Thermodynamics

1, The first low of thermodynamics is

$$dU + L = Q \,, \qquad (1)$$

where dU is the energy variation of the system, L is the work performed by the system, and Q is the heat absorbed in the system.

Now, we know L and Q. Thus, we get

$$
\begin{aligned}
dU &= Q - L \\
&= 32[\text{cal}] - 3.4 \times 10^8[\text{erg}] \\
&= 32 \times 4.19[\text{J}] - 3.4 \times 10^8 \times 10^{-7}[\text{J}] \\
&= 1.0 \times 10^2[\text{J}] \,.
\end{aligned}
$$

2, The energy of an ideal gas depends on temperature only. Since the process is isothermal, equation (1) yields

$$
\begin{aligned}
Q &= L \\
&= \int_{V_i}^{V_f} dV\, p \\
&= \int_{p_i}^{p_f} dp\, \frac{dV}{dp} p \,, \qquad (2)
\end{aligned}
$$

where the suffix i means an initial state and f means a final state. In the ideal gas, the equation of state is written
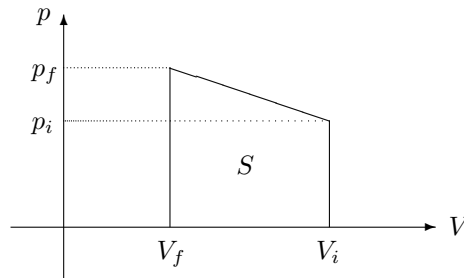
$$pV = nRT \,. \qquad (3)$$

Substituting (3) for (2), we find

$$Q = \int_{p_i}^{p_f} dp \frac{dV}{dp} p$$

$$= \int_{p_i}^{p_f} dp \left(-\frac{nRT}{p^2} \right) p$$

$$= -nRT \int_{p_i}^{p_f} dp \frac{dp}{p}$$

$$= -nRT \ln \frac{p_f}{p_i}$$

$$= nRT \ln \frac{p_i}{p_f}$$

$$= 3[\text{mol}] \cdot 2.0[\text{cal}/(\text{K} \cdot \text{mol})] \cdot (0 + 273)[\text{K}] \cdot \ln \frac{5[\text{atm}]}{3[\text{atm}]}$$

$$= 8 \times 10^2 [\text{cal}] \,.$$

3, From (3), the initial pressure p_i and the final pressure p_f become

$$p_i = \frac{nRT_i}{V_i}$$

$$= \frac{1[\text{mol}] \cdot 8.3[\text{J}/(\text{K} \cdot \text{mol})] \cdot 291[\text{K}]}{21000[\text{cc}]}$$

$$= 1 \times 10^5 [\text{Pa}] \,,$$

$$p_f = \frac{nRT_f}{V_f}$$

$$= \frac{1[\text{mol}] \cdot 8.3[\text{J}/(\text{K} \cdot \text{mol})] \cdot 305[\text{K}]}{12700[\text{cc}]}$$

$$= 2 \times 10^5 [\text{Pa}] \,.$$

The work L performed by the system is represented as the area S of the trapezoid on the figure. However, L is negative because of compressive

process. Accordingly, the work L is

$$L = -\frac{1}{2}(p_i + p_f)(V_i - V_f)$$
$$= -1 \times 10^3 [\text{J}] \, .$$

Next, internal energy for a diatomic ideal gas was written

$$U = \frac{5}{2}nRT \, ,$$

because a diatomic molecule has degree of freedom 5. Using this, we can calculate energy variation

$$dU = \frac{5}{2}nRT_f - \frac{5}{2}nRT_i$$
$$= \frac{5}{2}nR(T_f - T_i)$$
$$= \frac{5}{2} \cdot 1[\text{mol}] \cdot 8.3[\text{J}/(\text{K} \cdot \text{mol})] \cdot (305 - 291)[\text{K}]$$
$$= 3 \times 10^2 [\text{J}] \, .$$

From (1), we obtain the heat absorbed in the system

$$Q = dU + L$$
$$= -1 \times 10^3 [\text{J}] \, .$$

4, In adiabatic process for ideal gas, we know

$$V_i^{\gamma - 1}T_i = V_f^{\gamma - 1}T_f \, ,$$

where $\gamma = C_p/C_V$ is specific-heat ratio. Thus,

$$T_f = \left(\frac{V_f}{V_i}\right)^{1-\gamma} T_i \, .$$

Since a diatomic ideal gas has $\gamma = 7/5$, we find

$$T_f = \left(\frac{V_f}{V_i}\right)^{1-\gamma} T_i$$
$$= (1.35)^{1-7/5} \times 291.15[\text{K}]$$
$$= 258[\text{K}] \, .$$