$e^{A\hat{n}} = ?$

$\hat{n} = c^\dagger c$, $\{c, c^\dagger\} = c c^\dagger + c^\dagger c = 1$, $c^2 = 0$, $(c^\dagger)^2 = 0$

$1 + (e^A - 1) n$

$n^2 = c^\dagger c \cdot c^\dagger c = c^\dagger \underbrace{c c^\dagger}_{1 - c^\dagger c} c = c^\dagger (1 - c^\dagger c) c$

$= c^\dagger c - c^\dagger c^\dagger \underbrace{c c}_{\to 0} = c^\dagger c = n$

$n^2 = n$, $n^3 = n^2 = n$

$e^{An} = \sum_{k=0}^{\infty} \frac{(An)^k}{k!} = 1 + \sum_{k=1}^{\infty} \frac{A^k n^{k=n}}{k!} = 1 + \left(\sum_{k=1}^{\infty} \frac{A^k}{k!} \right) n$

$= 1 + \left(1 + \underbrace{\sum_{k=0}^{\infty} \frac{A^k}{k!}}_{= e^A} \right) n - n$

$= 1 + (e^A - 1) n$

$A = 0 \Rightarrow e^0 = 1$, $1 + (\underbrace{e^0}_{= 0} - 1) n = 1$ o.k