e}_3{}') = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\underbrace{(\mathbb{e}_1{}', \mathbb{e}_2{}', \mathbb{e}_3{}')}$

$\vec{e}_i \cdot \vec{e}_j = \delta_{ij}$　$\mathbb{e}_i{}' \cdot \mathbb{e}_j{}' = \tilde{\mathbb{e}}_i{}' \mathbb{e}_j{}'$　T: 3×3行列

$$\tilde{T} \cdot T = \begin{pmatrix} \tilde{\mathbb{e}}_1{}' \\ \tilde{\mathbb{e}}_2{}' \\ \tilde{\mathbb{e}}_3{}' \end{pmatrix}(\mathbb{e}_1{}', \mathbb{e}_2{}', \mathbb{e}_3{}') = \begin{pmatrix} \tilde{\mathbb{e}}_1{}' \mathbb{e}_1{}' & & \\ & & \\ & & \end{pmatrix} \underset{\text{規格直交性}}{=} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = E_3$$

$$\underset{\text{God}}{\vec{v}} = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\underset{\text{human}}{\mathbb{v}} = (\vec{e}_1{}', \vec{e}_2{}', \vec{e}_3{}')\mathbb{v}' = (\vec{e}_1, \vec{e}_2, \vec{e}_3)T\mathbb{v}'$$

$\mathbb{v} = T\mathbb{v}'$　1つ1つ

すべてのベクトル量の成分は $\mathbb{v} = T\mathbb{v}'$ という1つのTで変換する。

Newton eq. $\vec{F} = m\vec{a}$　$m = m'$ 座標変換で不変（スカラー）

$(\vec{e}_1, \vec{e}_2, \vec{e}_3)\mathbb{F} = m(\vec{e}_1, \vec{e}_2, \vec{e}_3)\mathbb{a}$　$\mathbb{F} = m\mathbb{a}$ ← ある座標系でのNewton eq.

$\mathbb{F} = T\mathbb{F}'$　$\mathbb{a} = T\mathbb{a}'$　$T\mathbb{F}' = T(m\mathbb{a}')$

ベクトルの成分の変換則 $\mathbb{F}' = m\mathbb{a}'$ 座標変換後も成立

ベクトルの成分で表現したNewton eq.は座標系によらず成立

座標変換とは？

位置ベクトル $\vec{r} = (\vec{e}_x x + \vec{e}_y y + \vec{e}_z z) = \vec{e}_1 x_1 + \vec{e}_2 x_2 + \vec{e}_3 x_3$

$\vec{r} = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\mathbb{r} = (\vec{e}_1{}', \vec{e}_2{}', \vec{e}_3{}')\mathbb{r}'$　$\mathbb{r}' = \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix}$

$\mathbb{r} \leftrightarrow \mathbb{r}'$　$\mathbb{r} = T\mathbb{r}'$

$|\mathbb{r}|^2 = \mathbb{r} \cdot \mathbb{r} = \tilde{\mathbb{r}} \cdot \mathbb{r} = (\widetilde{T\mathbb{r}'})T\mathbb{r}' = \tilde{\mathbb{r}}'\tilde{T}T\mathbb{r}' = \tilde{\mathbb{r}}'\mathbb{r}' = |\mathbb{r}'|^2$　$\widetilde{AB} = \tilde{B}\tilde{A}$

長さは変化しない → 回転（反転、向きかえ）E_3

$\mathbb{r} = T\mathbb{r}'$：回転　T：直交変換

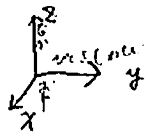
$(\mathbb{r})_i = x_i = (T\mathbb{r}')_i = T_{ij} x_j'$ 行列の積

$$\frac{\partial x_i}{\partial x_j'} = T_{ij} \quad x_i = \frac{\partial x_i}{\partial x_j'} x_j'$$

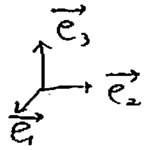
一般に $\mathbb{v} = T\mathbb{v}'$ $(\mathbb{v})_i' = v_i = (T v')_i = T_{ij} v_j'$

$$v_i = \frac{\partial x_i}{\partial x_j'} v_j' \qquad x_i = x_i' \quad \text{座標変換}$$

座標系の向き 右手系と左手系
こっちで議論



$(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ が右手系とは



外積

$$\left.\begin{array}{ll} \vec{e}_1 \times \vec{e}_2 = \vec{e}_3 & \vec{e}_2 \times \vec{e}_1 = -\vec{e}_3 \\ \vec{e}_2 \times \vec{e}_3 = \vec{e}_1 & \vec{e}_3 \times \vec{e}_2 = -\vec{e}_1 \\ \vec{e}_3 \times \vec{e}_1 = \vec{e}_2 & \vec{e}_1 \times \vec{e}_3 = -\vec{e}_2 \end{array}\right\} \to \vec{e}_i \times \vec{e}_j = \varepsilon_{ijk} \vec{e}_k$$

$\varepsilon_{123} = \varepsilon_{231} = \varepsilon_{312} = 1$
$\varepsilon_{132} = \varepsilon_{321} = \varepsilon_{213} = -1$

$$\varepsilon_{ijk} = \begin{cases} 1 & (ijk) = (1,2,3)\ (2,3,1)\ (3,1,2) \\ -1 & \qquad (1,3,2)\ (3,2,1)\ (2,1,3) \\ 0 \end{cases}$$

$$\vec{A} = \vec{e}_1 A_1 + \vec{e}_2 A_2 + \vec{e}_3 A_3 = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\mathbb{A}$$
$$\vec{B} = \vec{e}_1 B_1 + \vec{e}_2 B_2 + \vec{e}_3 B_3 = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\mathbb{B}$$

$$\begin{aligned} \vec{A} \times \vec{B} &= (\vec{e}_1 A_1, \vec{e}_2 A_2, \vec{e}_3 A_3) \times (\vec{e}_1 B_1, \vec{e}_2 B_2, \vec{e}_3 B_3) \\ &= \vec{e}_3 A_1 B_2 - \vec{e}_2 A_1 B_3 - \vec{e}_3 A_2 B_1 + \vec{e}_1 A_2 B_3 + \vec{e}_2 A_3 B_1 - \vec{e}_1 A_3 B_2 \\ &= \vec{e}_1 (A_2 B_3 - A_3 B_2) + \vec{e}_2 (A_3 B_1 - A_1 B_3) + \vec{e}_3 (A_1 B_2 - A_2 B_1) \end{aligned}$$

$$\vec{A} \times \vec{B} = (\vec{e}_1, \vec{e}_2, \vec{e}_3) \underbrace{\begin{pmatrix} A_2 B_3 - A_3 B_2 \\ A_3 B_1 - A_1 B_3 \\ A_1 B_2 - A_2 B_1 \end{pmatrix}}_{A \times B}$$

確認 $\varepsilon_{12k} \vec{e}_k = \overset{=0}{\varepsilon_{121}} \vec{e}_1 + \overset{=0}{\varepsilon_{122}} \vec{e}_2 + \varepsilon_{123} \vec{e}_3 = \vec{e}_3$ 他も同じ

$\vec{A} = \vec{e}_i A_i \quad \vec{B} = \vec{e}_j B_j \quad \vec{A} \times \vec{B} = (\vec{e}_i A_i) \times (\vec{e}_j B_j) = (\vec{e}_i \times \vec{e}_j) A_i B_j$

$\because \varepsilon_{ijk} \vec{e}_k$

$$\vec{A} \times \vec{B} = \varepsilon_{ijk} \vec{e}_k A_i B_j = \vec{e}_k \varepsilon_{ijk} A_i B_j$$

$k = i \quad i = j \quad j = k$

$$\vec{A} \times \vec{B} = \vec{e}_i \varepsilon_{jki} A_j B_k \quad \varepsilon_{jik} = (-)^2 \varepsilon_{ijk} = \vec{e}_i \varepsilon_{ijk} A_j B_k$$

$\vec{e}_i (\mathbb{A} \times \mathbb{B})_i$

$(\mathbb{A} \times \mathbb{B})_i = \varepsilon_{ijk} A_j B_k$

$$\begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \\ B_1 & B_2 & B_3 & B_4 \end{array} \qquad \begin{pmatrix} A_2 B_3 - A_3 B_2 \\ A_3 B_1 - A_1 B_3 \\ A_1 B_2 - A_2 B_1 \end{pmatrix}$$

$x_1 - x_3 - 10\%_4 = 0$

$\cancel{x_1} + 10(3x_2 + 2x_3 + 7\%_4 = 0$

$x_1 = \cancel{x_3} + 10$

x_2

$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \qquad \vec{A} \times \vec{A} = 0$

角運動量 $\vec{L} = \vec{r} \times \vec{p} \qquad \vec{p} = m\vec{v} = m\dot{\vec{r}}$ 運動量

Newton eq

$\vec{F} = m\vec{a} = \dot{\vec{p}} \qquad m\dot{\vec{v}} = m\vec{v} = \vec{p}$

$\vec{r} \times \vec{F} = \vec{r} \times \dot{\vec{p}}$

$\dot{\vec{L}} = \frac{d}{dt}\vec{L} = \frac{d}{dt}\vec{r} \times \vec{p} = \dot{\vec{r}} \times \vec{p} + \vec{r} \times \dot{\vec{p}} = \frac{1}{m}\vec{p} \times \vec{p} + \vec{r} \times \dot{\vec{p}} = \vec{r} \times \vec{F}$

$\vec{r} \times \vec{F} = \dot{\vec{L}}$

$\vec{N}$: 力のモーメント $\qquad \vec{N} = \dot{\vec{L}}$ 角運動量の方程式

$(\vec{F} = \dot{\vec{p}})$