# 力学レポート No 1

1.1 $\frac{\partial r}{\partial x} = \frac{1}{2\sqrt{x^2+y^2+z^2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2+z^2}} = \frac{x}{r}$ 同様に $\frac{\partial r}{\partial y} = \frac{y}{\sqrt{x^2+y^2+z^2}} = \frac{y}{r}$, $\frac{\partial r}{\partial z} = \frac{z}{\sqrt{x^2+y^2+z^2}} = \frac{z}{r}$

1.2 $V(r) = -\frac{\mu}{r}$ のとき

$$-\nabla V(\vec{r}) = -\frac{\mu}{r}\nabla = -\mu\begin{pmatrix}\frac{\partial r^{-1}}{\partial x}\\ \frac{\partial r^{-1}}{\partial y}\\ \frac{\partial r^{-1}}{\partial z}\end{pmatrix} = -\mu r^{-2}\begin{pmatrix}\frac{\partial r}{\partial x}\\ \frac{\partial r}{\partial y}\\ \frac{\partial r}{\partial z}\end{pmatrix} = \mu r^{-2}\begin{pmatrix}\frac{x}{r}\\ \frac{y}{r}\\ \frac{z}{r}\end{pmatrix} = \frac{\mu}{r^3}\vec{r} = \frac{\mu}{r^2}\hat{r}$$

1.3 $V(\vec{r}) = -\frac{\mu}{|\vec{r}-\vec{r}'|}$ $|\vec{r}-\vec{r}'| = R$ とおいて $\vec{R} = \vec{r}-\vec{r}'$

$V(\vec{r}) = -\frac{\mu}{R}$ 1.2より $-\nabla V(\vec{r}) = \frac{\mu}{R^2}\hat{R}$ $\vec{F} = \frac{\mu}{R^2}\hat{R}$

1.4 万有引力, クーロン力

1.5 $\vec{F} = -\nabla V_n(\vec{r}) = -\nabla \mu r^n = -\mu\begin{pmatrix}\frac{\partial r^n}{\partial x}\\ \frac{\partial r^n}{\partial y}\\ \frac{\partial r^n}{\partial z}\end{pmatrix}$

$$\frac{\partial r^n}{\partial x} = \frac{n}{2}(x^2+y^2+z^2)^{\frac{n}{2}-1}\cdot 2x = nx(x^2+y^2+z^2)^{\frac{n-2}{2}} = nx r^{n-2}$$

y, zについても同様に $\frac{\partial r^n}{\partial y} = ny r^{n-2}$ $\frac{\partial r^n}{\partial z} = nz r^{n-2}$

よって $\vec{F} = -\mu n r^{n-2}\vec{r}$

$n=2$ のとき $\vec{F} = -2\mu\vec{r}$

常に原点方向に力が加わる運動 単振動

1.6 $\vec{a} = (a, b, c)$ とする $V(\vec{r}) = \vec{a}\cdot\vec{r} = ax+by+cz$

$\vec{F} = -\nabla V(\vec{r}) = (-a, -b, -c) = -\vec{a}$

常に同じ方向に同じ大きさの力がはたらく。 重力

1.7 $E = K + V(\vec{r})$ K: 運動エネルギー なので

$E - V(\vec{r}) = K$ ここで $K \geqq 0$ より $E - V(\vec{r}) \geqq 0$

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II.1 二項定理より

$$(1+x)^n = 1 + nx + \underbrace{\frac{n(n-1)}{2}x^2 + \cdots\cdots}$$

$$x^2\left(\frac{n(n-1)}{2} + \cdots\cdots\right) \quad ここ = \varphi(x) \ とする$$

$$\lim_{x\to 0}\frac{\varphi(x)}{x} = 0 \ , \quad \lim_{x\to 0}\frac{\varphi(x)}{x^2} = \frac{n(n-1)}{2}$$

よって $\varphi(x) = o(x)$ , $O(x^2)$

以上より $(1+x)^n = 1 + nx + o(x) = 1 + nx + O(x^2)$

II.2 $\delta f = \delta f(x) + \delta f(y) + \delta f(z)$ $\left(\begin{array}{l} \delta f(x) = f(x+\delta x, y, z) - f(x, y, z) \\ \delta f(y) = f(x, y+\delta y, z) - f(x, y, z) \\ \delta f(z) = f(x, y, z+\delta z) - f(x, y, z) \end{array}\right)$

$f(x, y, z) = (x+y+z)^3$

$$\delta f = f(x+\delta x, y+\delta y, z+\delta z) - f(x, y, z) = (x+y+z+\delta x+\delta y+\delta z)^3 - (x+y+z)^3$$

$$= (x+y+z)^3 + 3(x+y+z)^2(\delta x+\delta y+\delta z) + 3(x+y+z)(\delta x+\delta y+\delta z)^2 + (\delta x+\delta y+\delta z)^3 - (x+y+z)^3$$

$$= 3(x+y+z)^2(\delta x+\delta y+\delta z) + 3(x+y+z)(\delta x+\delta y+\delta z)^2 + (\delta x+\delta y+\delta z)^3 \quad — ①$$

次の頁1の式で $\delta f = 3(x+y+z)^2\delta x + 3(x+y+z)^2\delta y + 3(x+y+z)^2\delta z + o(\delta x, \delta y, \delta z)$

$$= 3(x+y+z)^2(\delta x+\delta y+\delta z) + o(\delta x, \delta y, \delta z) \quad — ②$$

ここまででokです

ここで ①の $3(x+y+z)(\delta x+\delta y+\delta z)^2 + (\delta x+\delta y+\delta z)^3$ を変形して

$$= (\delta x+\delta y+\delta z)^2\{3(x+y+z) + \delta x+\delta y+\delta z\}$$

$$= o(\delta x+\delta y+\delta z) \quad (\delta x\to 0, \delta y\to 0, \delta z\to 0)$$

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II.3 $$\delta f = \frac{\partial f}{\partial X}\delta X + \frac{\partial f}{\partial Y}\delta Y = \frac{\partial f}{\partial X}\left(\frac{\partial X}{\partial x}\delta x + \frac{\partial X}{\partial y}\delta y\right) + \frac{\partial f}{\partial Y}\left(\frac{\partial Y}{\partial x}\delta x + \frac{\partial Y}{\partial y}\delta y\right)$$

$$= \delta x\left(\frac{\partial f}{\partial X}\frac{\partial X}{\partial x} + \frac{\partial f}{\partial Y}\frac{\partial Y}{\partial x}\right) + \delta y\left(\frac{\partial f}{\partial X}\frac{\partial X}{\partial y} + \frac{\partial f}{\partial Y}\frac{\partial Y}{\partial y}\right) \quad \text{—①}$$

次に $f$ を $x$、$y$ の関数として見ると

$$\delta f = \frac{\partial f}{\partial x}\delta x + \frac{\partial f}{\partial y}\delta y \quad \text{—②}$$

①②より $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial X}\frac{\partial X}{\partial x} + \frac{\partial f}{\partial Y}\frac{\partial Y}{\partial x}$ $\quad \frac{\partial f}{\partial y} = \frac{\partial f}{\partial X}\frac{\partial X}{\partial y} + \frac{\partial f}{\partial Y}\frac{\partial Y}{\partial y}$

II.4 $f(X_i) = f(X_1, X_2, \cdots, X_n)$ $\quad X_i(x_j) = X_i(x_1, x_2, x_3, \cdots, x_m)$

アインシュタインの記法なので $\frac{\partial f}{\partial x_j} = \frac{\partial f}{\partial x_1} + \frac{\partial f}{\partial x_2} + \frac{\partial f}{\partial x_3} + \cdots + \frac{\partial f}{\partial x_m}$ と変形できる

II.3より $$\frac{\partial f}{\partial x_1} = \frac{\partial f}{\partial X_1}\frac{\partial X_1}{\partial x_1} + \frac{\partial f}{\partial X_2}\frac{\partial X_2}{\partial x_1} + \cdots + \frac{\partial f}{\partial X_n}\frac{\partial X_n}{\partial x_1}$$

$$= \frac{\partial f}{\partial X_i}\frac{\partial X_i}{\partial x_1}$$

よって $$\frac{\partial f}{\partial x_j} = \frac{\partial f}{\partial X_i}\frac{\partial X_i}{\partial x_1} + \frac{\partial f}{\partial X_i}\frac{\partial X_i}{\partial x_2} + \cdots + \frac{\partial f}{\partial X_i}\frac{\partial X_i}{\partial x_m}$$

$$= \frac{\partial f}{\partial X_i}\frac{\partial X_i}{\partial x_j}$$

II.5 $\vec{r} = (x, y, z)$ とおくと $\vec{v} = \dot{\vec{r}}$ より $\vec{v} = (\dot{x}, \dot{y}, \dot{z})$

$f$ を $x, y, z$ の関数と考えて

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial t}$$

$x, y, z$ はそれぞれ $t$ のみによって定まるものなので

$$\frac{\partial x}{\partial t} = \frac{dx}{dt} = \dot{x} \quad \frac{\partial y}{\partial t} = \frac{dy}{dt} = \dot{y} \quad \frac{\partial z}{\partial t} = \frac{dz}{dt} = \dot{z}$$

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よって $$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\dot{x} + \frac{\partial f}{\partial y}\dot{y} + \frac{\partial f}{\partial z}\dot{z}$$
$$= \left(\frac{\partial}{\partial x}\dot{x} + \frac{\partial}{\partial y}\dot{y} + \frac{\partial}{\partial z}\dot{z}\right) f$$
$$= (\vec{v} \cdot \vec{\nabla}) f$$

II 6　ポテンシャル力なので $F(\vec{r}) = -\vec{\nabla} V(\vec{r})$ となる $V(\vec{r})$ が存在する

質点の位置 $\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$　速度：$\vec{v} = \dot{\vec{r}} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}$　加速度：$\vec{a} = \dot{\vec{v}}$

$$F(\vec{r})\vec{v} = m\vec{a}\cdot\vec{v} = m\dot{\vec{v}}\cdot\vec{v} = \frac{d}{dt}\frac{1}{2}m|\vec{v}|^2 = \frac{d}{dt}K$$ 　$K = \frac{1}{2}m|\vec{v}|^2$：運動エネルギー

$$\int_{t_1}^{t_2} F(\vec{r})\vec{v}\,dt = \int_{t_1}^{t_2} \frac{d}{dt}K\,dt = K_{(t_2)} - K_{(t_1)}$$ 　$\Delta K = K_{(t_2)} - K_{(t_1)}$ とする
$$= \Delta K$$

また $$\int_{t_1}^{t_2} F(\vec{r})\vec{v}\,dt = \int_{t_1}^{t_2} (-\vec{\nabla} V(\vec{r}))\cdot\vec{v}\,dt$$

ここで $\vec{\nabla} V(\vec{r}) = \begin{pmatrix} \frac{\partial V}{\partial x} \\ \frac{\partial V}{\partial y} \\ \frac{\partial V}{\partial z} \end{pmatrix}$ より

$$\int_{t_1}^{t_2} (-\vec{\nabla} V(\vec{r}))\cdot\vec{v}\,dt = -\int_{t_1}^{t_2}\left(\frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial y}\frac{dy}{dt} + \frac{\partial V}{\partial z}\frac{dz}{dt}\right)dt$$

微小量に関する関係式より $$\delta V = \frac{\partial V}{\partial x}\delta x + \frac{\partial V}{\partial y}\delta y + \frac{\partial V}{\partial z}\delta z$$

両辺 $\delta t$ でわると $$\frac{\delta V}{\delta t} = \frac{\partial V}{\partial x}\frac{\delta x}{\delta t} + \frac{\partial V}{\partial y}\frac{\delta y}{\delta t} + \frac{\partial V}{\partial z}\frac{\delta z}{\delta t}$$

$\delta t \to 0$ のとき $\delta x \to 0$, $\delta y \to 0$, $\delta z \to 0$ なので $\delta V \to 0$
$$\frac{dV}{dt} = \frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial y}\frac{dy}{dt} + \frac{\partial V}{\partial z}\frac{dz}{dt}$$

よって $$-\int_{t_1}^{t_2}\left(\frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial y}\frac{dy}{dt} + \frac{\partial V}{\partial z}\frac{dz}{dt}\right)dt = -\int_{t_1}^{t_2}\frac{dV}{dt}\,dt = -\left(V_{(t_2)} - V_{(t_1)}\right)$$

以上より $\Delta K = -\Delta V$　$\Delta K + \Delta V = 0$　$K + V = E$：力学的エネルギー より
$\Delta E = 0$　よって力学的エネルギーは保存量である