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力学A レポート

$(A \times B)_i = \varepsilon_{iab} A_a B_b$

$(C \times D)_i = \varepsilon_{icd} C_c D_d$

$(A \times B)\cdot(C \times D) = (A \times B)_i (C \times D)_i$
$= \varepsilon_{iab} A_a B_b \varepsilon_{icd} C_c D_d$
$= \varepsilon_{iab} \varepsilon_{icd} A_a B_b C_c D_d$

$(A\cdot C)(B\cdot D) = \delta_{ac} A_a C_c \delta_{bd} B_b D_d = (\delta_{ac}\delta_{bd}) A_a B_b C_c D_d$
$(A\cdot D)(B\cdot C) = \delta_{ad} A_a D_d \delta_{bc} B_b C_c = (\delta_{ad}\delta_{bc}) A_a B_b C_c D_d$

$\varepsilon_{iab}\varepsilon_{icd} A_a B_b C_c D_d = (\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}) A_a B_b C_c D_d$
ここで、$A_a B_b C_c D_d$:任意
$\Rightarrow \varepsilon_{iab}\varepsilon_{icd}$ 縮約

$= \delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}$

一般に $|\vec{A} \times \vec{B}|^2 = |A \times B|^2 = (A \times B)\cdot(A \times B)$
$= \varepsilon_{iab} A_a B_b \varepsilon_{i} A_c B_d$

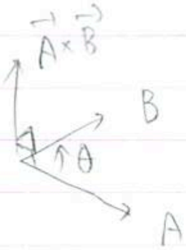
$= (\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}) A_a B_b A_c B_d$
$= A_a B_b A_a B_b - \boxed{A_a} B_b A_b \boxed{B_a}$
$= |A|^2 |B|^2 - (A\cdot B)^2 = |A|^2|B|^2 \cos^2\theta$
$= |A|^2 |B|^2 (1 - \cos^2\theta)$ AとBのなす角 θ
$= |A|^2 |B|^2 \sin^2\theta$

$(A \times B)\cdot A = (\varepsilon_{ijk} A_j B_k)\cdot A_i = \varepsilon_{ijk} A_i A_j B_k$
$= \frac{1}{2}\varepsilon_{ijk}(A_i A_j + A_j A_i) B_k$
$= \frac{1}{2}\varepsilon_{ijk} A_i A_j B_k + \frac{1}{2}\varepsilon_{ijk} A_j A_i B_k$
$= \frac{1}{2}(\varepsilon_{ijk} + \varepsilon_{jik}) A_i A_j B_k = 0$

$\parallel$
0

$A \times B \perp A$
$\perp B$

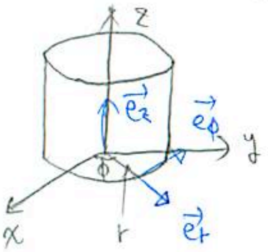
$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$
$\vec{A} \parallel \vec{B} \to \theta = 0, \pi \Rightarrow \vec{A} \times \vec{B} = 0$



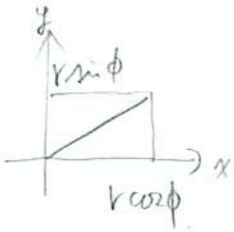
$\vec{e_1} \times \vec{e_2} = \vec{e_3}$ などを考えると
$\vec{A} \times \vec{B}$: $\vec{A} \to \vec{B}$ へ右ねじをまわす時ねじの進む方向

座標変換の例

円柱座標(2次の極座標)



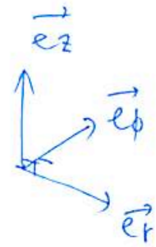
$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$



$x = r\cos\phi$

$y = r\sin\phi$

$z = z$



$$\vec{e}_r = \widehat{\frac{\partial \vec{r}}{\partial r}} = \frac{\frac{\partial \vec{r}}{\partial r}}{\left|\frac{\partial \vec{r}}{\partial r}\right|} \qquad \vec{r} = \vec{e}_1 x + \vec{e}_2 y + \vec{e}_3 z$$

$$= (\vec{e}_1, \vec{e}_2, \vec{e}_3)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = (\vec{e}_1\ \vec{e}_2\ \vec{e}_3)\begin{pmatrix} r\cos\phi \\ r\sin\phi \\ z \end{pmatrix}$$

$$\frac{\partial \vec{r}}{\partial r} = (\vec{e}_1\ \vec{e}_2\ \vec{e}_3)\begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix}$$

$$\left|\frac{\partial \vec{r}}{\partial r}\right|^2 = \cos^2\phi + \sin^2\phi + 0 = 1, \quad \vec{e}_r = (\vec{e}_1\ \vec{e}_2\ \vec{e}_3)\begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix} = \theta\,\mathbf{e}_r, \quad \mathbf{e}_r = \begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix}$$

$$\vec{e}_\phi = \widehat{\frac{\partial \vec{r}}{\partial \phi}} = \frac{\frac{\partial \vec{r}}{\partial \phi}}{\left|\frac{\partial \vec{r}}{\partial \phi}\right|}, \quad \frac{\partial \vec{r}}{\partial \phi} = (\vec{e}_1\ \vec{e}_2\ \vec{e}_3)\begin{pmatrix} -r\sin\phi \\ r\cos\phi \\ 0 \end{pmatrix}$$

$$\left|\frac{\partial \vec{r}}{\partial \phi}\right|^2 = \left|\frac{\partial (r)}{\partial \phi}\right|^2 = r^2\sin^2\phi + r^2\cos^2\phi + 0 = r^2, \quad \left|\frac{\partial \vec{r}}{\partial \phi}\right| = r$$

$$\vec{e}_\phi = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix}, \quad \mathbf{e}_\phi = \begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix} = \theta\,\mathbf{e}_\phi$$

$$\vec{e}_z = \vec{e}_3 = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \theta\,\mathbf{e}_z, \quad \mathbf{e}_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

一般にベクトル $\vec{v}$

$$\vec{v} = \vec{e}_1 v_x + \vec{e}_2 v_y + \vec{e}_3 v_z = (\vec{e}_1\ \vec{e}_2\ \vec{e}_3)\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \theta\,\mathbf{v}$$

$$(\vec{e}_r, \vec{e}_\phi, \vec{e}_z)\begin{pmatrix} v_r \\ v_\phi \\ v_z \end{pmatrix} = \vec{e}_r v_r + \vec{e}_\phi v_\phi + \vec{e}_z v_z$$

$\left(\begin{pmatrix} v_r \\ v_\phi \\ v_z \end{pmatrix}\text{: 円柱座標での成分}\right)$

$$\vec{e}_\phi = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix}, \quad \mathbf{e}_\phi = \begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix}$$

$$= \theta\,\mathbf{e}_\phi$$

$$\vec{e}_z = \vec{e}_3 = (\vec{e}_1\ \vec{e}_2\ \vec{e}_3)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \theta\,\mathbf{e}_z, \quad \mathbf{e}_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$(\vec{e}_r, \vec{e}_\phi, \vec{e}_z) = (\vec{e}_1, \vec{e}_2, \vec{e}_3)(\theta_1, \theta_2, \theta_3) = \theta T_{r\phi z}, \quad T_{r\phi z} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3×3行列

$$\tilde{T}T = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \ \end{pmatrix}$$

一般にベクトル$\vec{v}$

$$\vec{v} = \vec{e}_1 v_x + \vec{e}_2 v_y + \vec{e}_3 v_z = (\vec{e}_1, \vec{e}_2, \vec{e}_3)\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \theta \mathbb{v} = \theta T_{r\phi z}\begin{pmatrix} v_r \\ v_\phi \\ v_z \end{pmatrix}$$

$\vec{r}$に質点が存在している時の速度ベクトル$\vec{V}$

加速 〃 $\vec{a}$

は円柱座標系$\theta_{r\phi z} = (\vec{e}_r, \vec{e}_\phi, \vec{e}_z)$でどう表されるか?

以上を一般のベクトル

$$\vec{r} = \vec{e}_1 x + \vec{e}_2 y + \vec{e}_3 z = \theta \mathbb{r}$$

$$\vec{V} = \dot{\vec{r}} = \theta \dot{\mathbb{r}} = \theta \mathbb{V}$$

$$\mathbb{V} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}, \quad x = r\cos\phi, \quad y = r\sin\phi, \quad z = z$$

r, ϕは時間tの関数

$x = x(t) = r(t)\cos\phi(t)$

$$\dot{x} = \frac{d}{dt}x(t) = \dot{r}\cos\phi - r\dot{\phi}\sin\phi$$

$\dot{y} = \dot{r}\sin\phi + r\dot{\phi}\cos\phi$ よって一般のベクトルの変換則※※より

$$\begin{pmatrix} V_r \\ V_\phi \\ V_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \dot{r}\cos\phi - r\dot{\phi}\sin\phi \\ \dot{r}\sin\phi + r\dot{\phi}\cos\phi \\ \dot{z} \end{pmatrix} = \begin{pmatrix} \dot{r}\cos^2\phi + \dot{r}\sin^2\phi \\ r\dot{\phi}\sin^2\phi + r\dot{\phi}\cos^2\phi \\ \dot{z} \end{pmatrix}$$

$$= \begin{pmatrix} \dot{r} \\ r\dot{\phi} \\ \dot{z} \end{pmatrix}, \quad \vec{V} = (\vec{e}_r, \vec{e}_\phi, \vec{e}_z)\begin{pmatrix} \dot{r} \\ r\dot{\phi} \\ \dot{z} \end{pmatrix}$$

加速度ベクトル

$$\vec{a} = \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} \ddot{r}\cos\phi - \dot{r}\dot{\phi}\sin\phi - \dot{r}\dot{\phi}\sin\phi - r\ddot{\phi} - r\dot{\phi}\dot{\phi}\cos\phi \\ \ddot{r}\sin\phi + \dot{r}\dot{\phi}\cos\phi + r\ddot{\phi}\cos\phi - r\dot{\phi}\dot{\phi}\sin\phi \\ \ddot{z} \end{pmatrix}$$

$$
= \begin{pmatrix} (\ddot{r} - r\dot{\phi}^2)\cos\phi - (2\dot{r}\dot{\phi} + r\ddot{\phi})\sin\phi \\ (2\dot{r}\dot{\phi} + r\ddot{\phi})\cos\phi + (\ddot{r} - r\dot{\phi}^2)\sin\phi \\ \ddot{z} \end{pmatrix}, \quad a = \begin{pmatrix} (\ddot{r} - r\dot{\phi}^2)\cos\phi - (2\dot{r}\dot{\phi} + r\ddot{\phi})\sin\phi \\ (\ddot{r} - r\dot{\phi}^2)\sin\phi + (2\dot{r}\dot{\phi} + r\ddot{\phi})\cos\phi \\ \ddot{z} \end{pmatrix}
$$

＊＊より

$$
\begin{pmatrix} a_r \\ a_\phi \\ a_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \ddot{r} - r\dot{\phi}^2 \\ 2\dot{r}\dot{\phi} + r\ddot{\phi} \\ \ddot{z} \end{pmatrix}
$$

$$
\vec{a} = (\vec{e_r}, \vec{e_\phi}, \vec{e_z}) \begin{pmatrix} \ddot{r} - r\dot{\phi} \\ 2\dot{r}\dot{\phi} + r\ddot{\phi} \\ \ddot{z} \end{pmatrix} \Rightarrow F = (\vec{e_r}, \vec{e_\phi}, \vec{e_z}) \begin{pmatrix} F_r \\ F_\phi \\ F_z \end{pmatrix}
$$

とすれば

$$
F_r = m(\ddot{r} - r\dot{\phi}), \quad F_\phi = m(2\dot{r}\dot{\phi} + r\ddot{\phi}), \quad F_z = m\ddot{z}
$$

円柱座標での運動方程式