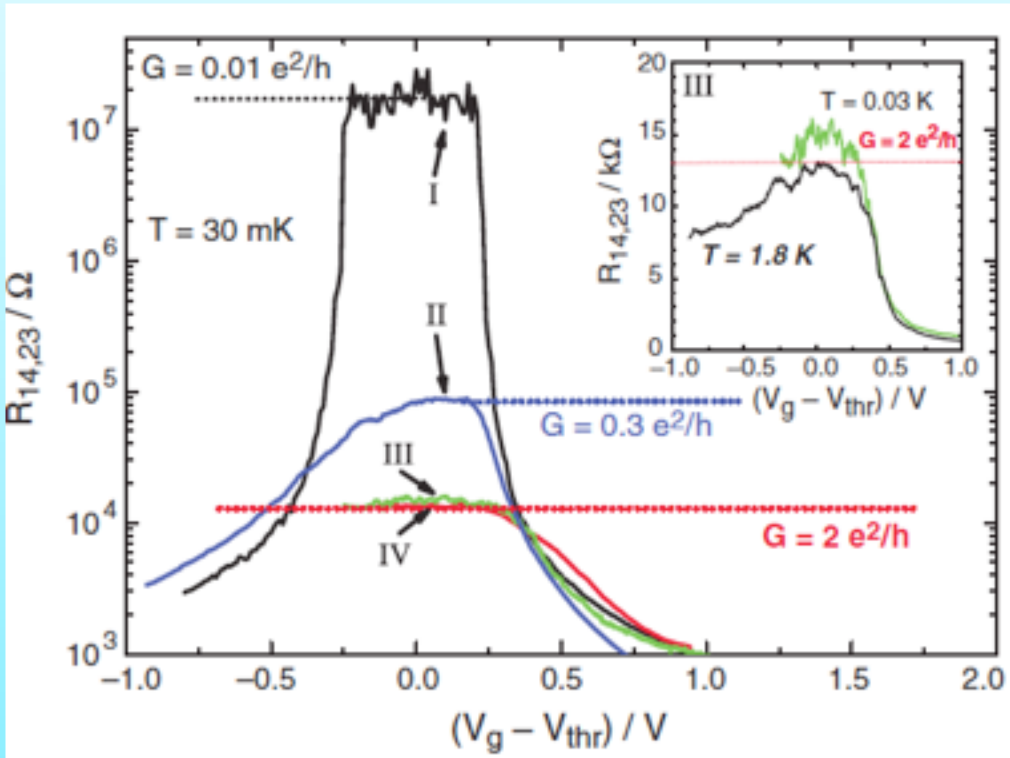
Topological Insulator

Topological insulator : Quantum Spin Hall state



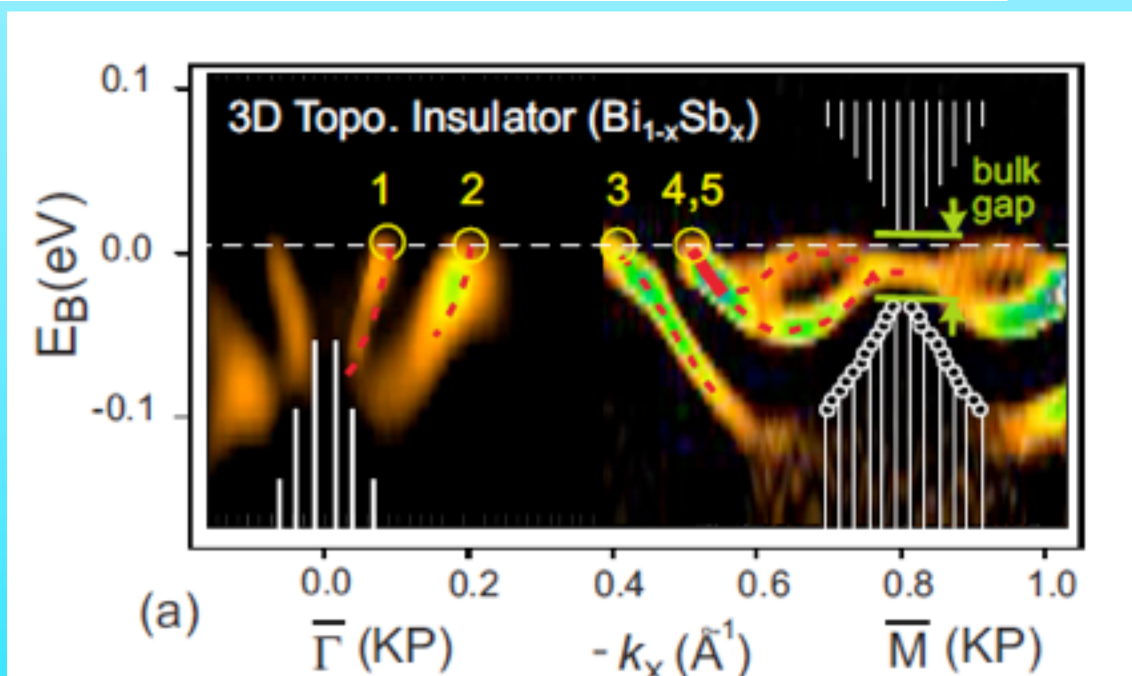
Quantum Spin Hall Insulator State in HgTe Quantum Wells

Markus König,[1] Steffen Wiedmann,[1] Christoph Brüne,[1] Andreas Roth,[1] Hartmut Buhmann,[1] Laurens W. Molenkamp,[1*] Xiao-Liang Qi,[2] Shou-Cheng Zhang[2]

Science **318**, 766 (2007)

2D

TNG



Hsieh, D., D. Qian, L. Wray, Y. Xia, Y. S. Hor, R. J. Cava, and M. Z. Hasan, 2008, Nature (London) **452**, 970.

TOPOLOGICAL INSULATORS 378 NATURE PHYSICS | VOL 5 | JUNE 2009 |

The next generation

Spin-orbit coupling in some materials leads to the formation of surface states tha
scattering. Theory and experiments have found an important new family of such

Joel Moore

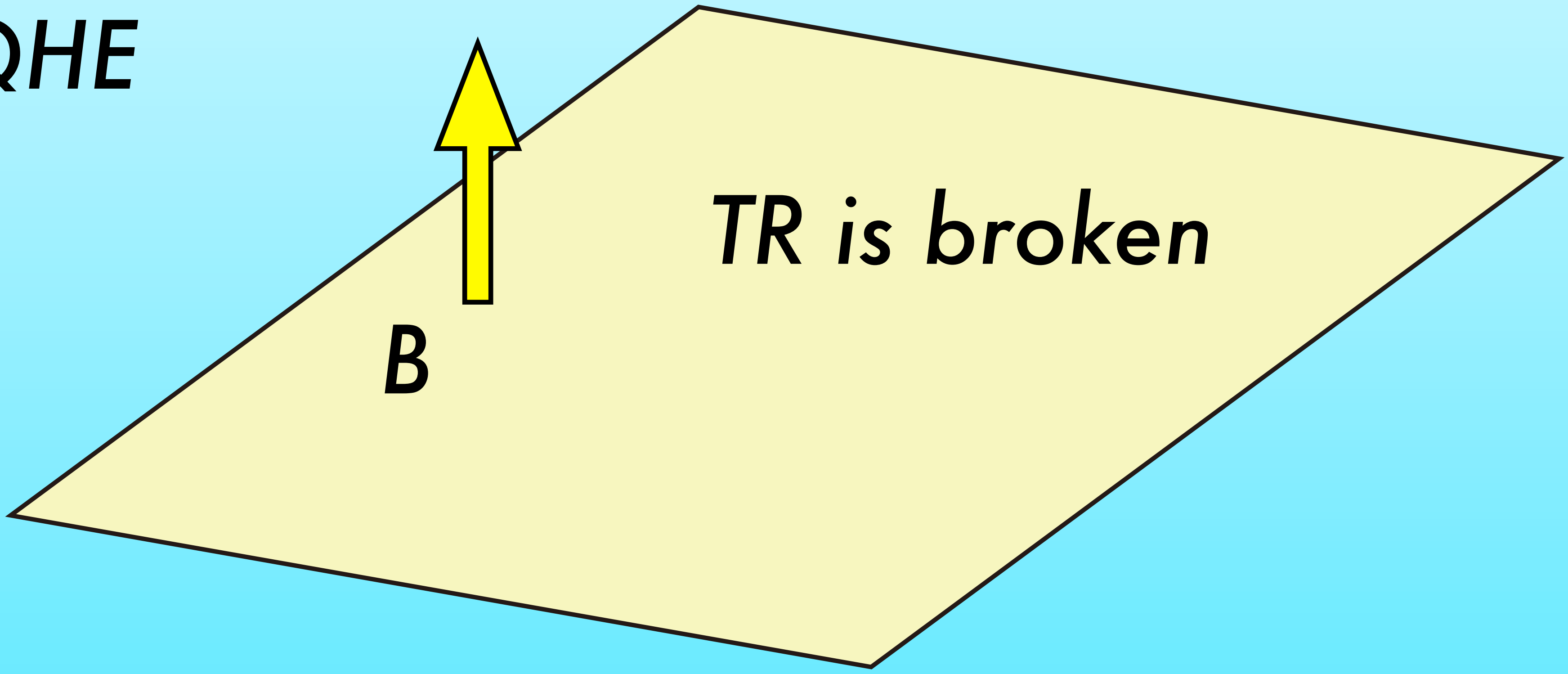
3D

"Quantum" Spin Hall Effect

= Quantum Hall Effect without magnetic field

= Quantum Hall Effect with time-reversal invariance

QHE

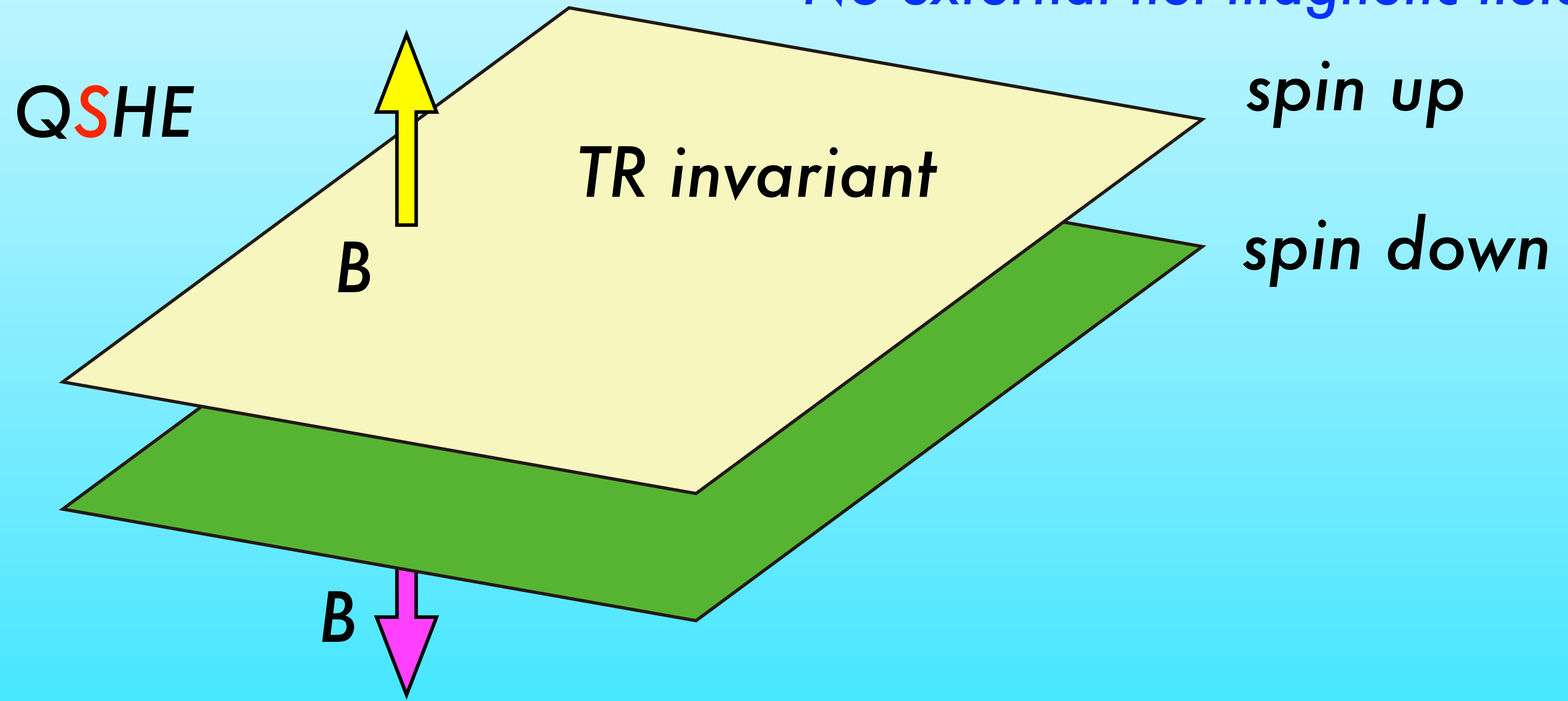


"Quantum" Spin Hall Effect

= Quantum Hall Effect *without magnetic field*

= Quantum Hall Effect *with time-reversal invariance*

No external net magnetic field



"Quantum" Spin Hall Effect

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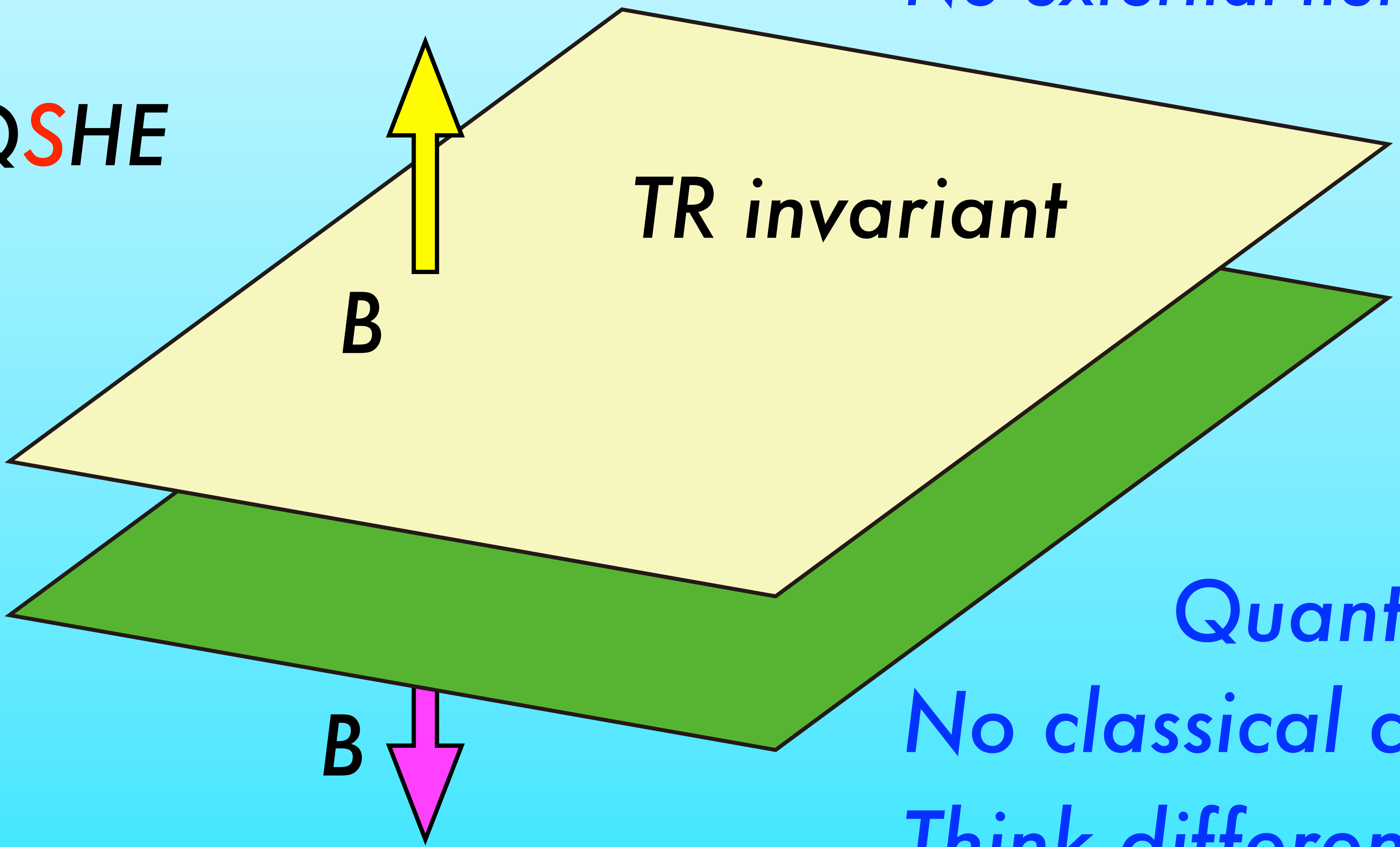
No external net magnetic field

QSHE



Quantum effects !

No classical correspondent !

Think different

Following the QHE

so-called Topological Insulator

Topological insulator : *Quantum Spin Hall state*

Need to undestand !

Time Reversal

Kramers degeneracy

Let me explain !

Spin Hall conductance is not quantized

Spin is not conserved (spin-orbit)

Time-Reversal (TR) symmetry & Kramers degeneracy

TR: Anti-Unitary Θ : $c_i = \begin{bmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{bmatrix} \rightarrow \begin{bmatrix} c_{i\downarrow} \\ -c_{i\uparrow} \end{bmatrix} = Jc_i$

$$\mathcal{H} = c_i^\dagger H_{ij} c_j$$

& complex conjugate

$$J = i\sigma_y = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

TR invariance

$$[\Theta, \mathcal{H}] = 0$$

→ $JH^*J^{-1} = H \qquad \{H\}_{ij} = H_{ij}$

J H * J^{-1}

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}^* \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} a^* & b^* \\ c^* & d^* \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} c^* & d^* \\ -a^* & -b^* \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} d^* & -c^* \\ -b^* & a^* \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

H

$$a = d^*, \; b = -c^*$$

$$H = \begin{bmatrix} t & \Delta \\ -\Delta^* & t^* \end{bmatrix}$$

t : *Spin independent hopping*

Δ : *Spin-orbit, Rashba term, etc.*

Time-Reversal (TR) symmetry & Kramers degeneracy

TR: Anti-Unitary Θ : $c_i = \begin{bmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{bmatrix} \rightarrow \begin{bmatrix} c_{i\downarrow} \\ -c_{i\uparrow} \end{bmatrix} = J c_i$

$\mathcal{H} = c_i^\dagger H_{ij} c_j$

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$J = i\sigma_y = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

TR invariance

$[\Theta, \mathcal{H}] = 0$ → $J H^* J^{-1} = H \qquad \{H\}_{ij} = H_{ij}$

J **H *** **J-1**

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}^* \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} a^* & b^* \\ c^* & d^* \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

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H

$$a = d^*, \; b = -c^*$$

$$H = \begin{bmatrix} t & \Delta \\ -\Delta^* & t^* \end{bmatrix}$$

$t^\dagger = t$ ***hermite***

& hermiticity $\Delta^\dagger = -\Delta^* \rightarrow \tilde{\Delta} = -\Delta$

anti-symmetric

Time-Reversal (TR) symmetry & Kramers degeneracy

Schrödinger Equation

$$\begin{bmatrix} t & \Delta \\ -\Delta^* & t^* \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = E \begin{bmatrix} u \\ v \end{bmatrix}$$

$$tu + \Delta v = Eu$$
$$-\Delta^* u + t^* v = Ev$$

$$tv^* + \Delta(-u^*) = Ev^*$$
$$-\Delta^* v^* + t^*(-u^*) = E(-u^*)$$

Time-Reversal (TR) symmetry & Kramers degeneracy

Schrödinger Equation

$$\begin{bmatrix} t & \Delta \\ -\Delta^* & t^* \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = E \begin{bmatrix} u \\ v \end{bmatrix}$$

↓

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Ah !

$H \begin{bmatrix} v^* \\ -u^* \end{bmatrix} = E \begin{bmatrix} v^* \\ -u^* \end{bmatrix}$, $\begin{bmatrix} v^* \\ -u^* \end{bmatrix}$ *is also an eigen state with the same energy*

→ $\begin{bmatrix} u_\Theta \\ v_\Theta \end{bmatrix} = \begin{bmatrix} v^* \\ -u^* \end{bmatrix}$ & $\begin{bmatrix} u \\ v \end{bmatrix}$ *: the same energy, degenerate ?*

Time-Reversal (TR) symmetry & Kramers degeneracy

Schrödinger Equation

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→ $\begin{bmatrix} u_\Theta \\ v_\Theta \end{bmatrix} = \begin{bmatrix} v^* \\ -u^* \end{bmatrix}$ & $\begin{bmatrix} u \\ v \end{bmatrix}$: the same energy, degenerate ?

Not yet !

the same state ??

Time-Reversal (TR) symmetry & Kramers degeneracy

Schrödinger Equation

$$\begin{bmatrix} t & \Delta \\ -\Delta^* & t^* \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = E \begin{bmatrix} u \\ v \end{bmatrix}$$

↓

$$tu + \Delta v = Eu$$
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↗↘

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Not yet !

the same state ??

OK ! surely different !

Orthogonal ! $\begin{bmatrix} u \\ v \end{bmatrix}^\dagger \begin{bmatrix} u_\Theta \\ v_\Theta \end{bmatrix} = u^* v^* + v^*(-u^*) = 0$

Time-Reversal (TR) symmetry & Kramers degeneracy

Schrödinger Equation

$$\begin{bmatrix} t & \Delta \\ -\Delta^* & t^* \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = E \begin{bmatrix} u \\ v \end{bmatrix}$$

Kramers degeneracy

Any one particle state is doubly degenerate

$$tu + \Delta v = Eu$$
$$-\Delta^* u + t^* v = Ev$$

$$tv^* + \Delta(-u^*) = Ev^*$$
$$-\Delta^* v^* + t^*(-u^*) = E(-u^*)$$

Ah !

$$H \begin{bmatrix} v^* \\ -u^* \end{bmatrix} = E \begin{bmatrix} v^* \\ -u^* \end{bmatrix}, \quad \begin{bmatrix} v^* \\ -u^* \end{bmatrix}$$ is also an eigen state with the same energy

→ $\begin{bmatrix} u_\Theta \\ v_\Theta \end{bmatrix} = \begin{bmatrix} v^* \\ -u^* \end{bmatrix}$ & $\begin{bmatrix} u \\ v \end{bmatrix}$: the same energy, degenerate ?

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the same state ??

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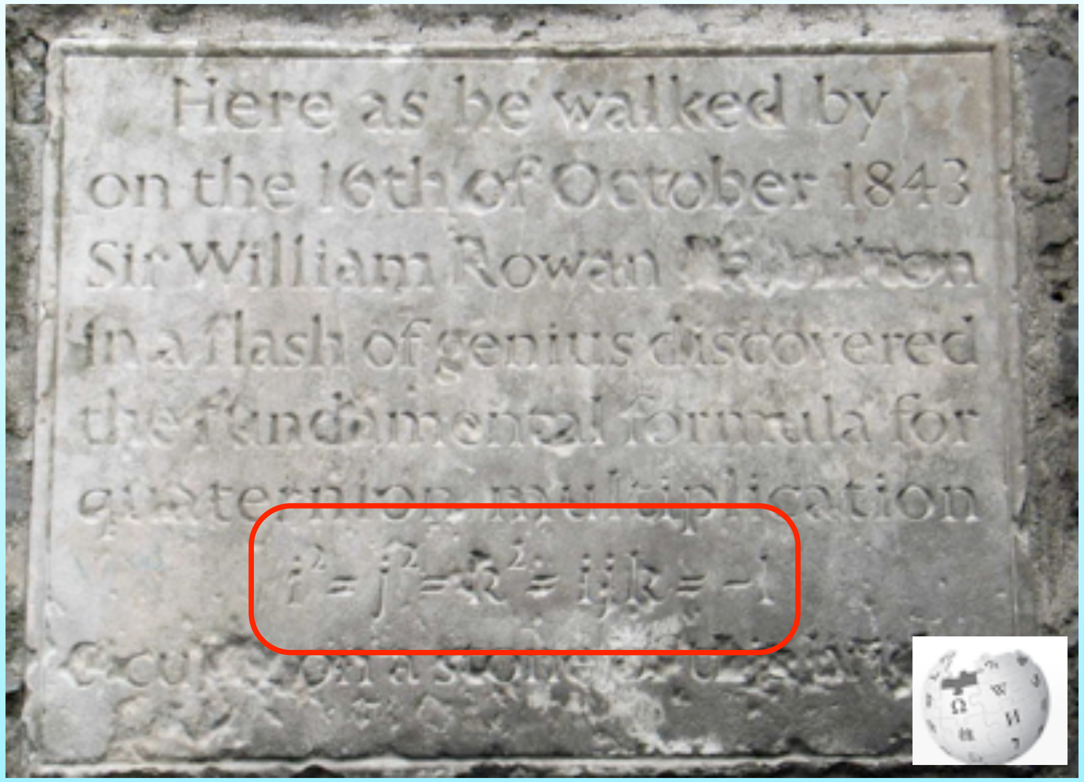
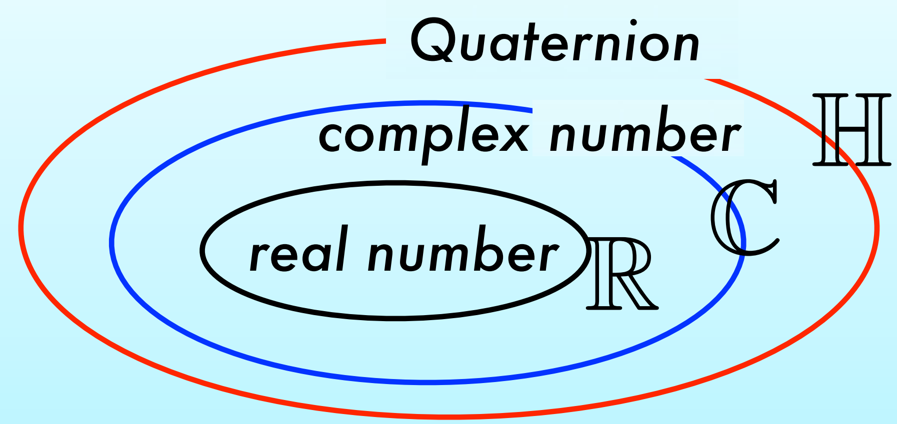
Time Reversal & Quaternions

Classical to Quantum



Hamilton 四元数発見の碑

F.J.Dyson '61–



No magic, neither crazy

just Pauli matrices

$$H = \begin{bmatrix} t & \Delta \\ -\Delta^* & t^* \end{bmatrix} = (\mathrm{Re}t) I_2 + (\mathrm{Im}t)\, i\sigma_z + (\mathrm{Re}\Delta)\, i\sigma_y + (\mathrm{Im}\Delta)\, i\sigma_x$$

$$\cong (\mathrm{Re}t)\, 1 + (\mathrm{Im}t) i_{\mathbb{H}} + (\mathrm{Re}\Delta)\, j_{\mathbb{H}} + (\mathrm{Im}\Delta)\, k_{\mathbb{H}}$$

: Quaternion (四元数)

$$i_{\mathbb{H}} \cong i\sigma_z, \;\; j_{\mathbb{H}} \cong i\sigma_y, \;\; k_{\mathbb{H}} \cong i\sigma_z$$

$$i_{\mathbb{H}}^2 = k_{\mathbb{H}}^2 = k_{\mathbb{H}}^2 = i_{\mathbb{H}} j_{\mathbb{H}} k_{\mathbb{H}} = -1$$

Quaternion 2×2 Matrix, Yang Monopole & quantization: YH, NJP12, 065004 (2010)

Classical to Quantum

Time-Reversal, Spins & Spinors

spin

spinor

$$\hat{\boldsymbol{S}} = \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} = \boldsymbol{c}^\dagger \boldsymbol{S} \boldsymbol{c}\,, \quad \boldsymbol{S} = \frac{\boldsymbol{\sigma}}{2}\,, \quad \boldsymbol{c} = \begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix}$$

$$\Theta \hat{\boldsymbol{S}} \Theta^{-1} = \boldsymbol{c}^\dagger \boldsymbol{S}^\Theta \boldsymbol{c}$$

$$\boldsymbol{S}^\Theta = \boldsymbol{J} \boldsymbol{S}^* \boldsymbol{J}^{-1}$$

$$\Theta\, \boldsymbol{c}\, \Theta^{-1} = \boldsymbol{J} \boldsymbol{c}$$

$$\sigma_x^\Theta = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^* \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -\sigma_x$$

$$\sigma_y^\Theta = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}^* \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -\sigma_y$$

$$\sigma_z^\Theta = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^* \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -\sigma_z$$

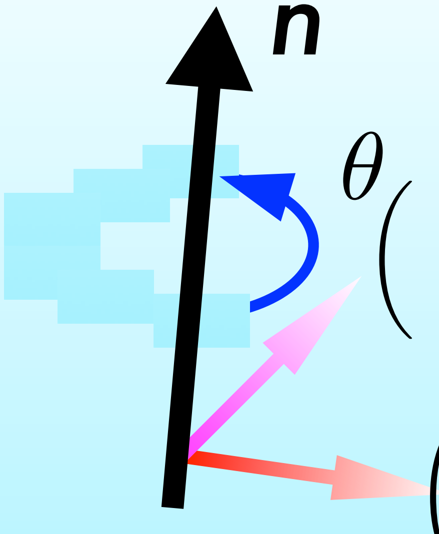
$$\boldsymbol{S}^\Theta = -\boldsymbol{S}$$

$$\boldsymbol{B} \cdot \boldsymbol{S} \to -\boldsymbol{B} \cdot \boldsymbol{S}$$

Magnetic field

Zeeman term breaks TR

Rotation: Spin & Spinor



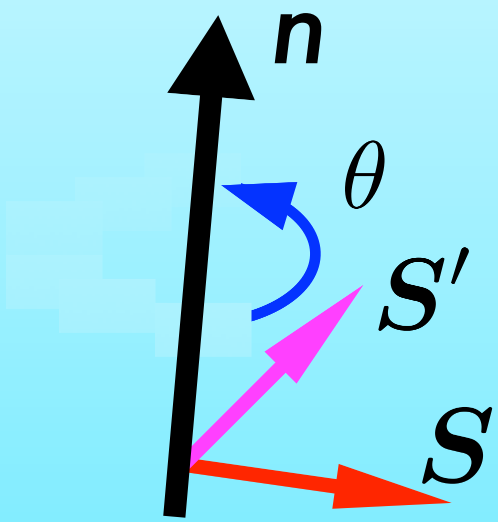
$\begin{pmatrix} c'_\uparrow \\ c'_\downarrow \end{pmatrix}$ $\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix}$

spinor

$$\begin{pmatrix} c'_\uparrow \\ c'_\downarrow \end{pmatrix} = \boldsymbol{U}(\theta) \begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} \qquad \boldsymbol{U} \in SU(2)$$

$$\boldsymbol{U}(\theta) = e^{-i\boldsymbol{S}\cdot\boldsymbol{n}\theta} = \cos\frac{\theta}{2} - i\boldsymbol{n}\cdot\boldsymbol{\sigma}\sin\frac{\theta}{2}$$

$$\det \boldsymbol{U} = e^{-i(\mathrm{Tr}\,\boldsymbol{S})\cdot\boldsymbol{n}\theta} = e^0 = 1$$



spin

$$\begin{pmatrix} S'_x \\ S'_y \\ S'_z \end{pmatrix} = \boldsymbol{U}\boldsymbol{S}\boldsymbol{U}^\dagger = \begin{pmatrix} & & \\ & Q(\theta) & \\ & & \end{pmatrix} \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}$$

$\boldsymbol{S}'$: hermite $\quad \mathrm{Tr}\,\boldsymbol{S}' = \mathrm{Tr}\,\boldsymbol{S}\boldsymbol{U}^\dagger\boldsymbol{U} = \boldsymbol{0}$

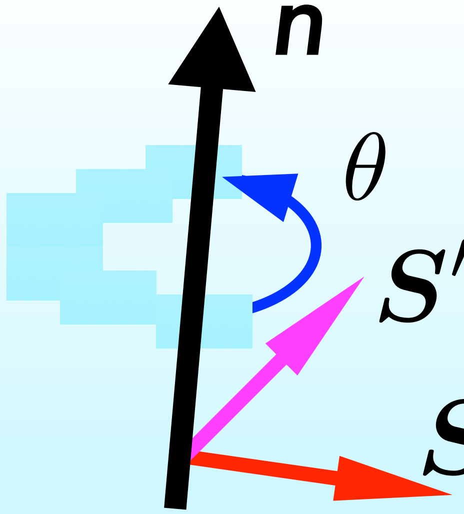
Expand by Pauli matrices with real coefficients

$$\begin{aligned} \mathrm{Tr}\, S'_\alpha S'_\beta &= \mathrm{Tr}\, S_\alpha S_\beta = \frac{1}{2}\delta_{\alpha\beta} \\ \mathrm{Tr}\, S'_\alpha S'_\beta &= Q_{\alpha\alpha'} Q_{\beta\beta'} \mathrm{Tr}\, S_{\alpha'} S_{\beta'} \\ &= \frac{1}{2} Q_{\alpha\gamma} Q_{\beta\gamma} = \frac{1}{2}(\boldsymbol{Q}\tilde{\boldsymbol{Q}})_{\alpha\beta} \end{aligned}$$

$$\boldsymbol{Q}\tilde{\boldsymbol{Q}} = \boldsymbol{E}_3 \qquad \boldsymbol{Q} \in SO(3)$$

continiously connected to E_3

Rotation: Spin & Spinor



spin

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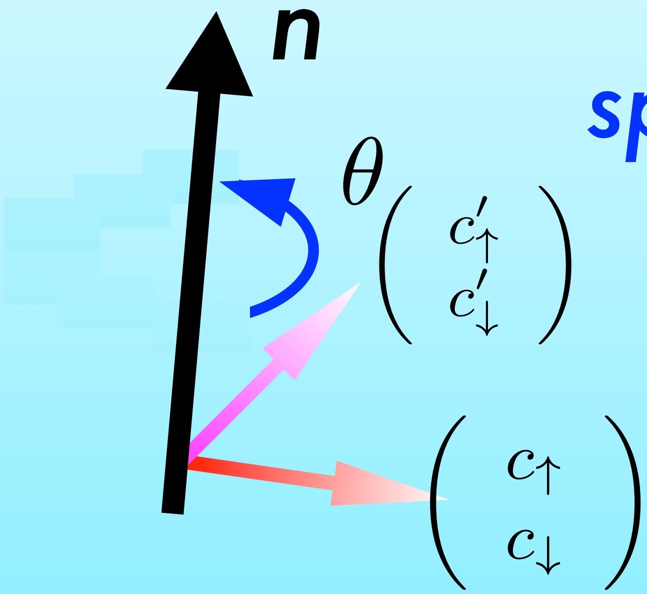
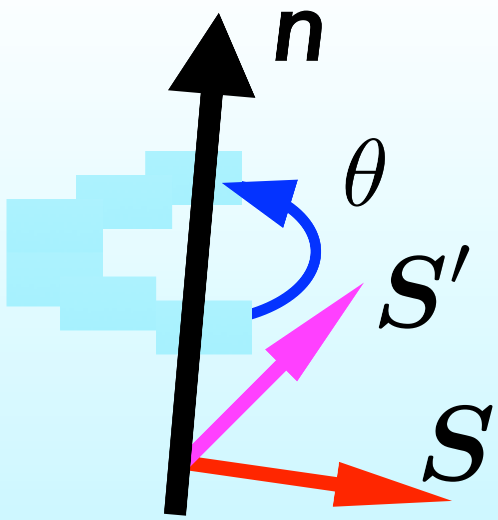


spinor

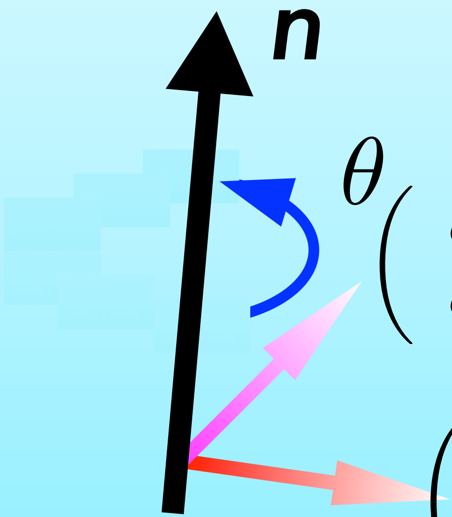
$$\begin{pmatrix} c'_\uparrow \\ c'_\downarrow \end{pmatrix} = \boldsymbol{U}(\theta) \begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix}$$

$$\boldsymbol{U}(\theta) = e^{-i\boldsymbol{S}\cdot\boldsymbol{n}\theta} = \cos\frac{\theta}{2} - i\boldsymbol{n}\cdot\boldsymbol{\sigma}\sin\frac{\theta}{2}$$

Rotation: Spin & Spinor



spin $$\begin{pmatrix} S'_x \\ S'_y \\ S'_z \end{pmatrix} = \boldsymbol{U}\boldsymbol{S}\boldsymbol{U}^\dagger = \begin{pmatrix} \boldsymbol{Q}(\theta) \end{pmatrix} \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}$$



$$\begin{pmatrix} c'_\uparrow \\ c'_\downarrow \end{pmatrix} \qquad \begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix}$$

spinor $$\begin{pmatrix} c'_\uparrow \\ c'_\downarrow \end{pmatrix} = \boldsymbol{U}(\theta) \begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix}$$

$$\boldsymbol{U}(\theta) = e^{-i\boldsymbol{S}\cdot\boldsymbol{n}\theta} = \cos\frac{\theta}{2} - i\boldsymbol{n}\cdot\boldsymbol{\sigma}\sin\frac{\theta}{2}$$

Spin goes

Spinor does not go

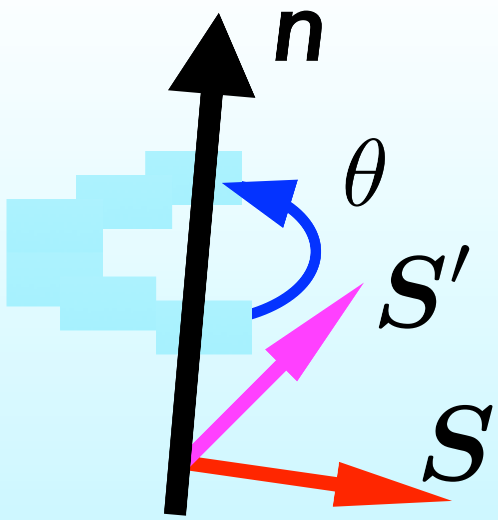
back by 2π rotation

$$\boldsymbol{Q}(\theta + 2\pi) = +\boldsymbol{Q}(\theta)$$

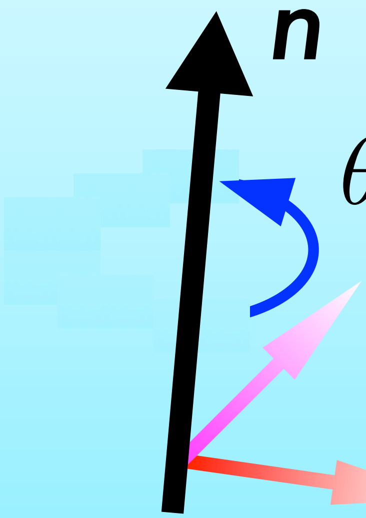
$$\boldsymbol{U}(\theta + 2\pi) = -\boldsymbol{U}(\theta)$$

Classical to Quantum

Rotation: Spin & Spinor



spin
$$\begin{pmatrix} S'_x \\ S'_y \\ S'_z \end{pmatrix} = \boldsymbol{U}\boldsymbol{S}\boldsymbol{U}^\dagger = \begin{pmatrix} & & \\ & Q(\theta) & \\ & & \end{pmatrix} \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}$$



$$\begin{pmatrix} c'_\uparrow \\ c'_\downarrow \end{pmatrix} \qquad \begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix}$$

spinor
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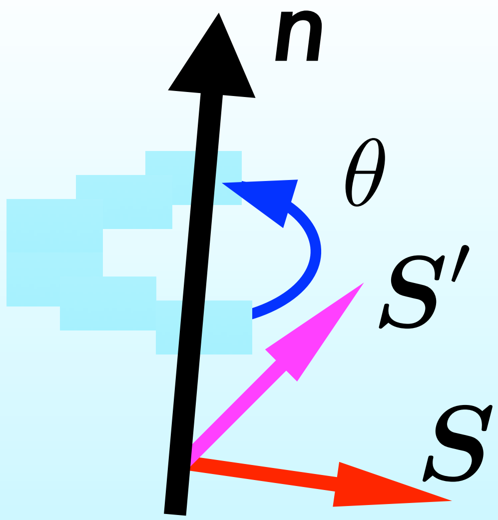
Spin goes
Spinor does not go
back by 2π rotation

$$\boldsymbol{Q}(\theta + 2\pi) = +\boldsymbol{Q}(\theta)$$
$$\boldsymbol{U}(\theta + 2\pi) = -\boldsymbol{U}(\theta)$$

4π is always OK :
continuously deformed
to 0 rotation c.f. 2D

Rotation: Spin & Spinor



spin

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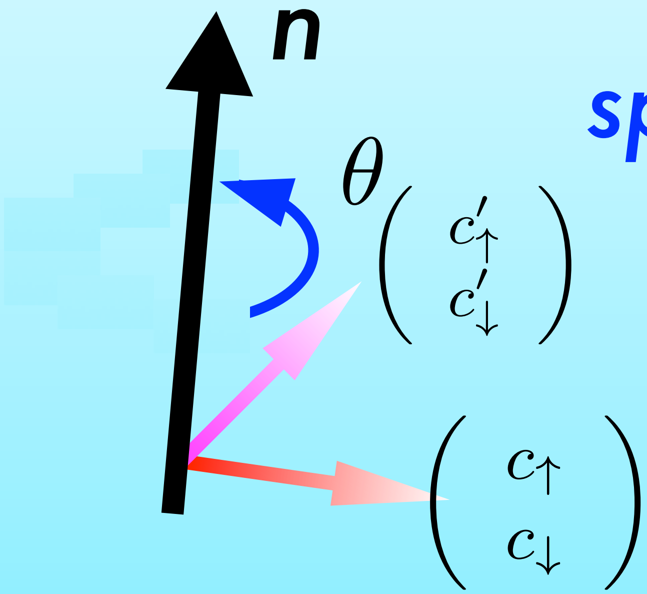

spinor

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Spin goes back by 2π rotation

Spinor does not go

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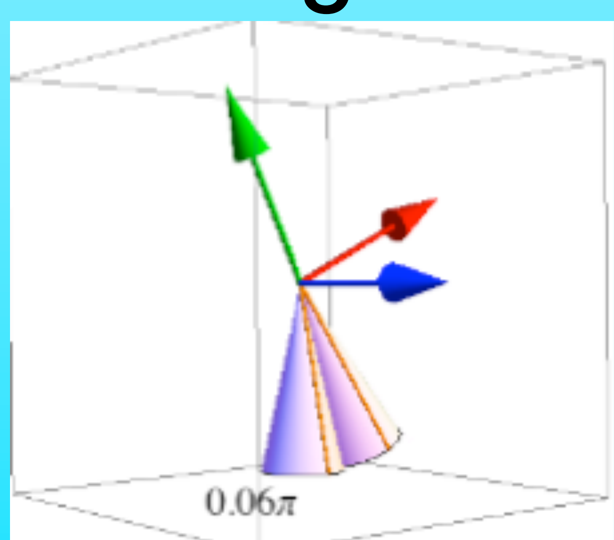
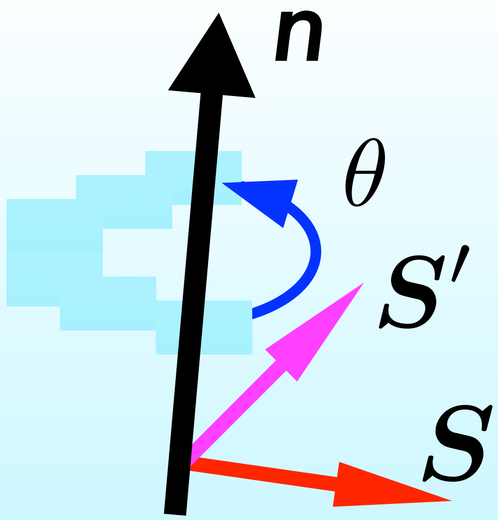


4π is always OK : continuously deformed to 0 rotation c.f. 2D

Rotation: Spin & Spinor



spin

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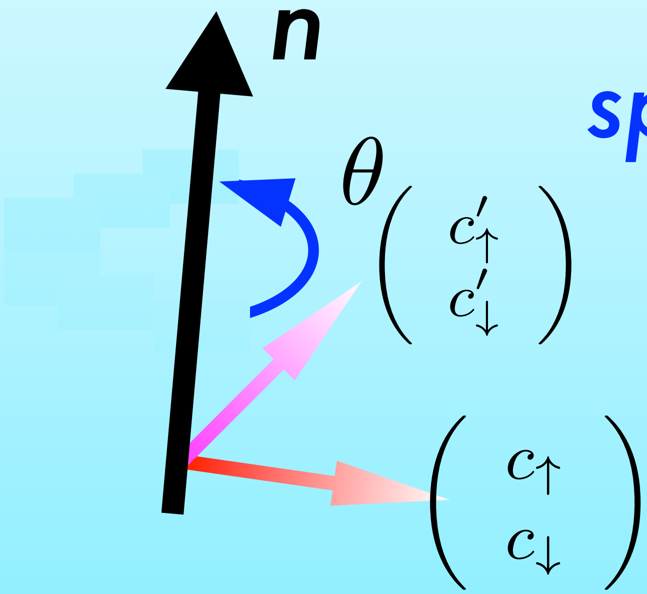

spinor

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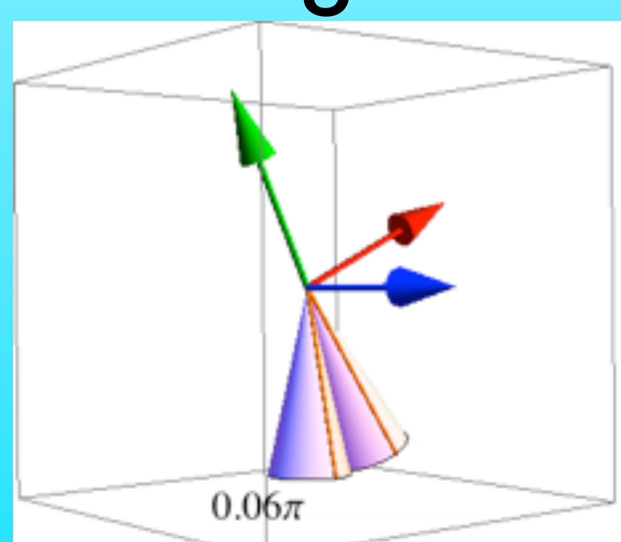
$$\boldsymbol{U}(\theta) = e^{-i\boldsymbol{S}\cdot\boldsymbol{n}\theta} = \cos\frac{\theta}{2} - i\boldsymbol{n}\cdot\boldsymbol{\sigma}\sin\frac{\theta}{2}$$

Spin goes back by 2π rotation
Spinor does not go back by 2π rotation

$$\boldsymbol{Q}(\theta + 2\pi) = +\boldsymbol{Q}(\theta)$$
$$\boldsymbol{U}(\theta + 2\pi) = -\boldsymbol{U}(\theta)$$

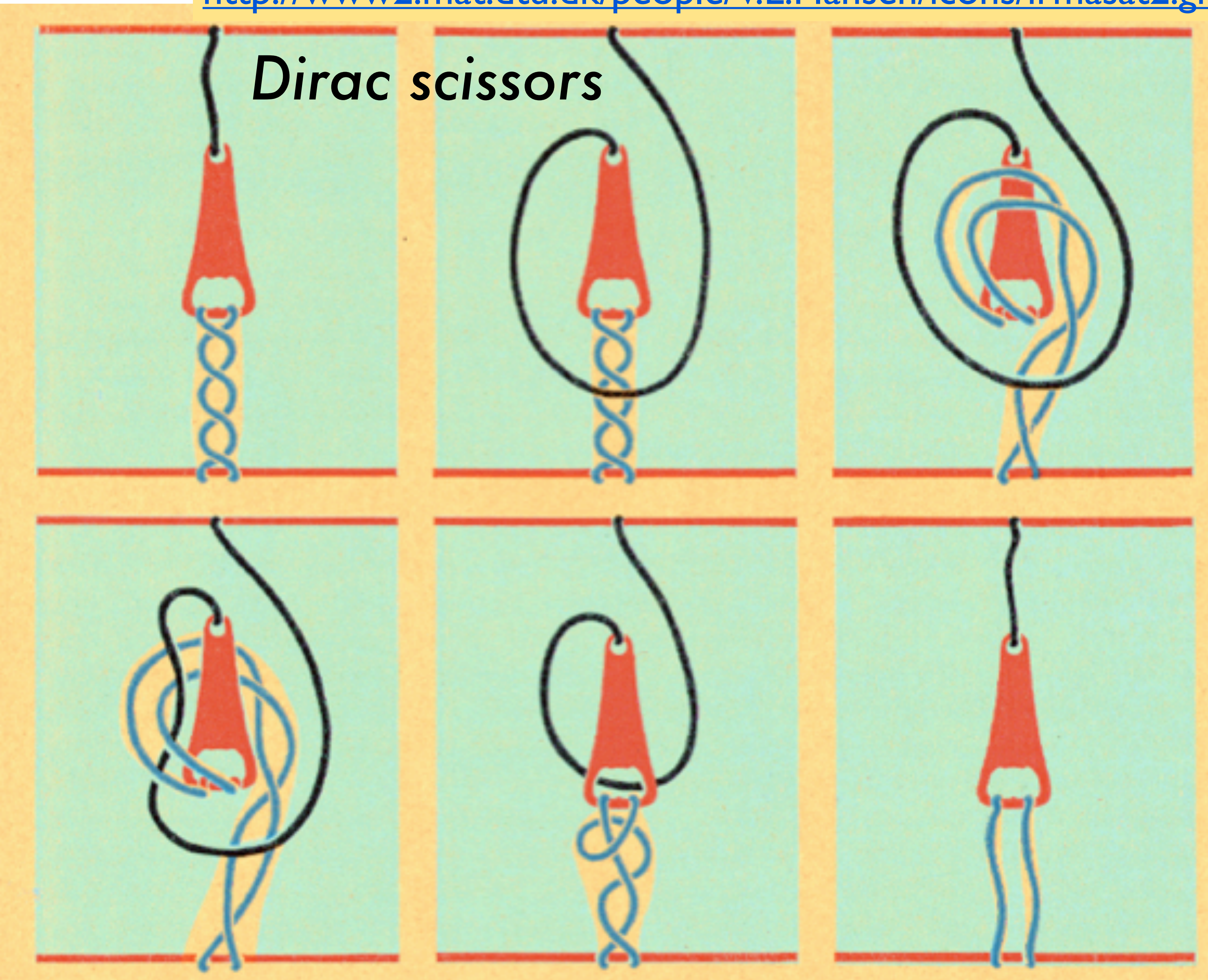


H. Weyl
Penrose-Rindler

4π is always OK :
continuously deformed
to 0 rotation c.f. 2D

& Spinor

http://www2.mat.dtu.dk/people/V.L.Hansen/icons/irmasat2.gif



$$U^\dagger = \begin{pmatrix} Q(\theta) \end{pmatrix} \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}$$

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix}$$

$$\theta = \cos\frac{\theta}{2} - i\boldsymbol{n}\cdot\boldsymbol{\sigma}\sin\frac{\theta}{2}$$

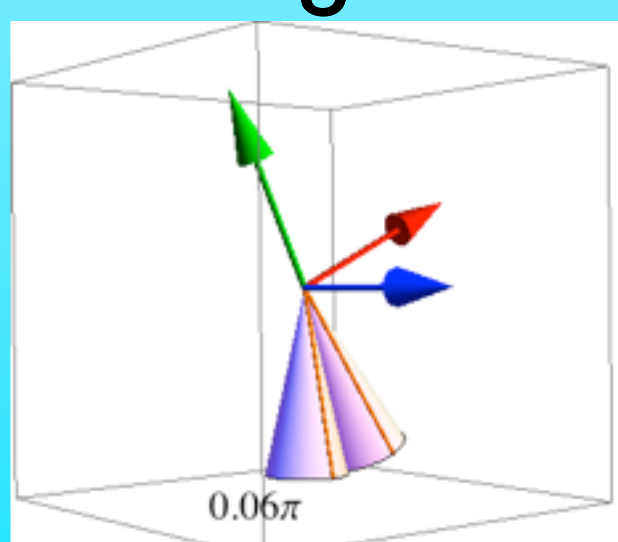
Spin goes back by 2π rotation

Spinor does not go

$$\boldsymbol{Q}(\theta + 2\pi) = +\,\boldsymbol{Q}(\theta)$$

$$\boldsymbol{U}(\theta + 2\pi) = -\,\boldsymbol{U}(\theta)$$

H. Weyl
Penrose-Rindler



4π is always OK :
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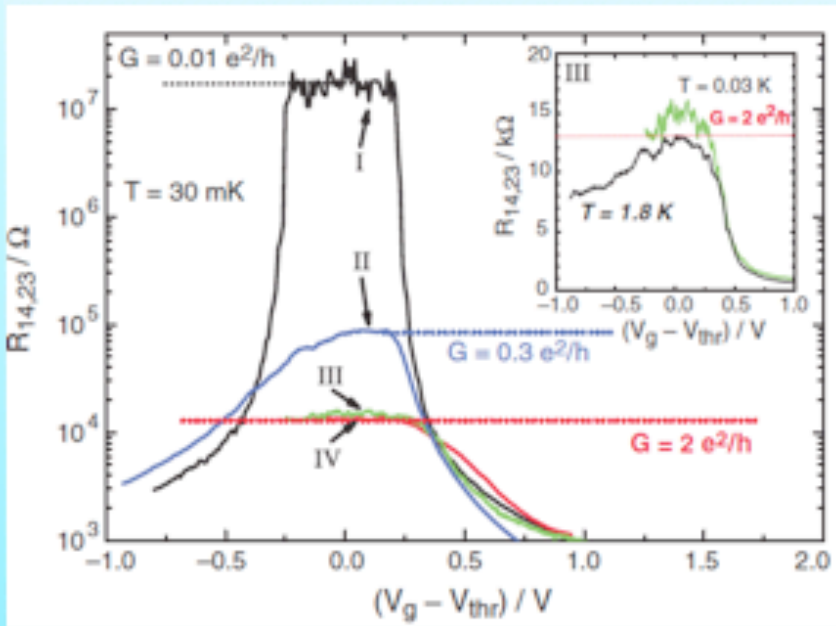
Quantum Spin Hall effect ??

Topological insulator : Quantum Spin Hall state



Quantum Spin Hall Insulator State in HgTe Quantum Wells

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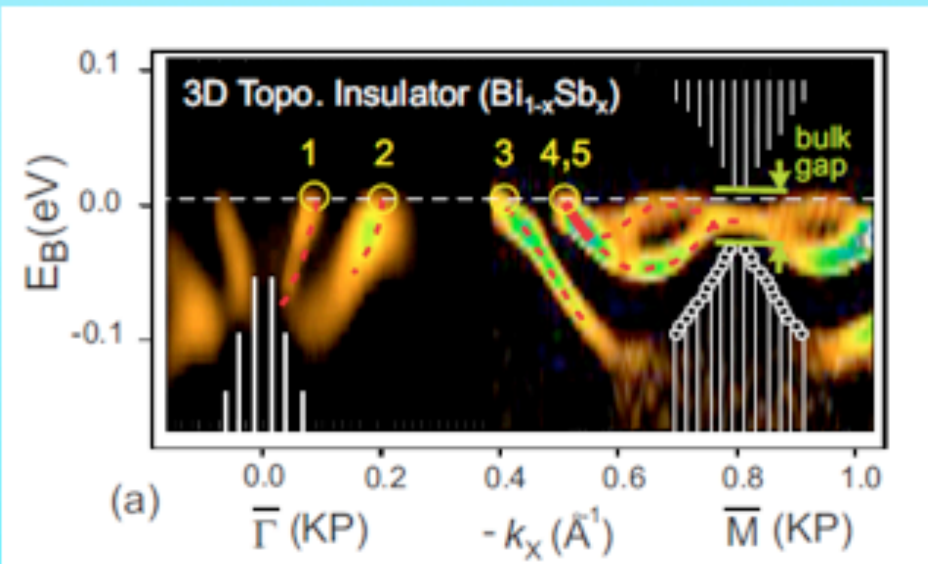
Science **318**, 766 (2007)

2D

TNG



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The next generation

Spin-orbit coupling in some materials leads to the formation of surface states th
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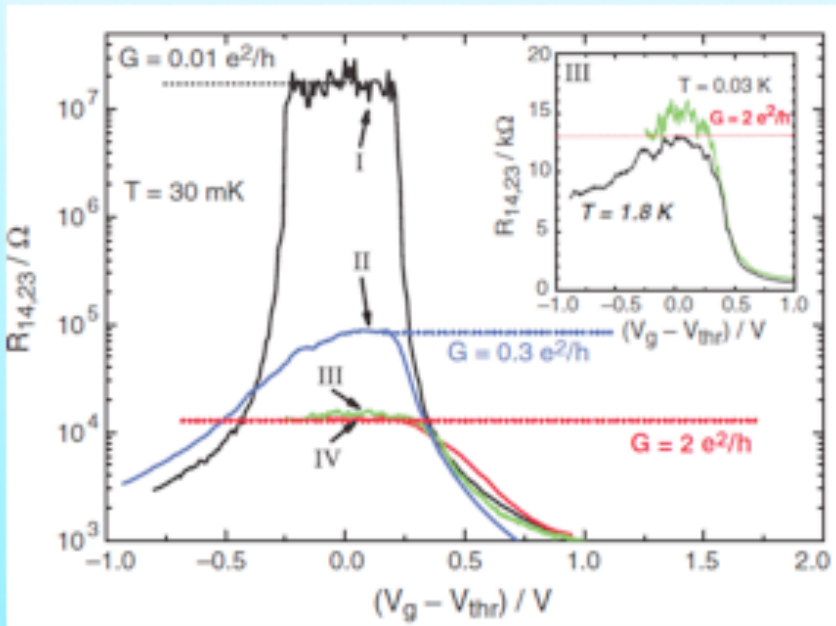
3D

Quantum Spin Hall effect ??

Topological insulator : *Quantum "Spinor" Hall state*



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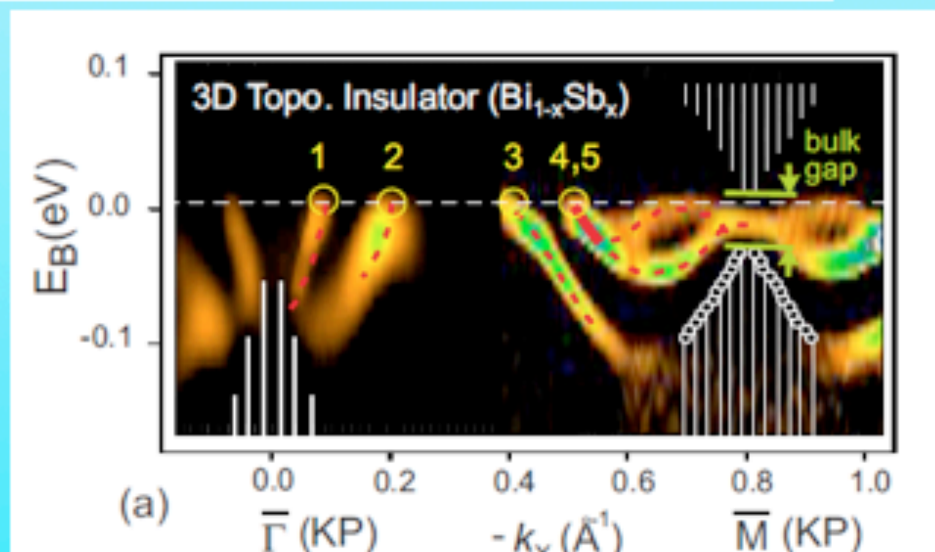

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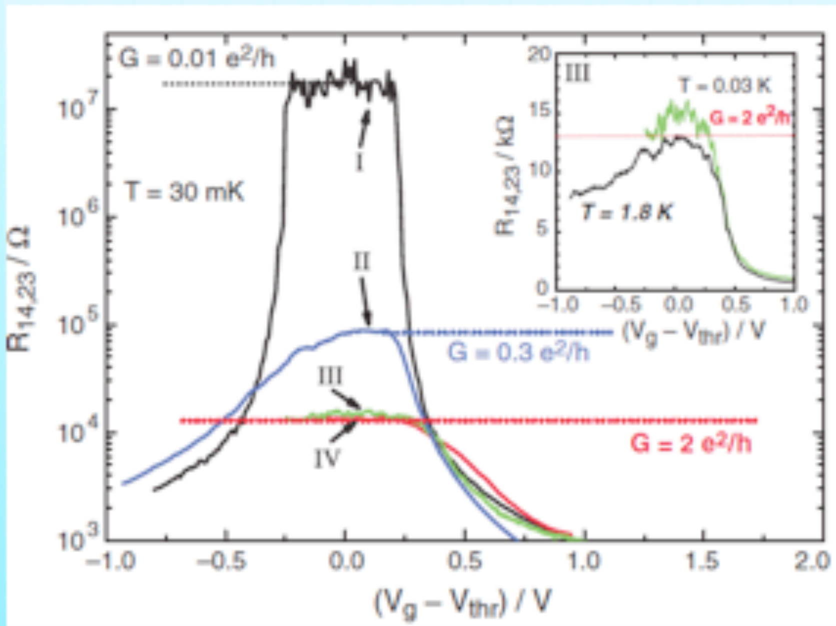

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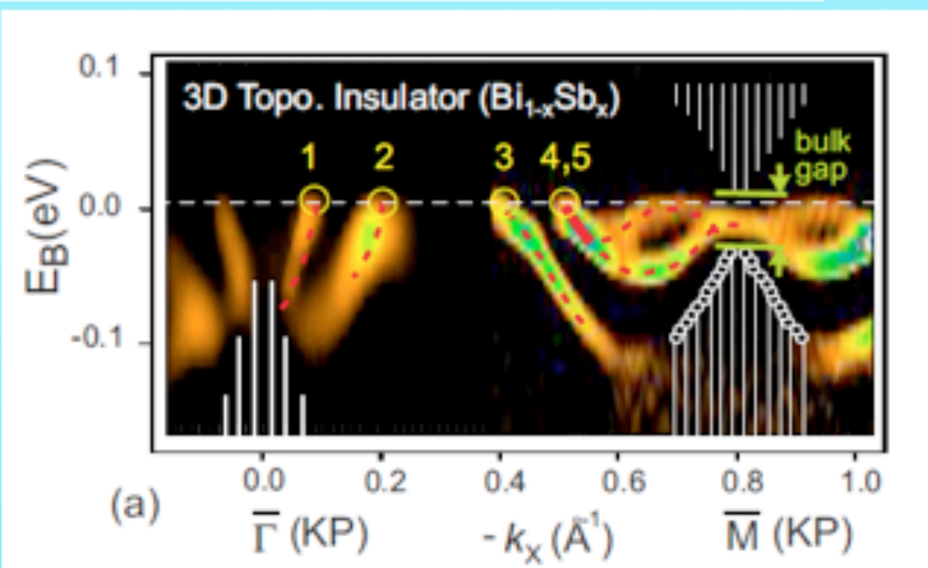

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Purely quantum mechanical !