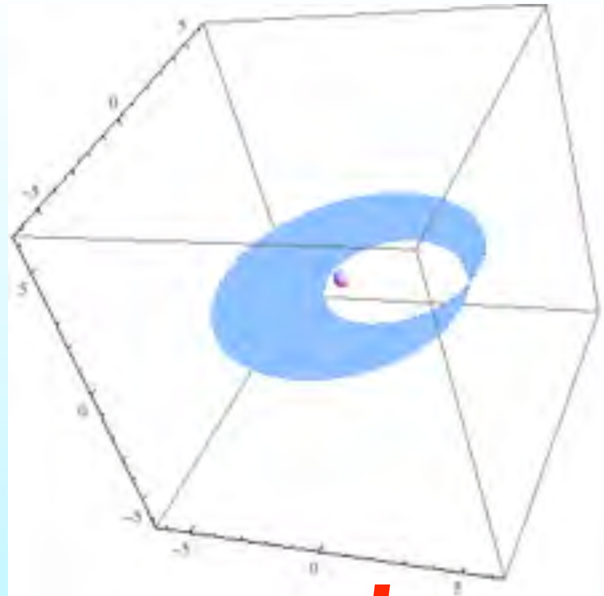
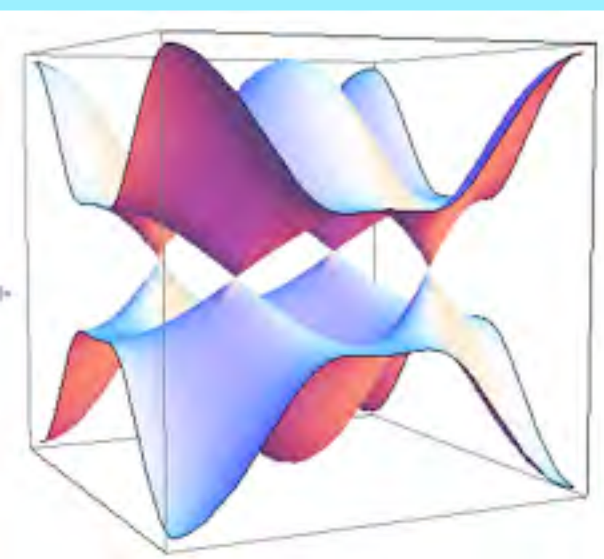
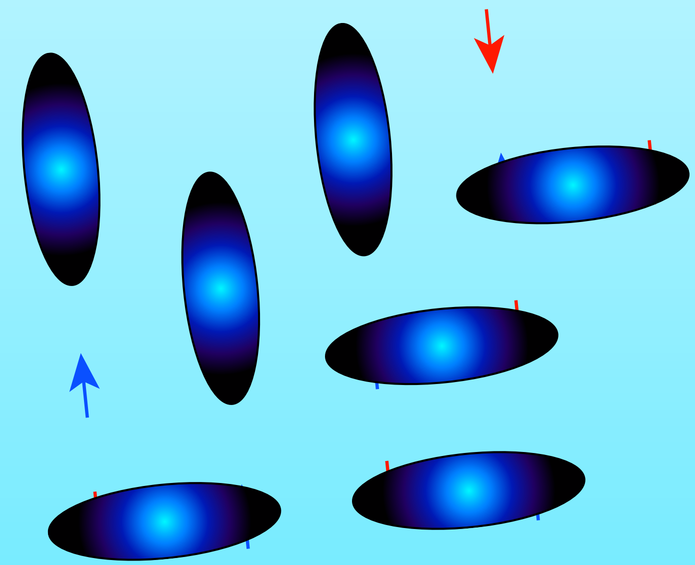
千葉大学理学部集中講義　2015年7月9日-10日



Symmetry protections for topological phases





Y. Hatsugai
Institute of Physics, Univ. Tsukuba

Plan

- Why topological ?
  - Novel phases without symmetry breaking
  - Why Symmetry ?
- Symmetry protection of gap nodes
  - Dimension & co-dimension
  - Anisotropic superconductivity/fluidity & graphene
- Topological order parameter by quantum interference
  - Berry connection: Z_2 Berry phases & Chern number
  - Successful examples

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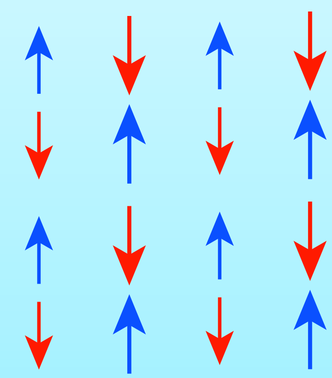
  - Successful examples
- Zoo of edge states as topological order parameters
  - Bulk-edge correspondence
  - Examples : Zoo of what we care.

Why do we care topological phases ?

Characterization of phases

too much success

- Ginzburg-Landau theory
  - Local order parameter: $\langle \boldsymbol{S}(r) \rangle$
  - Symmetry breaking $\langle \boldsymbol{S}(r) \rangle \neq 0$



Magnetism ... perconductivity, charge/orbital ordering ...

Is ... isfactory ?

Quantum ... ids

Absence of symmetry b... aking

need something more:

Topological !

Quantum/Spin Liquids ?

- Quantum Liquids in Low Dimensional Quantum Systems
  - Low Dimensionality, Quantum Fluctuations
  - No (fundamental) Symmetry Breaking
  - No Local Order Parameter

New Type of Order
Topological Order!
X.-G.Wen '89

- Quantum Liquids in Condensed Matter
  - Integer & Fractional Quantum Hall States
  - Dimer Models of Fermions and Spins
  - Half filled Kondo Lattice
  - Kitaev model & Levin-Wen model
  - Anisotropic superfluids/superconductors (ABM, BW, p-wave)
  - Graphene, Weyl semi-metal
  - Topological insulators : quantum spin Hall states
  - Photonic crystals & Some of cold atoms ..

Quantum Liquids ?

- Quantum Liquids in Low Dimensional Quantum Systems
  - Low Dimensionality, Quantum Fluctuations
  - N[illegible]ntal) Sy[illegible]king
  - N[illegible]der Para[illegible]

Gapped

Gapless

New Type of Order

Topological Order!

X.-G.Wen '89

- Quantum Liquids in Condensed Matter
  - Integer & Fractional Quantum Hall States — Gapped
  - Dimer Models of Fermions and Spin[illegible] — Gapped
  - Half filled Kondo Lattice — Gapped
  - Kitaev model & Levin-Wen model — Gapped/Gapless
  - Anisotropic superfluids/superconductors (ABM, BW, p-wave)
  - Graphene, Weyl semi-metal — Gapless
  - Topological insulators : quantum spin Hall states — Gapped
  - Photonic crystals & Some of cold atoms ..

Quantum Liquids are Featureless !!

A phase without symmetry breaking is interesting ?

Are there something to be learned ?

Too much general is boring !

Nothing to be characterized

in sufficiently high dimensions

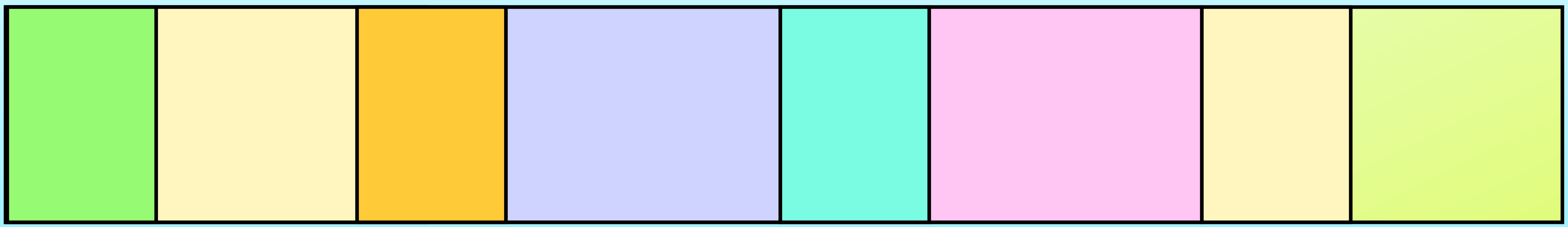
SYMMETRY & DIMENSION constrains !

Symmetry protection of
Topological Phases
without symmetry breaking

"TRULY GENERIC" phase without any symmetry breaking

topologically single phase (too simple ?)

With some symmetry A, B, C



YH, '06

Chen-Gu-Wen, '10
Pollmann et al., '10

1. Discrete symmetry
   - ☑ Time reversal
   - ☑ Charge conjugation
   - ☑ Space inversion
   - ☑ Reflection
2. Gauge symmetry
   - ☑ U(1) : QHE (TR ×)
   - ☑ Sp(1) : QSHE (TR ○)

How to characterize the phase Without Symmetry Breaking ?

Try to show overview

Stability against for perturbation !

Gapless

Gapped

✓ Nodes structures

point nodes, line nodes,...

✓ Adiabatic invariants

Chern numbers, Z_Q Berry phases

Protected by symmetry

✓ Bulk-edge correspondence

geometrically induced gapless excitations in gapped phase

Plan

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Gapless

- Topological order parameter by quantum interference
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  - Successful examples
- Zoo of edge states as topological order parameters
  - Bulk-edge correspondence
  - Examples : Zoo of what we care.

Gapless

Topological !

Single particle problem (mean field)

Nodes structures

point nodes, line nodes,...

protected by symmetry

gapless : generic 2 levels near the gap

von Neumann-Wigner '29
Berry '84

$$H(\boldsymbol{k}) = \boldsymbol{R}(\boldsymbol{k}) \cdot \boldsymbol{\sigma} = \begin{pmatrix} R_z & R_x - iR_y \\ R_y + iR_y & -R_z \end{pmatrix}$$

expanded by Pauli matrices

$$E = \pm |\boldsymbol{R}(\boldsymbol{k})|$$

3 parameters

$$(R_x, R_y, R_z)$$

gapless point $\boldsymbol{R} = 0$

To be gapless: 3 parameters to be tuned

co-dimension=3 (3 conditions)

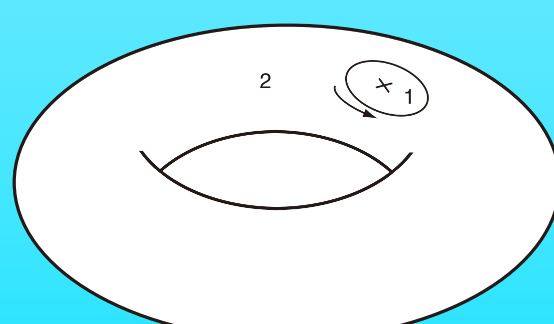
2-D closed surface in 3D

ex.

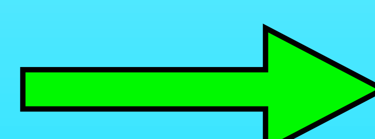
2D Brillouin zone

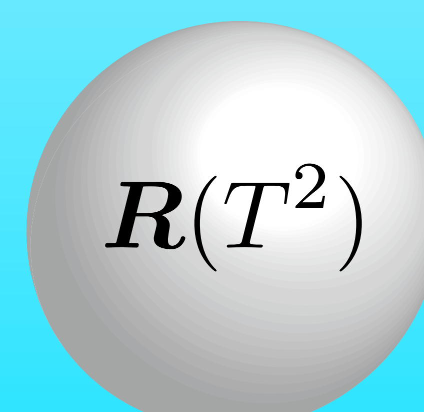
2D Torus T^2

:periodic in k_x & k_y



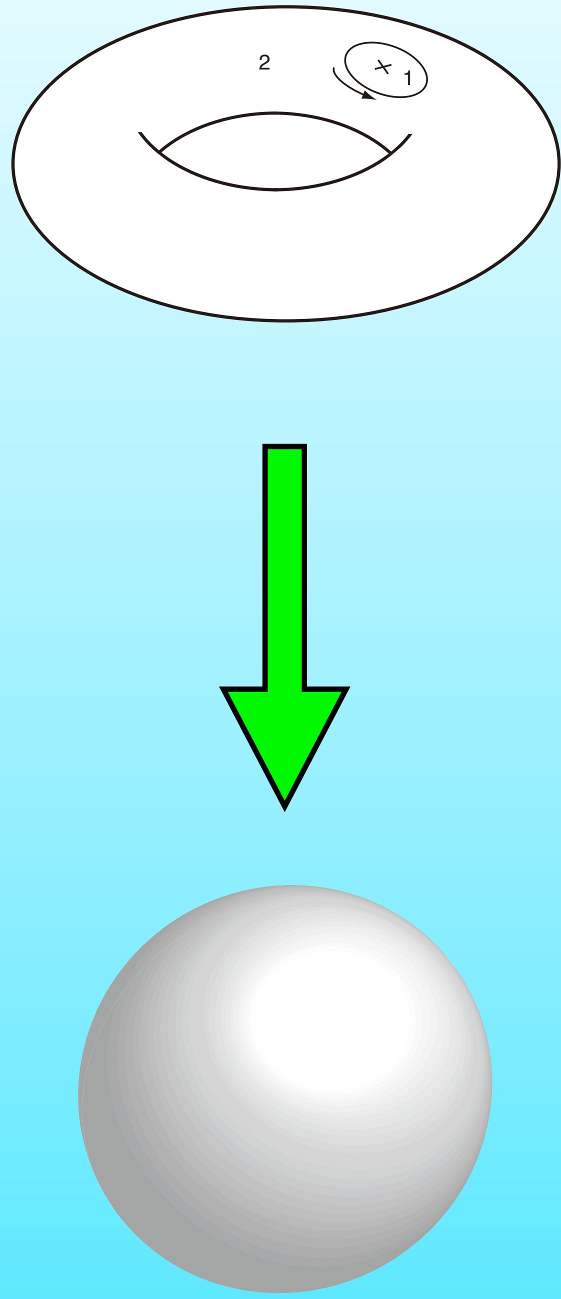
map





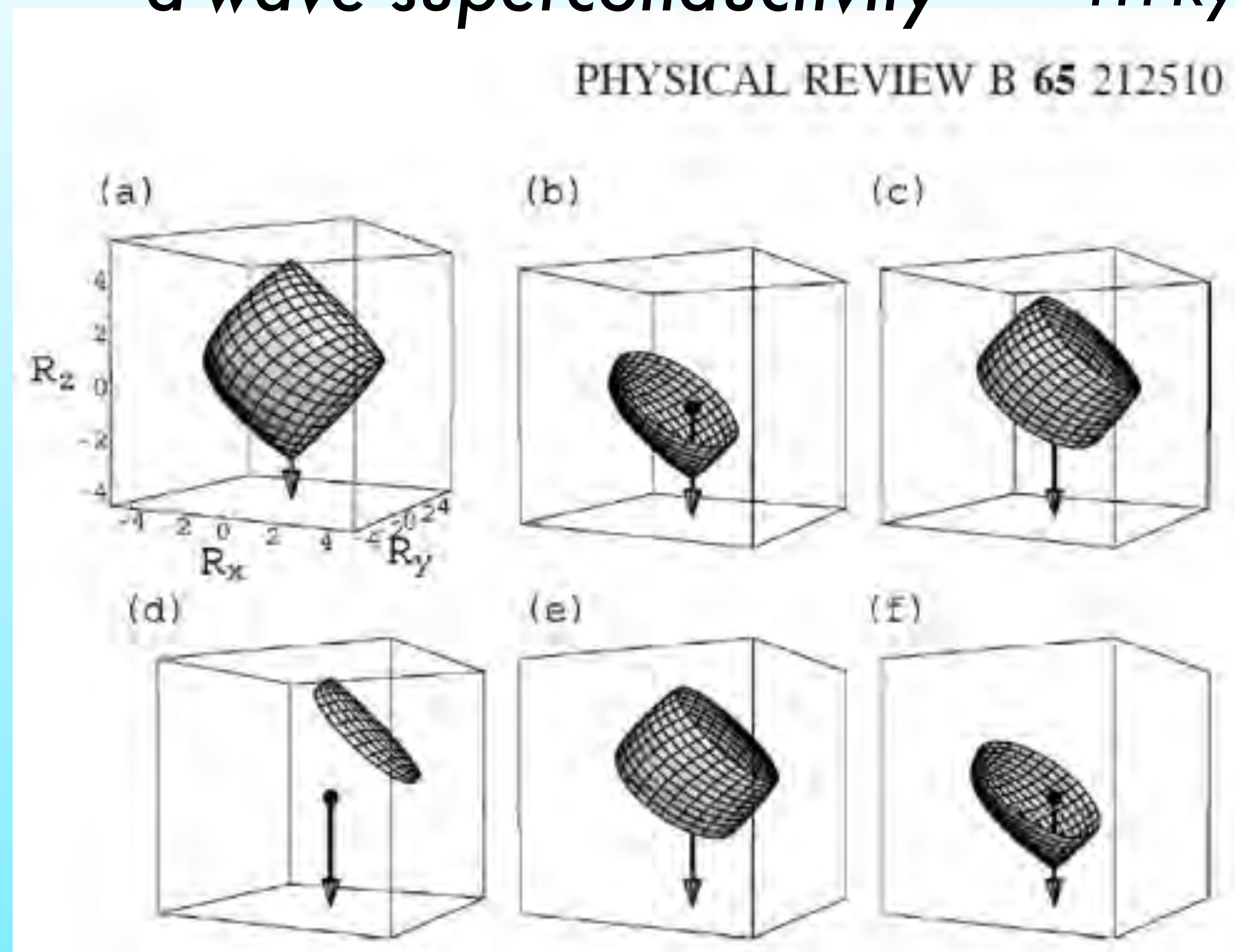
2D examples

2D Brillouin zone

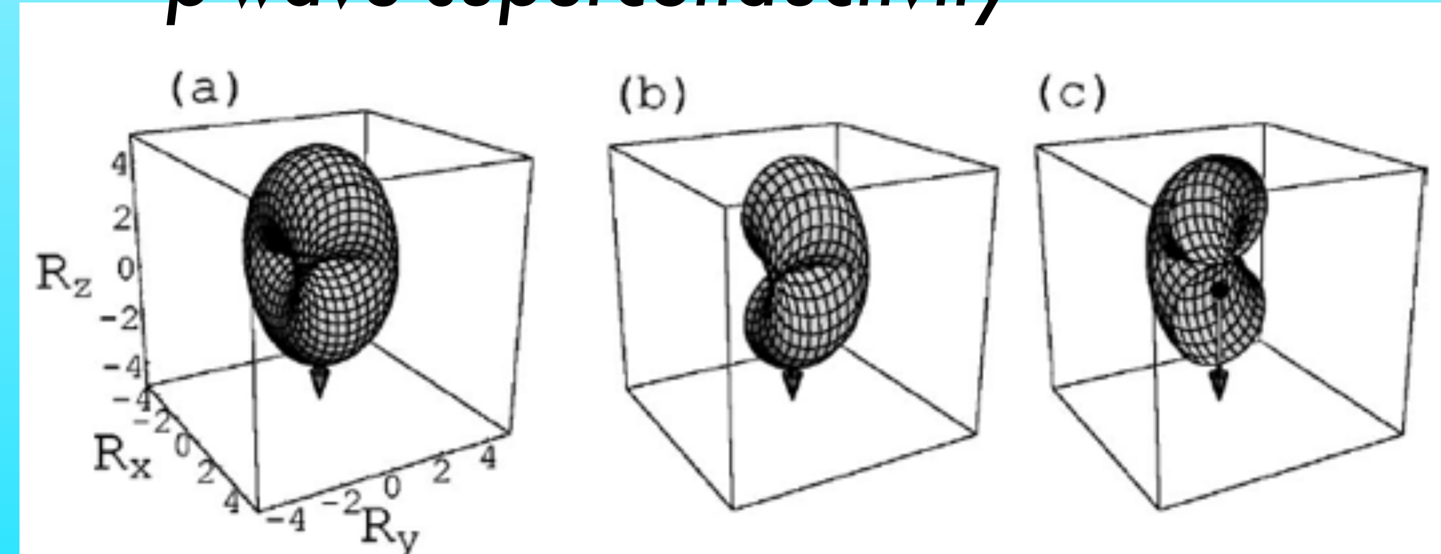


$$H(\boldsymbol{k}) = \boldsymbol{R}(\boldsymbol{k}) \cdot \boldsymbol{\sigma} = \begin{pmatrix} R_z & R_x - iR_y \\ R_y + iR_y & -R_z \end{pmatrix}$$

d-wave superconductivity *YH-Ryu, '02*

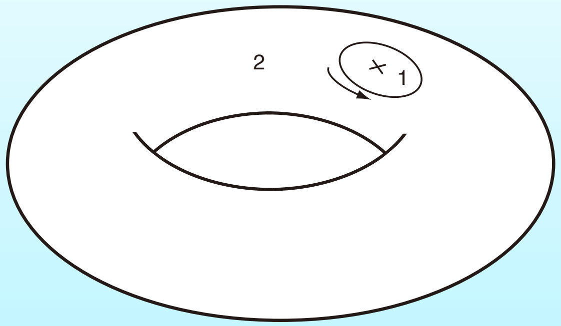


p-wave superconductivity

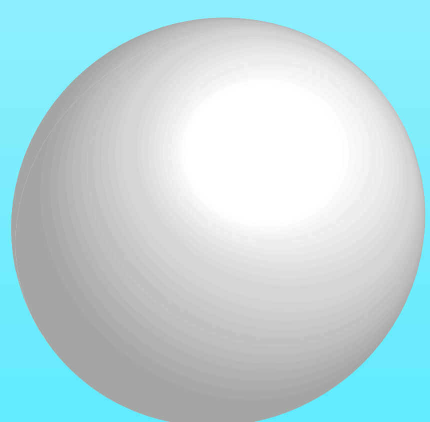


ABM states & Dirac mono pole

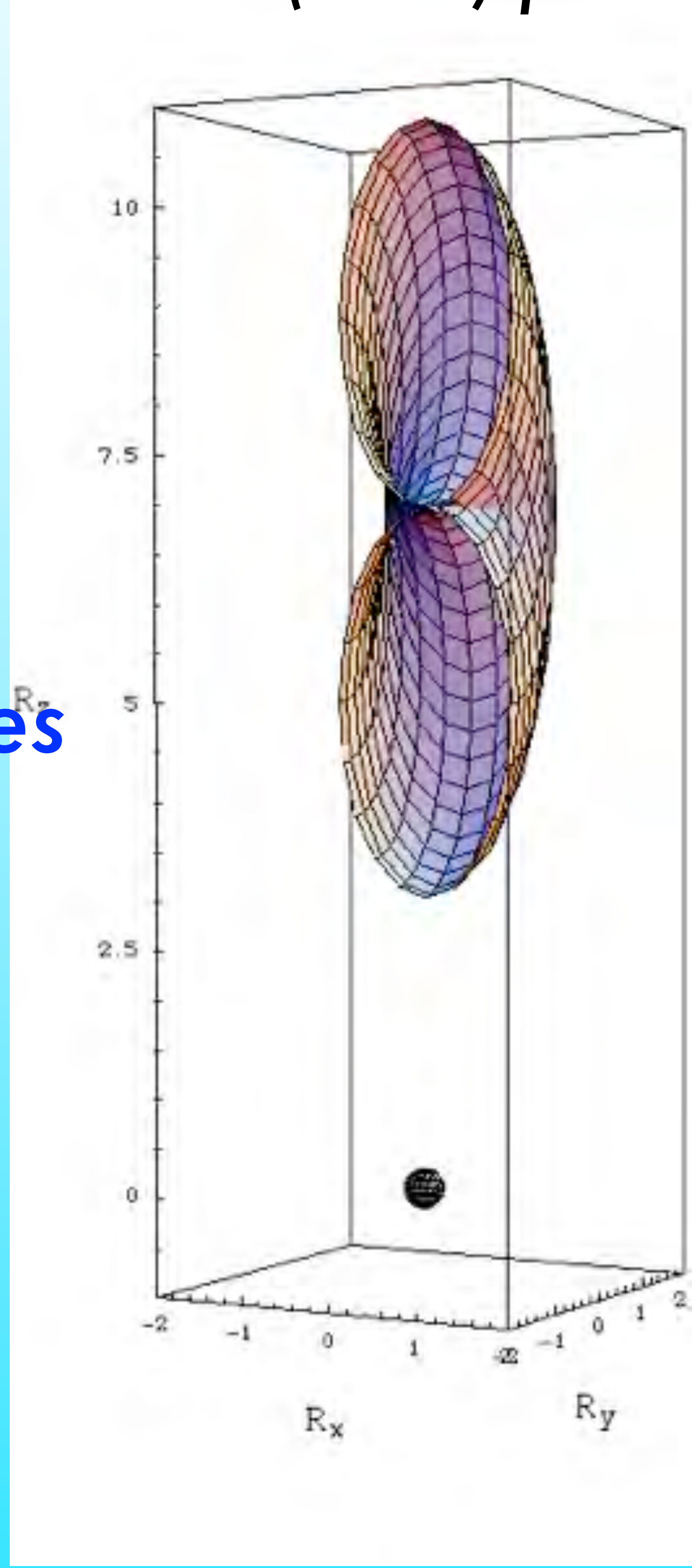
2D Brillouin zone



3rd momentum: time line



Anderson-Brinkman-Morel (ABM) phase of He



YH-Ryu-Kohmoto, '04

co-dimension 3

In 3D, 3–3=0 : point nodes

topological stability

$$H(\boldsymbol{k}) = \boldsymbol{R}(\boldsymbol{k}) \cdot \boldsymbol{\sigma} = \begin{pmatrix} R_z & R_x - iR_y \\ R_y + iR_y & -R_z \end{pmatrix}$$

Geometrical meaning of Chiral symmetry

$$H(\boldsymbol{k}) = \boldsymbol{R}(\boldsymbol{k})\cdot\boldsymbol{\sigma} = \begin{pmatrix} R_z & R_x - iR_y \\ R_y + iR_y & -R_z \end{pmatrix} \qquad E = \pm|\boldsymbol{R}(\boldsymbol{k})|$$

3D (R_x, R_y, R_z)

Chiral symmetry

$$\{H_{\rm eff}, {}^{\exists}\gamma\} = H_{\rm eff}\gamma + \gamma H_{\rm eff} = 0 \qquad \gamma^2 = 1$$

$$\gamma = \begin{cases} \sigma_z & \text{: bipartite lattice \& hopping between them} \\ \sigma_y & \text{: real } H_{\rm eff} \text{ : Time reversal \& Inversion} \end{cases}$$

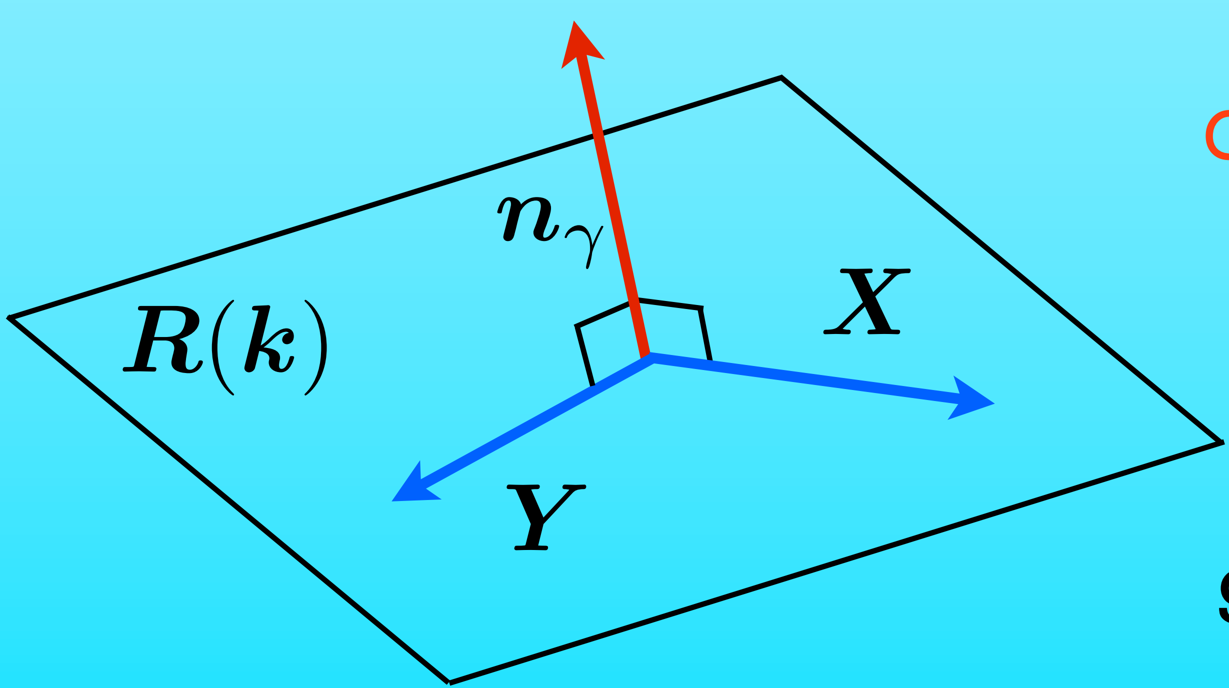
$R_z = 0$

$R_y = 0$

Generically

$$\gamma = \boldsymbol{n}_\gamma\cdot\boldsymbol{\sigma} \quad \{H_{\rm eff}, \gamma\} = 0 \rightleftarrows \boldsymbol{n}_\gamma \perp \boldsymbol{R}$$

Zero gap condition: Dirac dispersion $\quad H_{\rm eff}\to 0,\ \boldsymbol{k}\to\boldsymbol{k}_0$



Chiral Symmetry

$$\{H, {}^{\exists}\gamma\} = 0, \quad \gamma^2 = 1$$

co-dimension of Dirac cones=2

graphene, d-wave superconductor in 2D

Topological stability of the Doubled Dirac cones

c.f. 4D graphene & chiral symmetry, M. Creutz '08
also with TR inv. 5D YH, '10

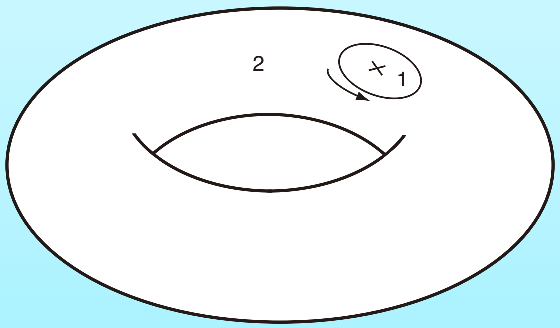
2D Nielsen-Ninomiya theorem

$$H(\boldsymbol{k}) = \boldsymbol{R}(\boldsymbol{k}) \cdot \boldsymbol{\sigma} = \begin{pmatrix} R_z & R_x - iR_y \\ R_y + iR_y & -R_z \end{pmatrix}$$

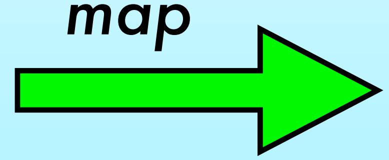
3D (R_x, R_y, R_z)

2D Brillouin zone :periodic in k_x & k_y

2D Torus T^2

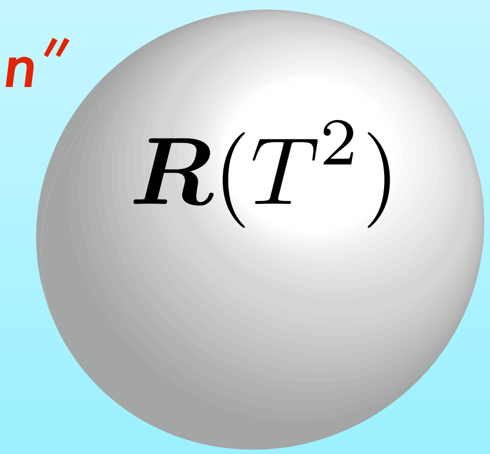


map



Generically 2-D closed surface in 3D

"balloon"



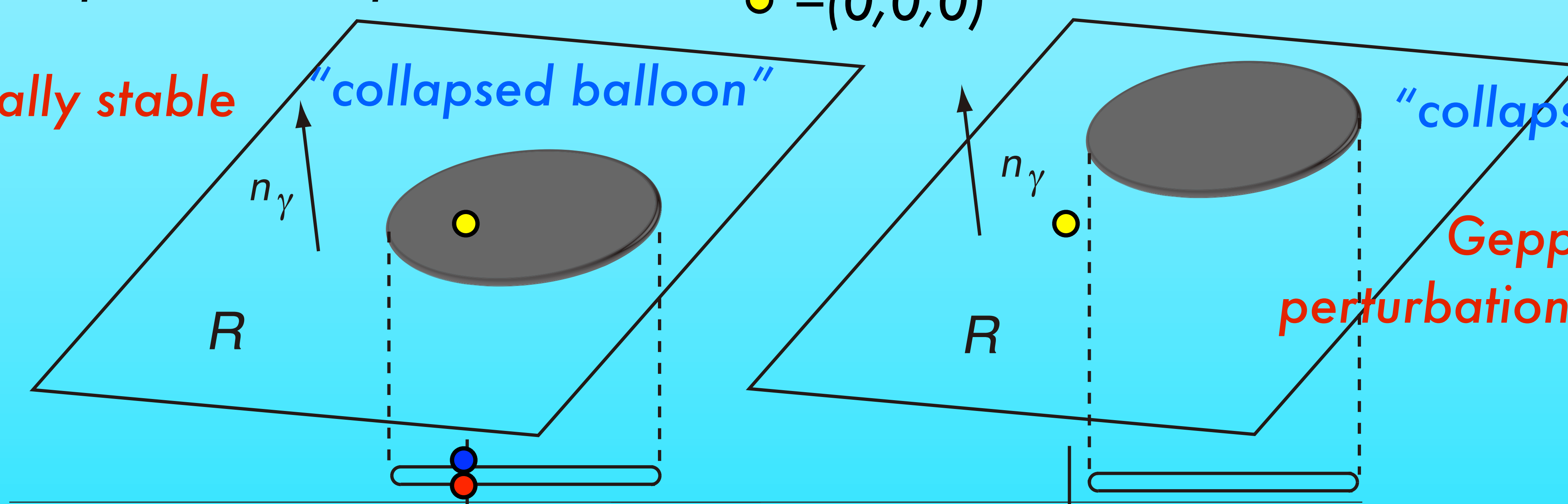
Chiral symmetry $\{H, \gamma\} = 0 \rightleftarrows \boldsymbol{n}_\gamma \perp \boldsymbol{R}$

$\gamma = \boldsymbol{n}_\gamma \cdot \boldsymbol{\sigma}$ $\boldsymbol{R}(\boldsymbol{k})$ is on a plane normal to $\boldsymbol{n}_\gamma$

$\boldsymbol{R}(T^2)$ is collapsed on the plane

=(0,0,0)

Topologically stable

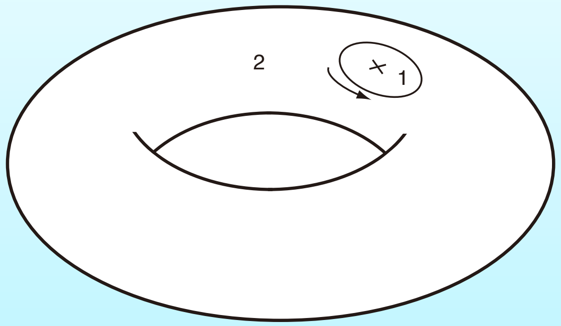


doubled Dirac cones

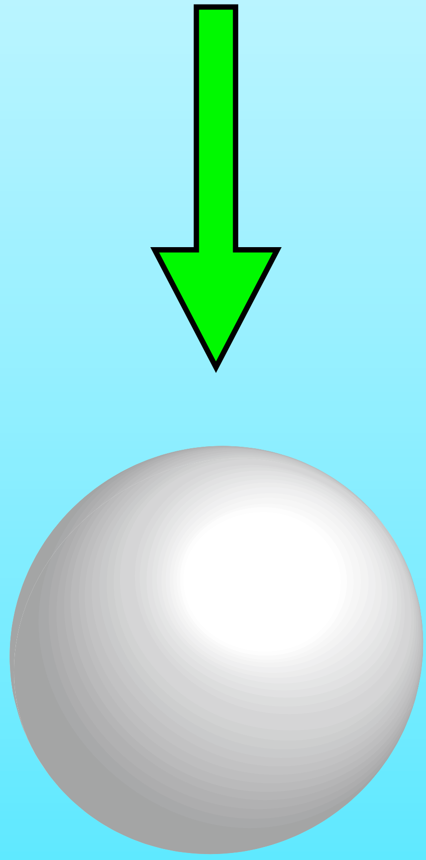
YH-Fukui-Aoki, '06

Graphene with deformation

2D Brillouin zone

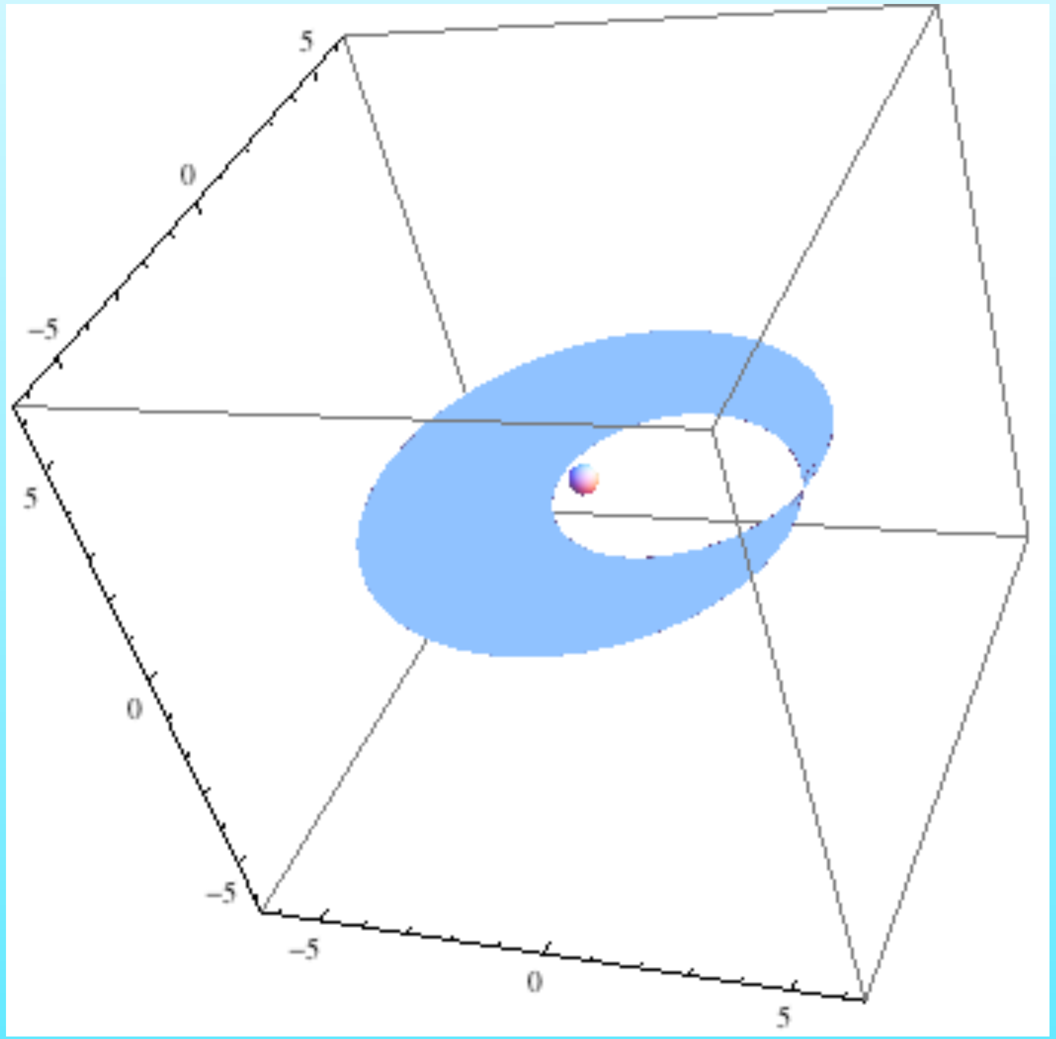


deformation of the system: time line



co-dimension 2

YH-Fukui-Aoki, '06



topological stability in 2D

In 2D with chiral symmetry, 2−2=0 Dirac cones of graphene

d-wave superconductor

Possible Line Nodes (A Model)

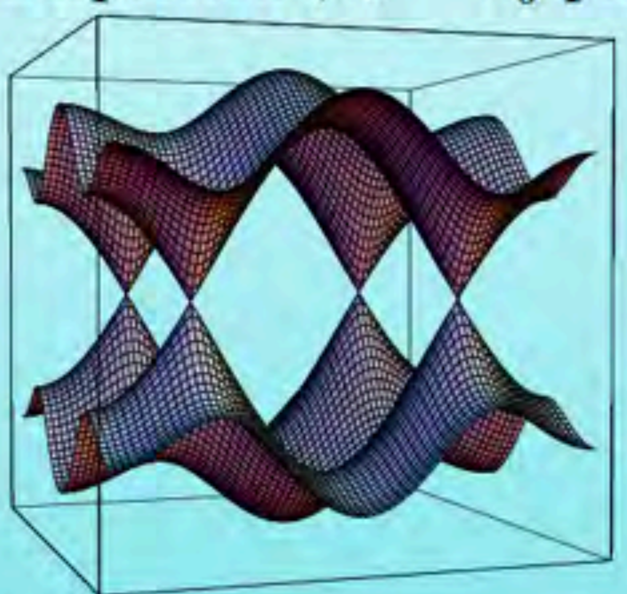
$$\boldsymbol{R} = \boldsymbol{R}(\boldsymbol{k}) = (\mathrm{Re}\,\Delta(\boldsymbol{k}), -\mathrm{Im}\,\Delta(\boldsymbol{k}), \epsilon(\boldsymbol{k}))$$

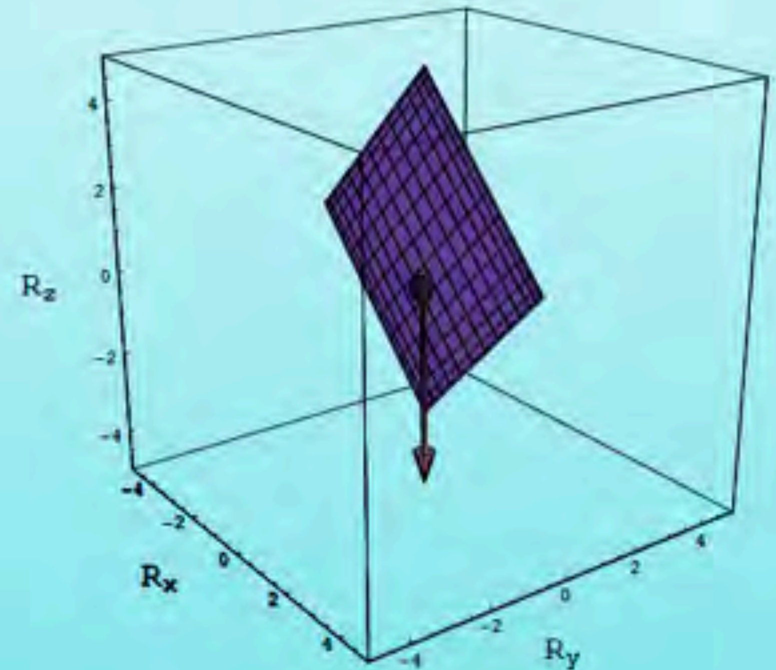
$$\Delta(\boldsymbol{k}) = 2\Delta_{x_2-y_2}(\cos k_x - \cos k_y)$$:Real!

$$\epsilon(\boldsymbol{k}) = -2(\cos k_x + \cos k_y + \cos k_z) - \mu, \Delta_{x^2-y^2} = 1, t = 1, \mu = -3$$

Quasiparticle Dispersion ($k_x - k_y$ plane)

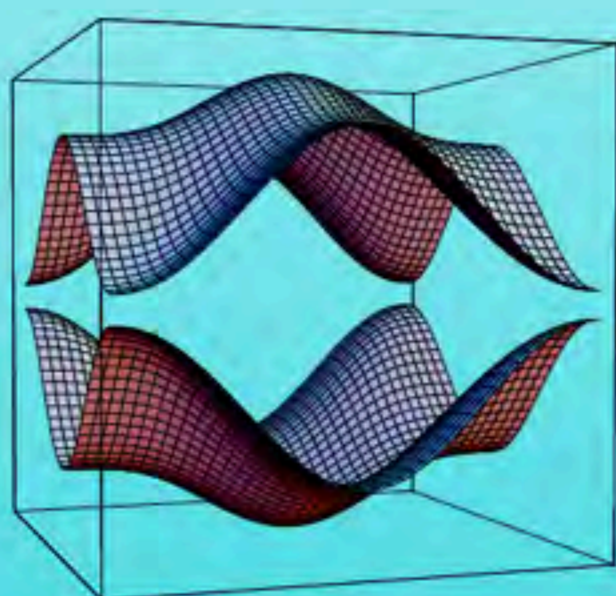
$k_z = 0$

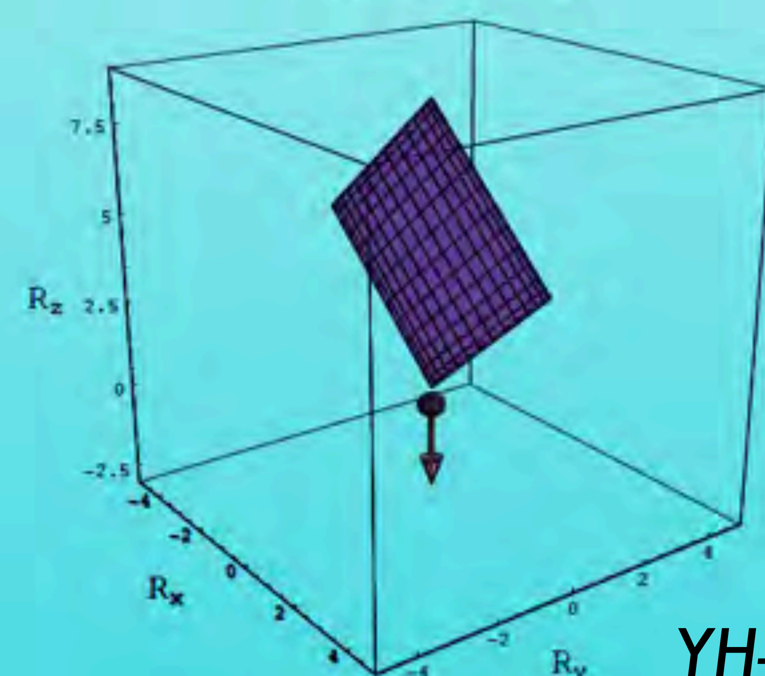




c.f. *Blount'85*

$k_z \approx 0.76\pi$





YH-Ryu-Kohmoto, '04

Gapless

Topological !

Nodes characterize the phase topologically

Generic

co-dimension 3

In 3D, 3–3=0 : point nodes :ABM state of He — Volovik '97

Weyl semi-metal — YH-Ryu'02, YH-Ryu-Kohmoto '04, Burkov-Balents '11

topological stabile Dirac point

with TR invariance/chiral symmetry

co-dimension 2

In 3D with TR invariance, 3–2=1 : line nodes super — Blount'85

In 2D with chiral symmety with TR invariance 2–2=0 : Dirac cones of graphene — Wallace'47

d-wave superconductor

YH-Ryu & Ryu-YH '02

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- Symmetry protection of gap nodes
  - Dimension & co-dimension
  - isotropic superfluidity & graphene

Gapped

- Topological order parameter by quantum interference
  - Berry connection: Z_2 Berry phases & Chern number
  - Successful examples
- Zoo of edge states as topological order parameters
  - Bulk-edge correspondence
  - Examples : Zoo of what we care.

Gapped — Are insulators boring ??

Gapped: Nothing in the gap : cf. Nambu-Goldstone boson

No low lying excitations

No Response against small perturbation

??

? ? ?

~~gapless modes:
acoustic phonons
zero sounds
spin waves~~

Absence of low energy excitations
Energy gap above the ground state

Lots of variety

Absence of fundamental symmetry breaking (mostly)

No responses against for small perturbation

Gapped

Topological !

Adiabatic invariants

Chern numbers, Z_Q Berry phases

protected by symmetry

Parameter dependent hamiltonian ⇨ Berry connection

★ Intrinsically quantized

- Chern numbers: 1st, 2nd, 3rd,....

$$C_1 = -\frac{1}{2\pi i}\int_{M^2} F$$

QHE ... Z ...,-2,-1,0,1,2,...

★ Symmetry protected quantization

- Berry phases & generalization:

$$\gamma_1 = -\frac{1}{2\pi i}\int_{M^1} A$$

Quantum spin chains, Spin-QHE ... Z_2 0 or 1/2

Gapped

Topological !

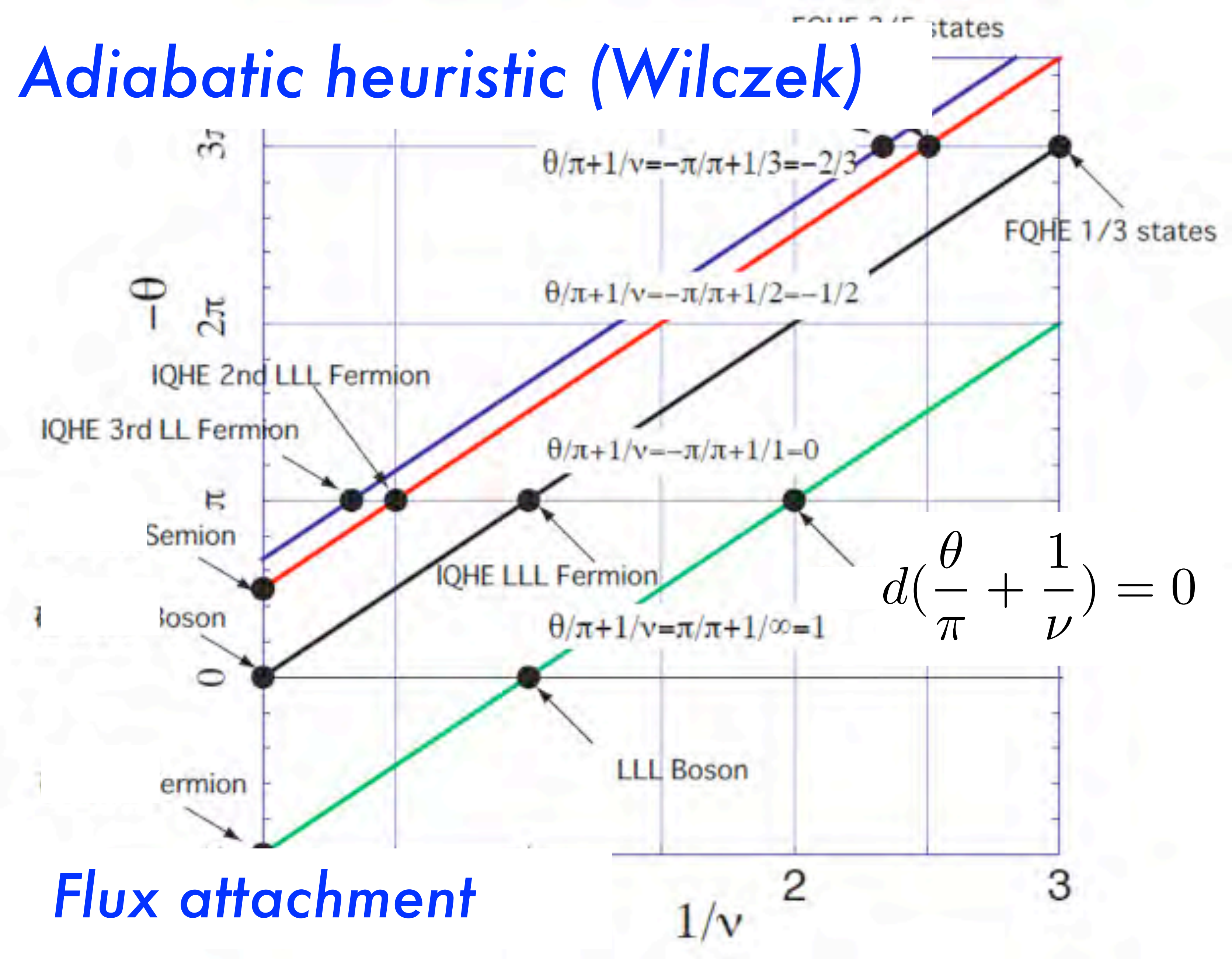
Adiabatic invariants

Chern numbers, Z_Q Berry phases

Fractional QHE

protected by symmetry

Adiabatic heuristic (Wilczek)



$$d(\frac{\theta}{\pi}+\frac{1}{\nu})=0$$

Flux attachment

Connect states by adiabatic process

Classical vs Quantum

☆ "Classical" Observables in Quantum Physics

$$\mathcal{O}_{cl} : H(\text{energy}), \quad \boldsymbol{p}(\text{momentum}), \quad n(\boldsymbol{r})(\text{charge density}) \ \cdots$$

$$\mathcal{O}_{cl} \xrightarrow{\text{quantization}} \mathcal{O}$$: Hermite Operator

$$\langle \mathcal{O} \rangle_G = \langle G|\mathcal{O}|G\rangle = \langle G'|\mathcal{O}|G'\rangle = \langle \mathcal{O} \rangle_{G'}$$

$$|G'\rangle = |G\rangle e^{i\phi}$$:Independent of the phase

☆ with N fold degeneracy $\boldsymbol{\Psi} = (|G_1\rangle, \cdots, |G_N\rangle)$

$$\langle \mathcal{O} \rangle_\Psi = \frac{1}{N} \sum_i \langle G_i|\mathcal{O}|G_i\rangle = \frac{1}{N} \mathrm{Tr}\, \boldsymbol{\Psi}^\dagger \mathcal{O} \boldsymbol{\Psi}$$

$$= \frac{1}{N} \mathrm{Tr}\, \boldsymbol{\Psi}'^\dagger \mathcal{O} \boldsymbol{\Psi}' = \langle \mathcal{O} \rangle_{\Psi'}$$

$$\boldsymbol{\Psi} = \boldsymbol{\Psi}' \boldsymbol{U}, \quad \boldsymbol{U} : \ \text{Unitary}$$

Unitary Invariant ?

YES!

Berry connection as quantum interference

- "Quantum" Observables !
  - No classical Correspondences
  - Quantum Interferences between 2 different states
    - Aharonov-Bohm Effects
    - Geometrical Phases
    - Berry connection & Phases

$$\langle G(t_A)|G(t_B)\rangle \neq \langle G'(t_A)|G'(t_B)\rangle$$

$$|G(t)\rangle = |G'(t)\rangle e^{i\phi(t)}$$

$$\langle G|G+dG\rangle = 1 + \langle G|dG\rangle$$

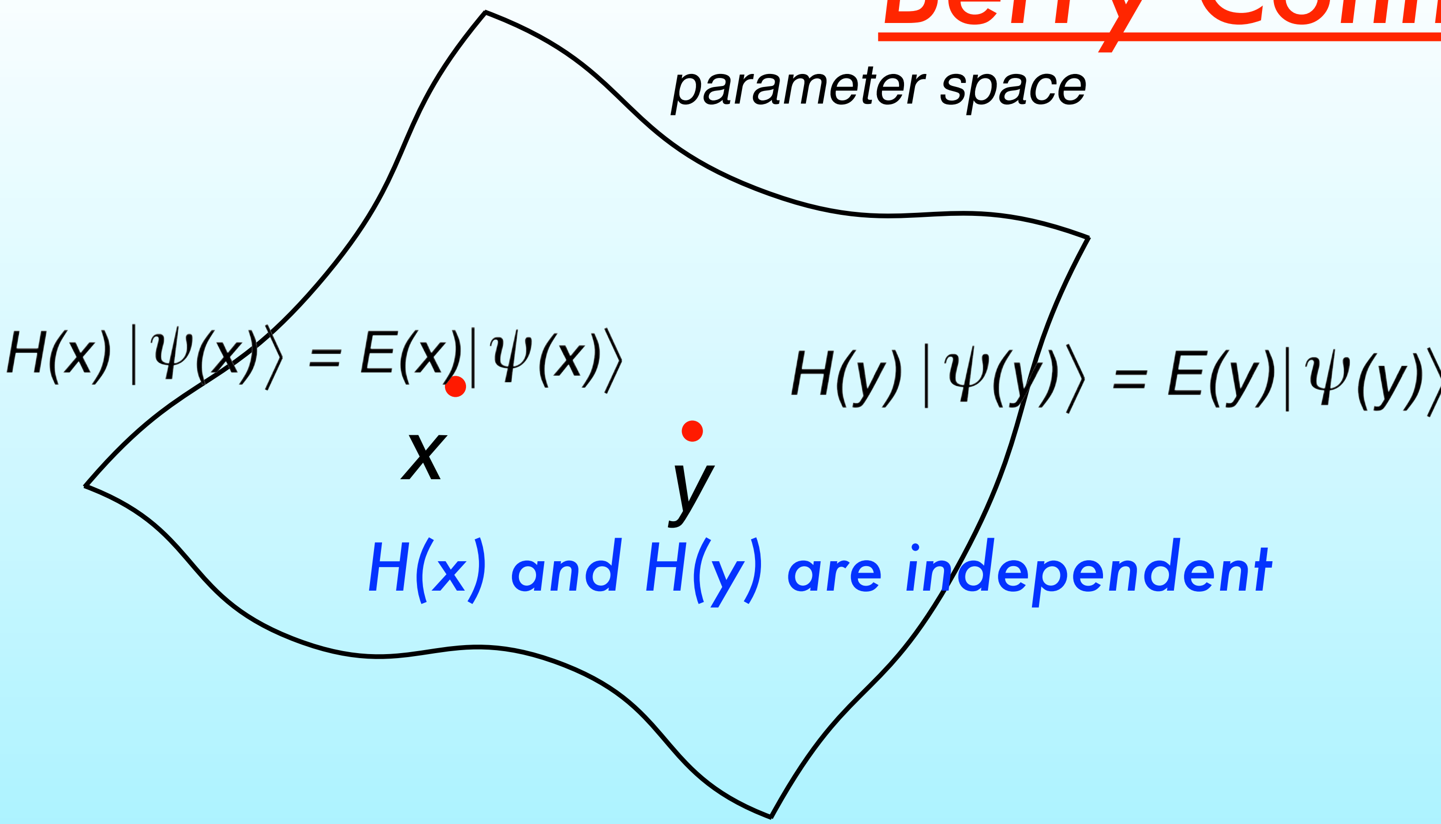
$$A = \langle G|dG\rangle$$:Berry Connection

$$i\gamma = \int A$$:Berry Phase

~~Unitary Invariant ?~~ NO ! Phase dependent

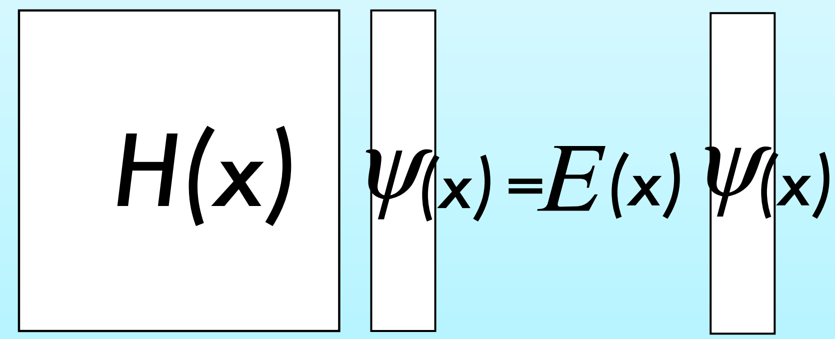
Berry Connection?

Berry '84



Eigenvectors (space)

with Parameters



(Abelian)

Information between nearby states

Fiber Bundle

Berry connection : $A_\psi = \langle\psi|d\psi\rangle = \langle\psi|\frac{d}{dx}\psi\rangle dx.$

gauge potential

Gauge Transformation

phase change=gauge transformations

$$|\psi(x)\rangle = |\psi'(x)\rangle e^{i\Omega(x)}$$

phase fix = gauge fix

$$A_\psi = A'_\psi + id\Omega = A'_\psi + i\frac{d\Omega}{dx}dx$$

Berry phase and its gauge dependence

- **Parameter Dependent Hamiltonian**

H(x) ψ(x) =E(x) ψ(x)

$$H(x)|\psi(x)\rangle = E(x)|\psi(x)\rangle, \ \langle\psi(x)|\psi(x)\rangle = 1.$$

- **Berry Connections** $A_\psi = \langle\psi|d\psi\rangle = \langle\psi|\frac{d}{dx}\psi\rangle dx.$
- **Berry Phases** $i\gamma_C(A_\psi) = \int_C A_\psi$

(Abelian)

- **Phase Ambiguity of the eigen state**

Gauge Transformation

$$|\psi(x)\rangle = |\psi'(x)\rangle e^{i\Omega(x)}$$

$$A_\psi = A'_\psi + i d\Omega = A'_\psi + i\frac{d\Omega}{dx}dx$$

- **Berry phases are not well-defined without specifying the gauge**

$$\gamma_C(A_\psi) = \gamma_C(A_{\psi'}) + \int_C d\Omega$$

$2\pi \times$ (integer) if $e^{i\Omega}$ is single valued

- **Well Defined up to mod 2π**

$$\gamma_C(A_\psi) \equiv \gamma_C(A_{\psi'}) \mod 2\pi$$

Anti-Unitary Operator and Berry Phases

Anti-Unitary Operator

(Time Reversal, Particle-Hole)

$$\Theta = KU_\Theta, \quad \begin{aligned} K &: \quad \text{Complex conjugate} \\ U_\Theta &: \quad \text{Unitary} \end{aligned}$$

(parameter independent)

$$|\Psi\rangle = \sum_J C_J \, |J\rangle \qquad \sum_J C_J^* C_J = \langle\Psi|\Psi\rangle = 1$$

$$|\Psi^\Theta\rangle = \Theta|\Psi\rangle = \sum_J C_J^* \, |J^\Theta\rangle, \quad |J^\Theta\rangle = \Theta|J\rangle$$

Berry Phases and Anti-Unitary Operation

$$A^\Psi = \langle\Psi|d\Psi\rangle = \sum_J C_J^* dC_J \qquad \sum_J dC_J^* C_J + \sum_J C_J^* dC_J = 0$$

$$A^{\Theta\Psi} = \langle\Psi^\Theta|d\Psi^\Theta\rangle = \sum_J C_J dC_J^* = -A^\Psi$$

$$\gamma_C(A^{\Theta\Psi}) = -\gamma_C(A^\Psi)$$

Anti-Unitary Invariant State and Quantized Berry Phases

- Anti-Unitary Symmetry $[H(x), \Theta] = 0$
- Invariant State $^{\exists}\varphi, \quad |\Psi^{\Theta}\rangle = \Theta|\Psi\rangle = |\Psi\rangle e^{i\varphi}$
  - ex. Unique Eigen State $\simeq |\Psi\rangle$ Gauge Equivalent(Different Gauge)
- To be compatible with the ambiguity,

  the Berry Phases have to be quantized as

$$\gamma_C(A^{\Psi}) = \begin{cases} 0 \\ \pi \end{cases} \mod 2\pi$$

$$\gamma_C(A^{\Psi}) = -\gamma_C(A^{\Theta\Psi}) \equiv -\gamma_C(A^{\Psi}), \ \mathrm{mod} 2\pi$$

Z_2 Berry phase as a topological order parameter

- Generic Heisenberg Models with possible frustration

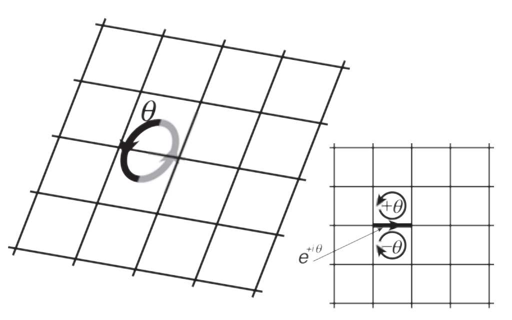
$$H = \sum_{ij} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Time Reversal Invariant

$$^\forall j,\ \boldsymbol{S}_j \to \Theta_T^{-1} \boldsymbol{S}_j \Theta_T = -\boldsymbol{S}_j$$

- U(1) twist as a Local Probe to define Berry Phases

$$\boldsymbol{S}_i \cdot \boldsymbol{S}_j \to \frac{1}{2}(e^{-i\theta} S_{i+} S_{j-} + e^{+i\theta} S_{i-} S_{j+}) + S_{iz} S_{jz}$$



$H(x = e^{i\theta})$

Parameter dependent Hamiltonian

$C = \{x = e^{i\theta} | \theta : 0 \to 2\pi\}$

Define Berry Phases by the Entire Many Spin Wavefunction

Excitation Gap! Time Reversal Invariance

Quantization

$$\gamma_C = \int_C A_\psi = \int_C \langle \psi | d\psi \rangle = \left\{ \begin{array}{c} 0 \\ \pi \end{array} \right. : \mod 2\pi$$

Z_2 Berry phase

Topological order parameter at the link <ij>

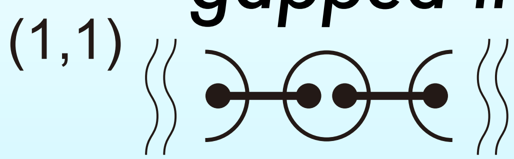
YH, J. Phys. Soc. Jpn. 75, 123601, '06

Quantum liquids

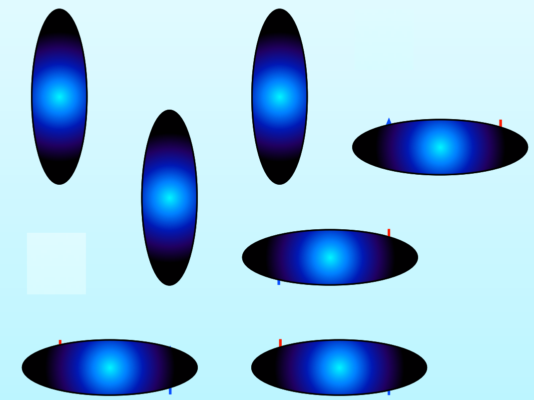
Short range entangled states

Ex.1) AKLT state

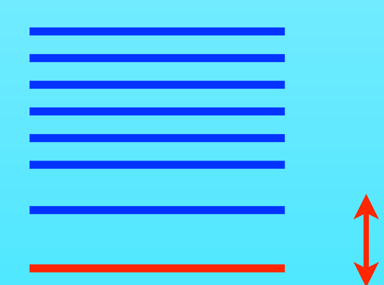
gapped integer spin chain



Ex.2) Collection of singlets



Something complicated
but gapped

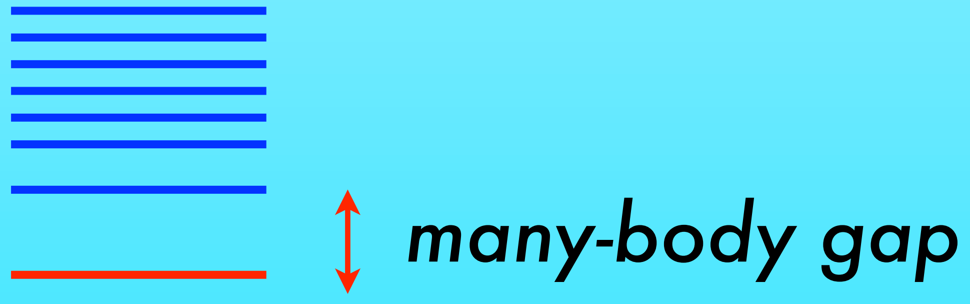


Quantum liquids

Short range entangled states

Adiabatic deformation !
gap remains open

Something complicated
but gapped

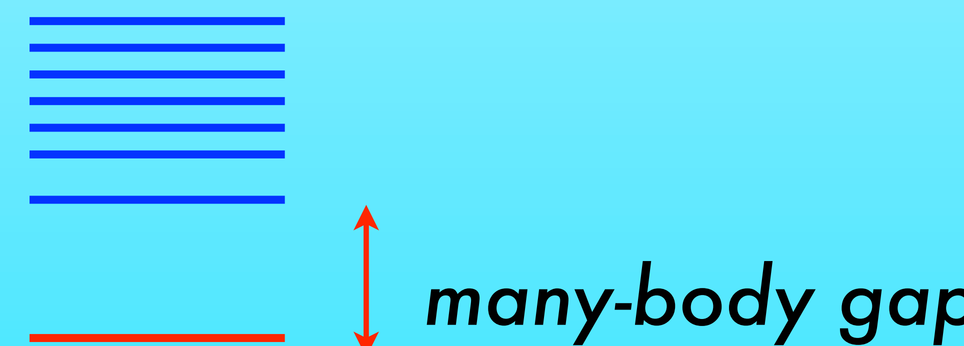


Quantum liquids

Short range entangled states

Adiabatic deformation !
gap remains open

Something complicated
but gapped



Quantum liquids

Short range entangled states

Adiabatic deformation !
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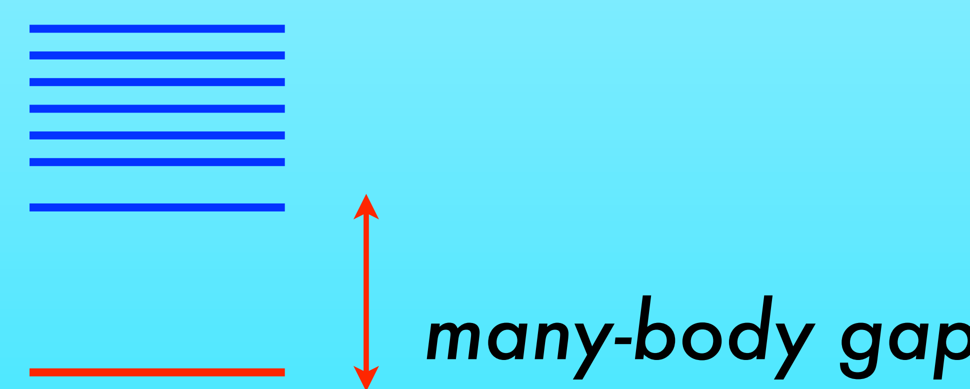
Something complicated
but gapped



Quantum liquids

Short range entangled states

Adiabatic deformation !
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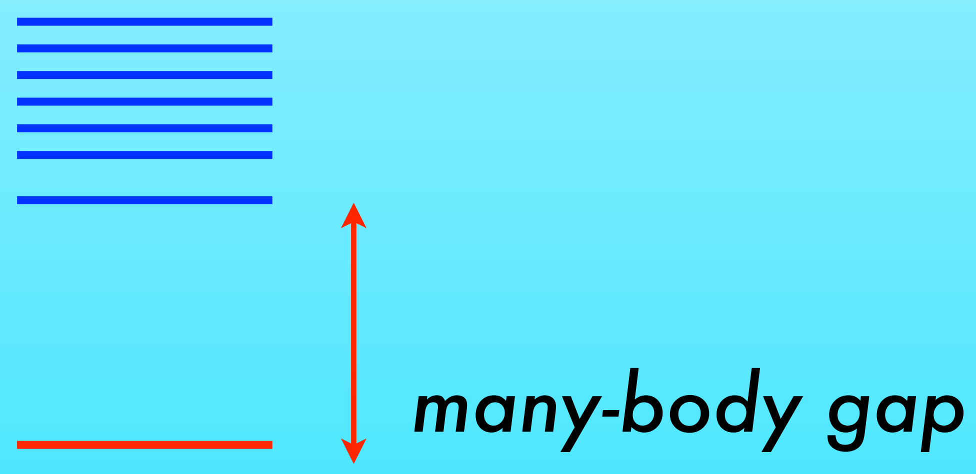
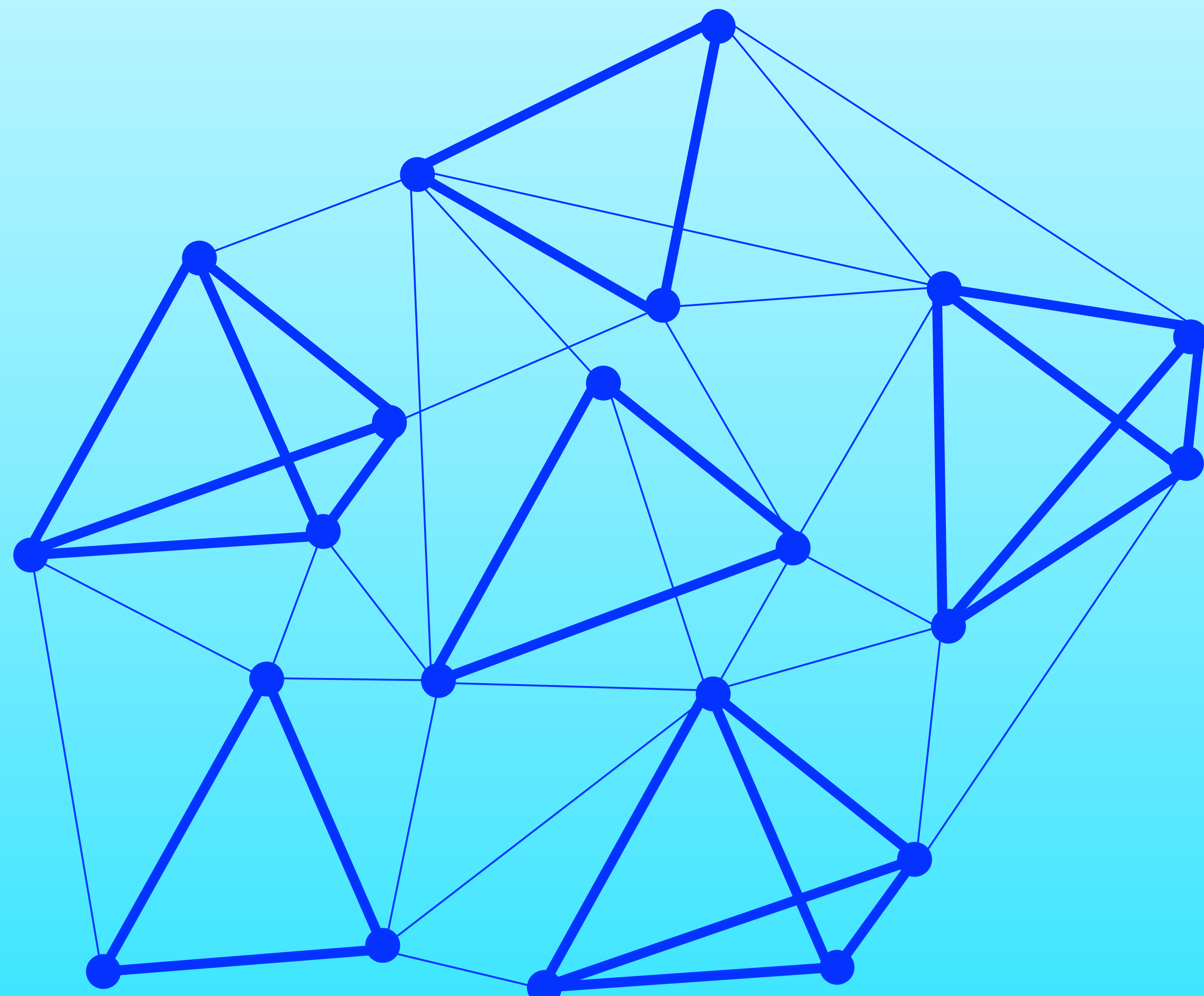
Something complicated
but gapped



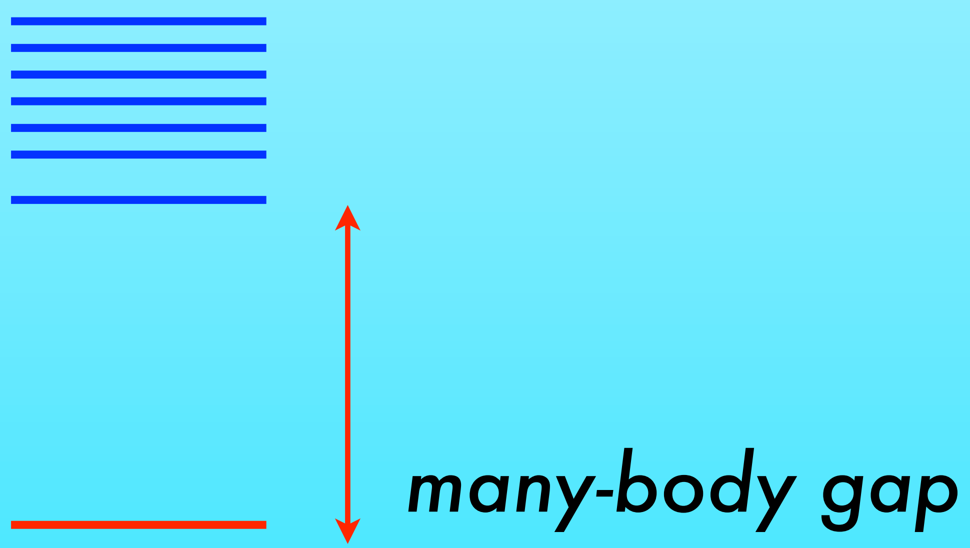
Quantum liquids

Short range entangled states

Adiabatic deformation !
gap remains open



Something complicated
but gapped

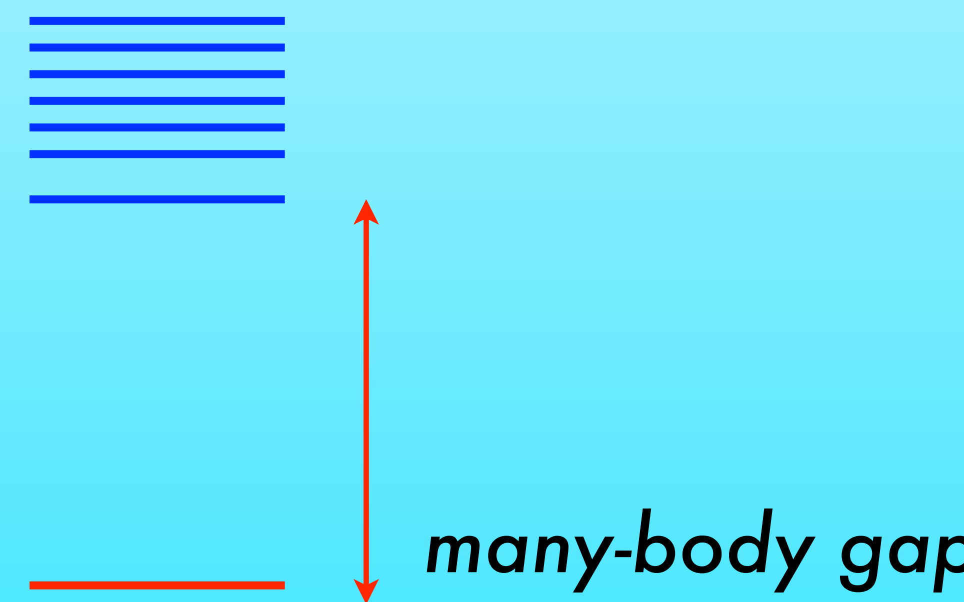


Quantum liquids

Short range entangled states

Adiabatic deformation !
gap remains open

Something complicated
but gapped



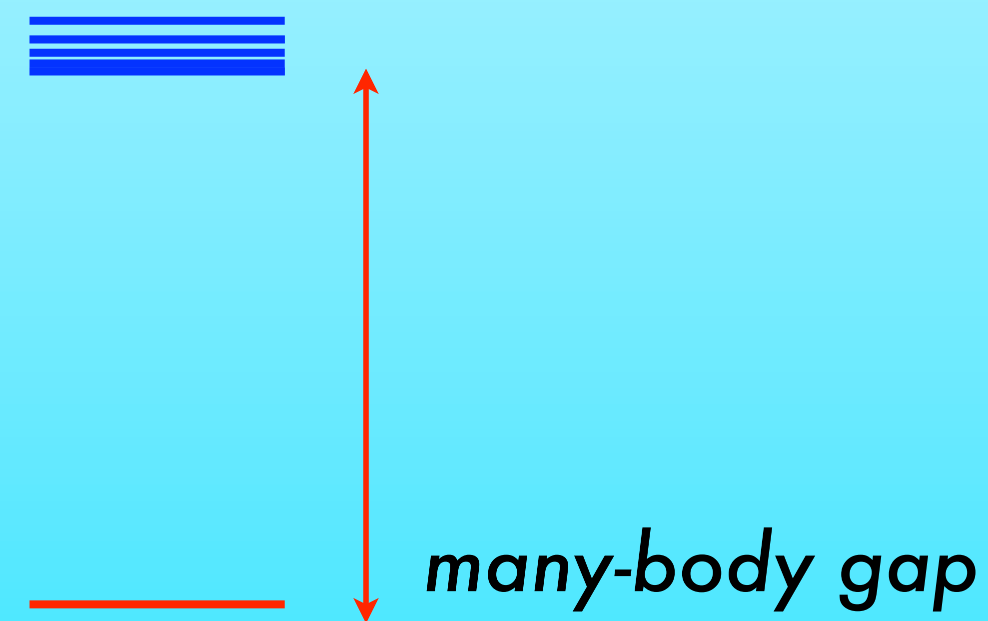
Quantum liquids

Short range entangled states

Adiabatic deformation !

gap remains open

Something complicated
but gapped

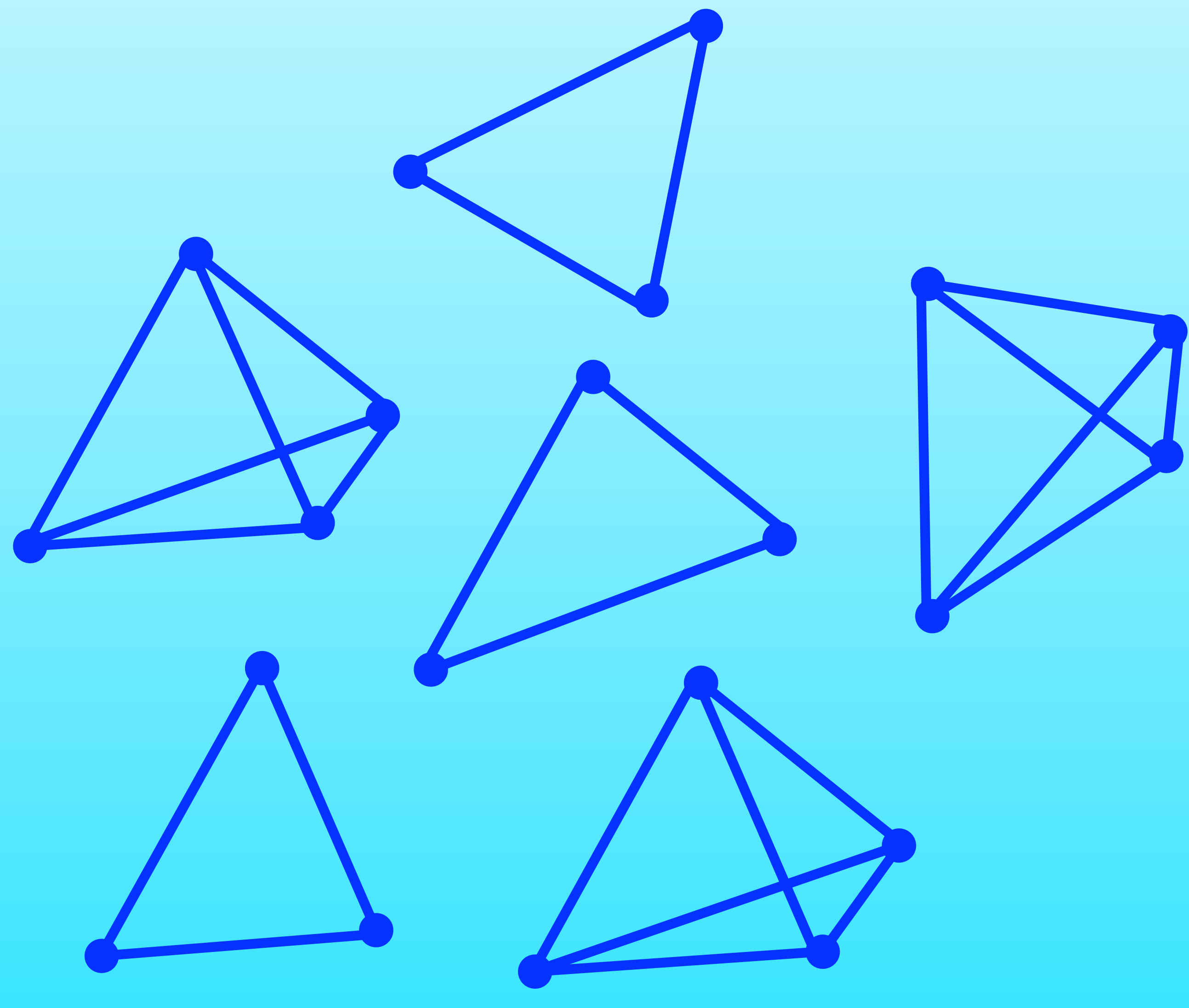


Quantum liquids

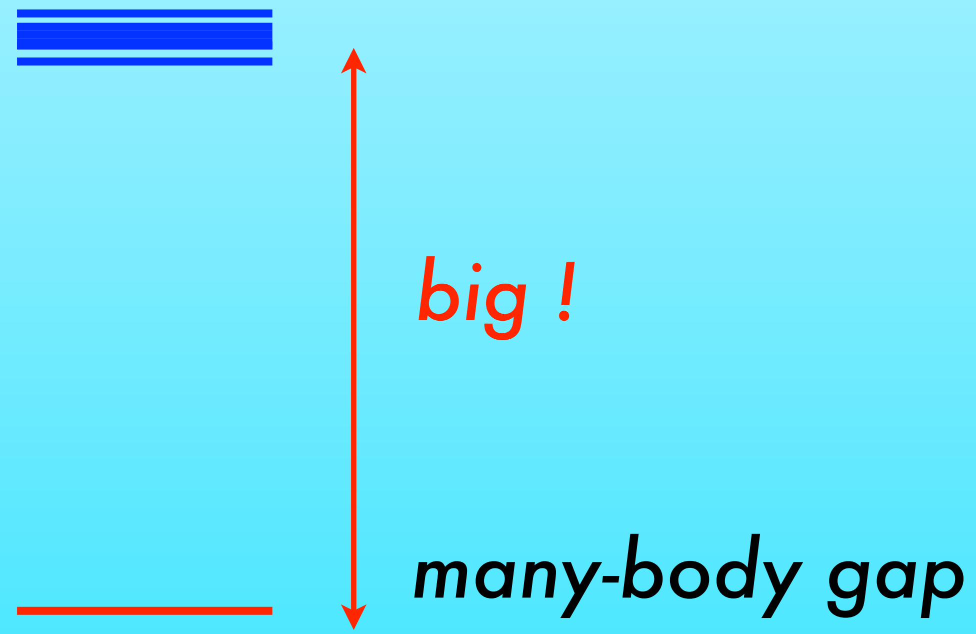
Short range entangled states

Adiabatic deformation !
gap remains open

Decoupled !



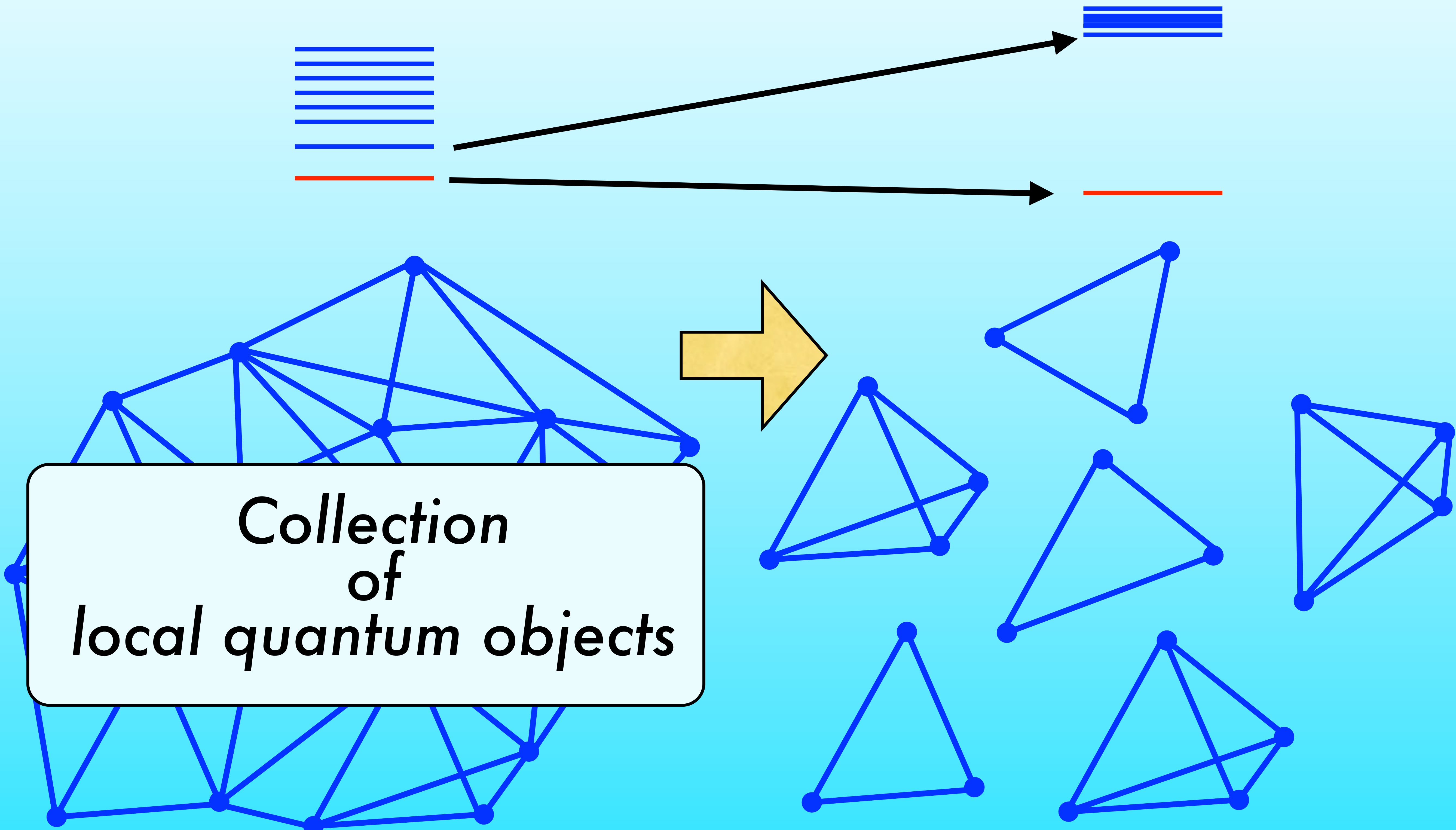
Something very simple
& gapped



Quantum liquids

Short range entangled states

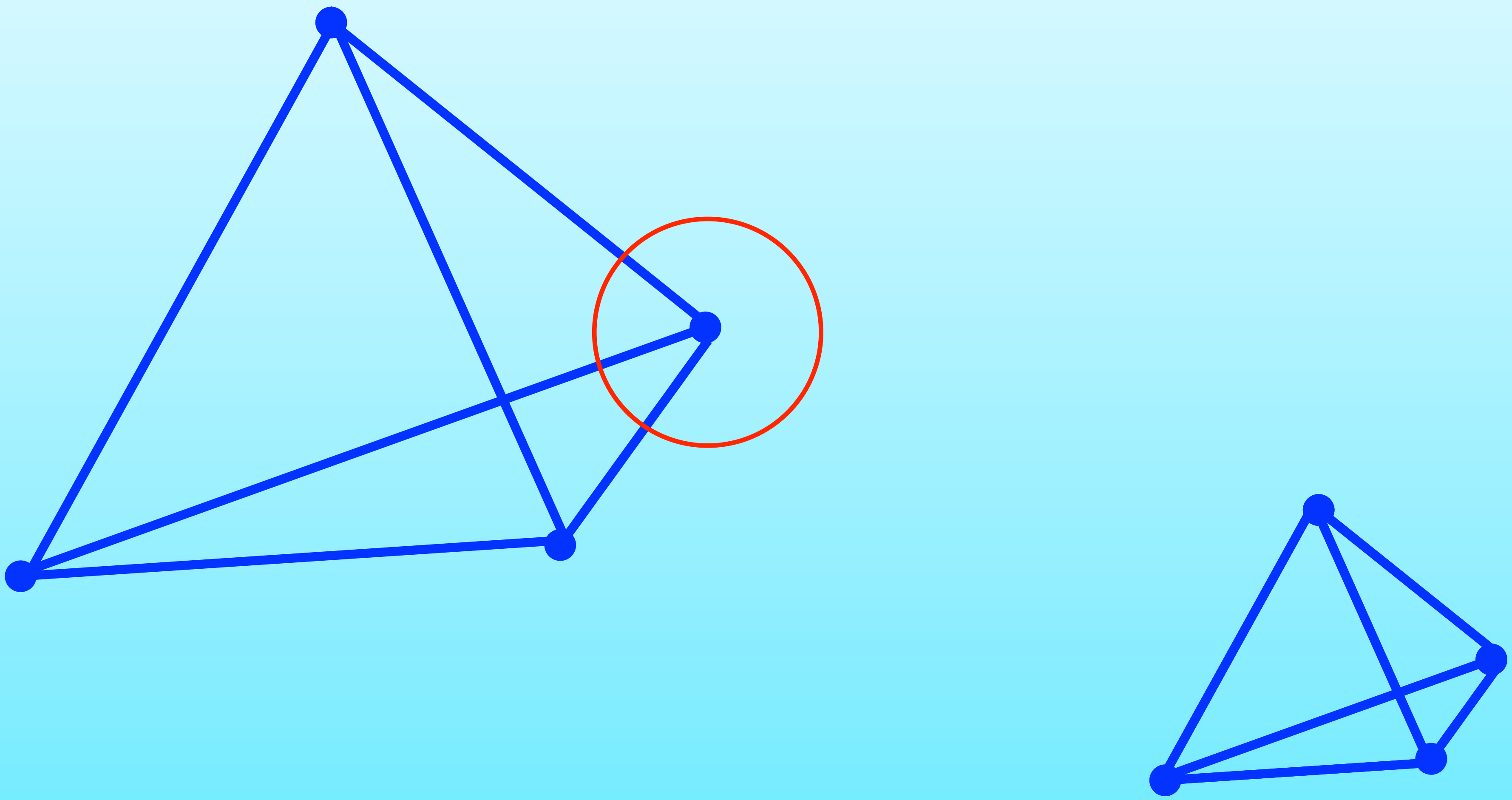
Adiabatic process to be decoupled: gap remains open



Def. of short range entangled state

How to characterize local object ?

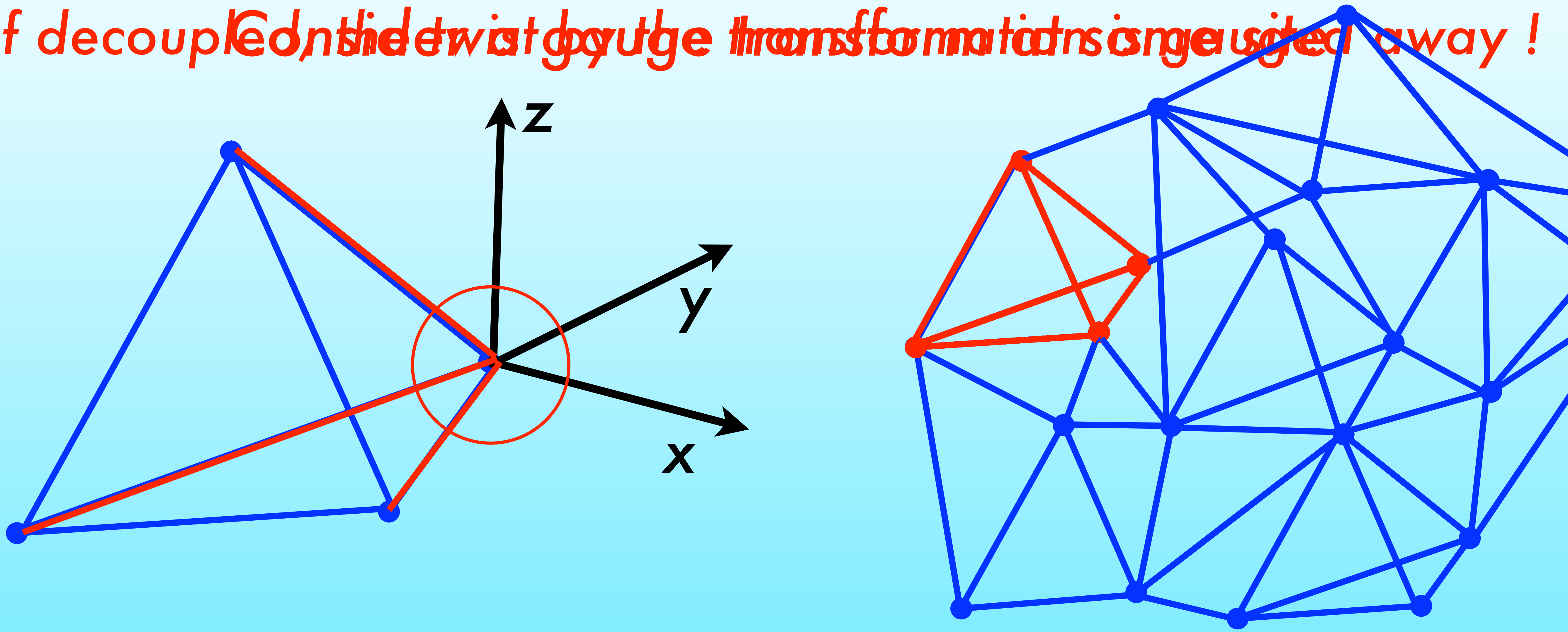
Consider a gauge transform at some site



How to characterize local object ?

If decoupled, the twist by the transformation is gauged away !

Consider a gauge transformation as a twist



It characterizes locality of the quantum object !

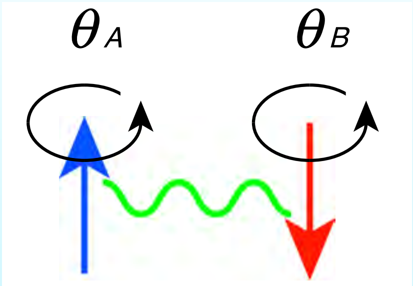
Answer !

How to see this locality by skipping the adiabatic deformation ?

Calculate the topological invariant as adiabatic invariant

Z_2 Berry phase of Singlet Pair

$\theta = \theta_A - \theta_B$



$$H_{AB} = (S_A^x, S_A^y, S_Z^z) \begin{pmatrix} \cos\theta & -\sin\theta & \\ \sin\theta & -\cos\theta & \\ & & 1 \end{pmatrix} \begin{pmatrix} S_B^x \\ S_B^y \\ S_B^z \end{pmatrix}$$
$$= \frac{1}{2}(e^{-i\theta} S_A^+ S_B^- + e^{i\theta} S_A^- S_B^+) + S_A^z S_B^z$$

- **Local Singlet Pair with the twist**

$$|\psi\rangle = \frac{1}{\sqrt{2}}(e^{i\theta/2}|\uparrow_A\downarrow_B\rangle - e^{-i\theta/2}|\downarrow_A\uparrow_B\rangle)$$

- **Berry phase of the twisted singlet pair**

$$A = \boldsymbol{\psi}^\dagger d\boldsymbol{\psi}$$

$$\psi = \begin{cases} i\frac{1}{\sqrt{2}}\, e^{i\mathrm{Arg}\,(a-be^{i\theta})} \begin{pmatrix} 1 \\ -e^{-i\theta} \end{pmatrix} & |a| > |b| \\ i\frac{1}{\sqrt{2}}\, e^{i\mathrm{Arg}\,(b-ae^{-i\theta})} \begin{pmatrix} -e^{i\theta} \\ 1 \end{pmatrix} & |a| < |b| \end{cases}$$

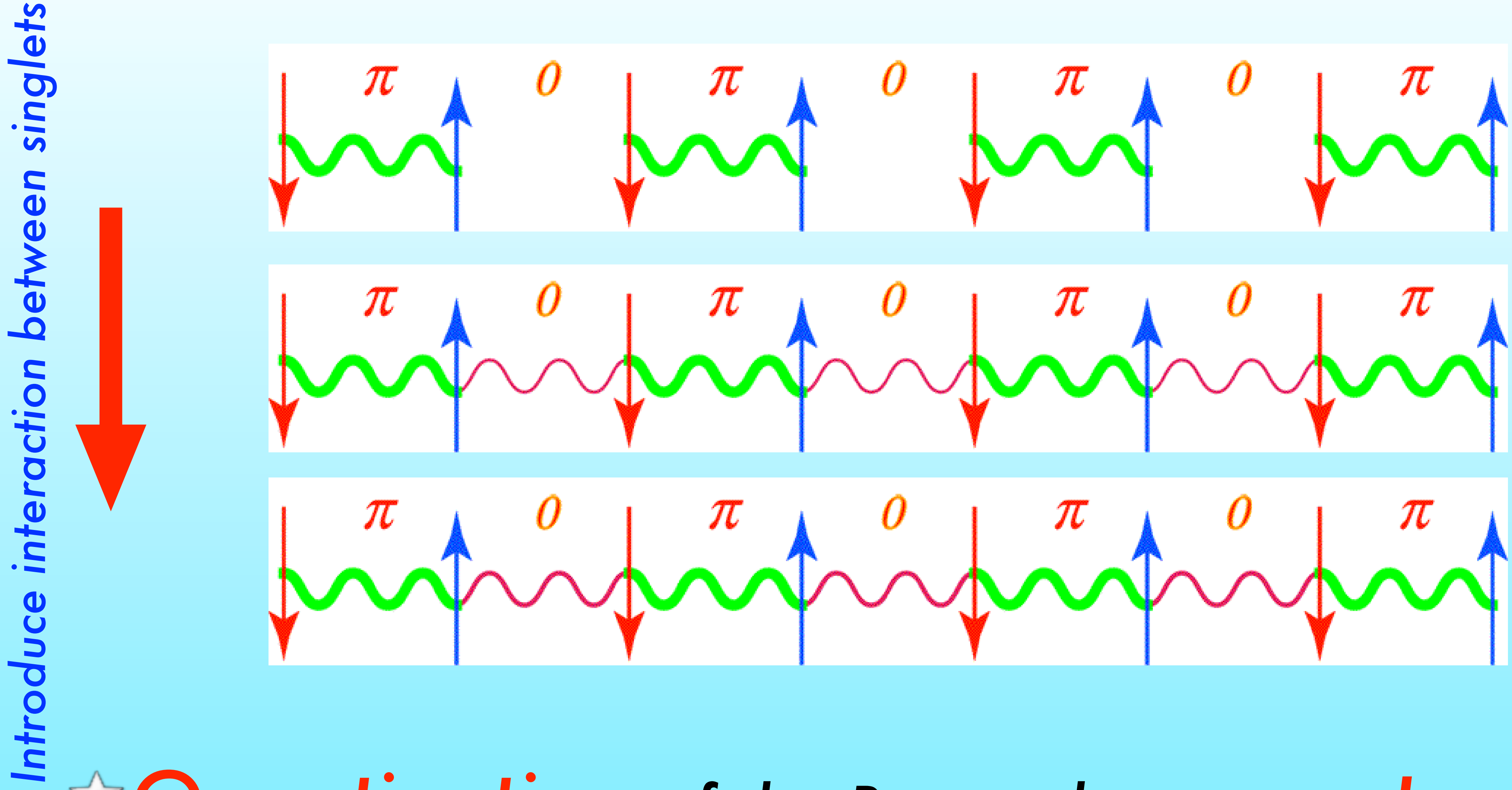
$$: \ a, b \in \mathbb{C}(\text{gauge parameters})$$

$$\gamma = -i\int A = \begin{cases} -\pi & |a| > |b| \\ \pi & |a| < |b| \end{cases}$$

$$\gamma_{\text{singlet pair}} = \pi \mod 2\pi$$

A singlet does not carry spin but does the Berry phase π

Adiabatic Continuation & the Quantization



☆ Quantization of the Berry phases protects from continuous change

Adiabatic Continuation in a gapped system

↕

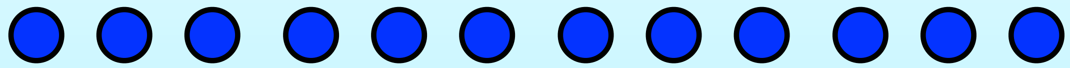
Renormalization Group in a gapless system

Metal/gapped : physicists & chemists ?

Sorry if I'm wrong

physicists

itinerant electrons

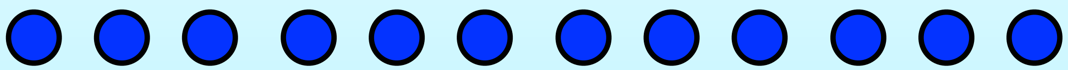


Metal/gapped : physicists & chemists ?

Sorry if I'm wrong

physicists

itinerant electrons



hopping

make energy band

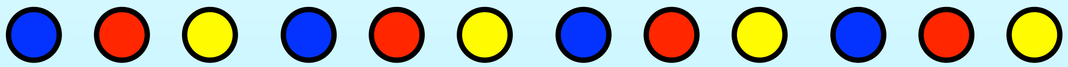
metal

Metal/gapped : physicists & chemists ?

Sorry if I'm wrong

physicists

itinerant electrons

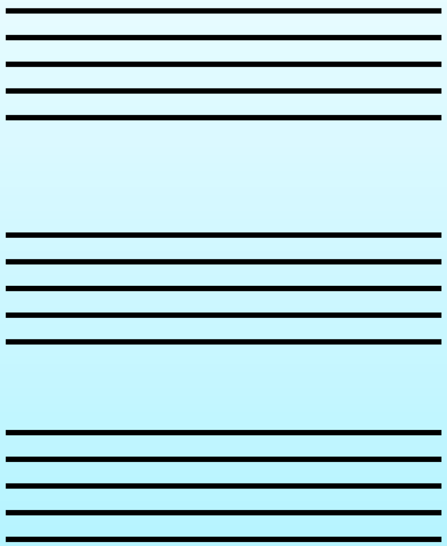


hopping

Peierls instability

Opening gap

stabilize



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physicists

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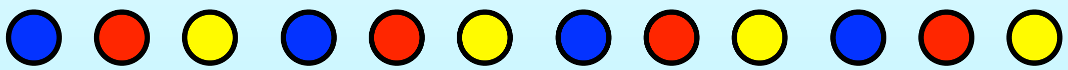
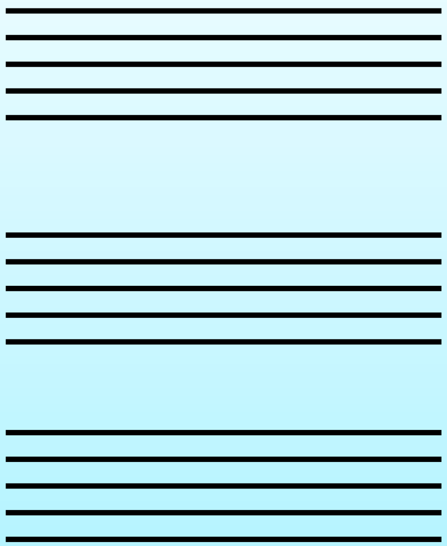


hopping

Peierls instability

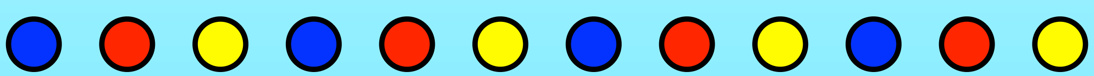
Opening gap

stabilize



chemists

form molecules first

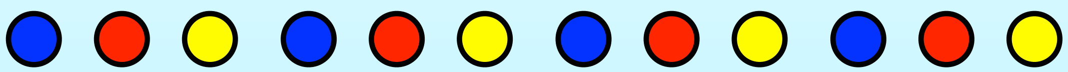


Metal/gapped : physicists & chemists ?

Sorry if I'm wrong

physicists

itinerant electrons

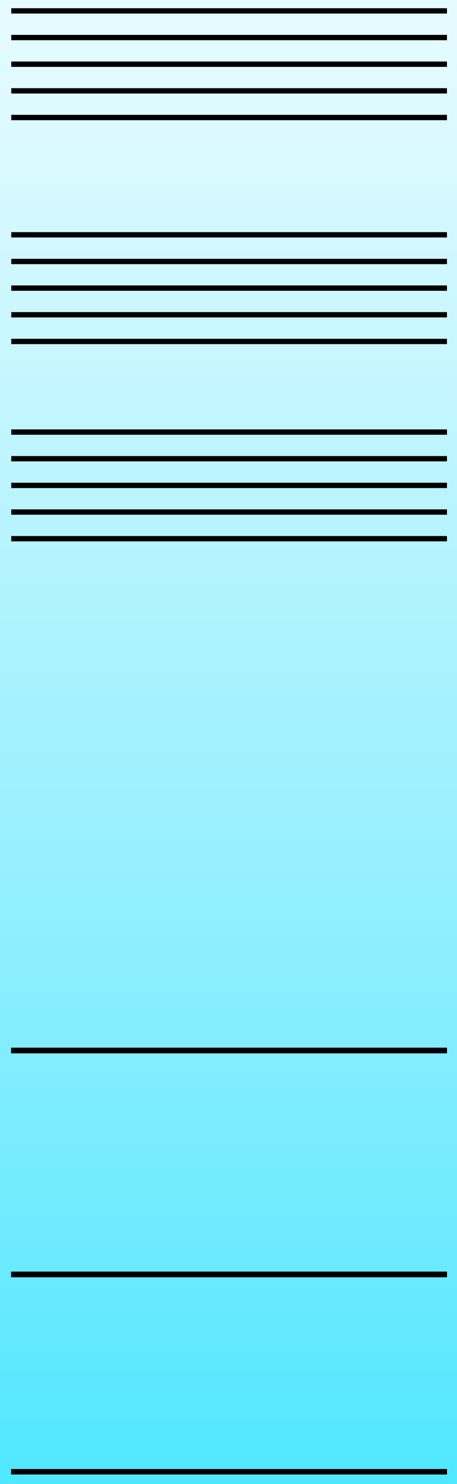


hopping

Peierls instability

Opening gap

stabilize



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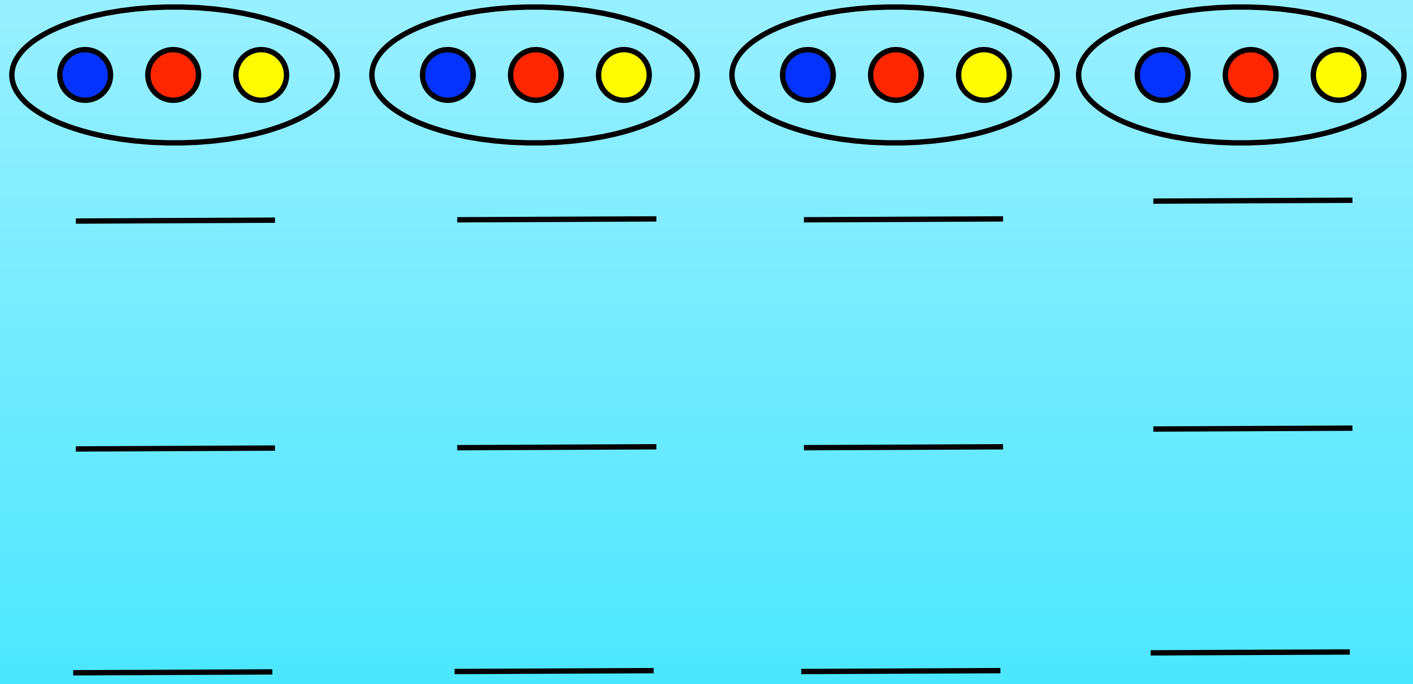
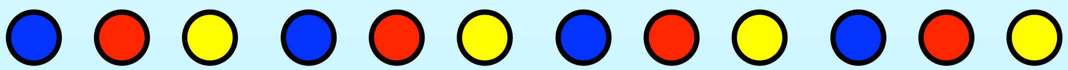


Metal/gapped : physicists & chemists ?

Sorry if I'm wrong

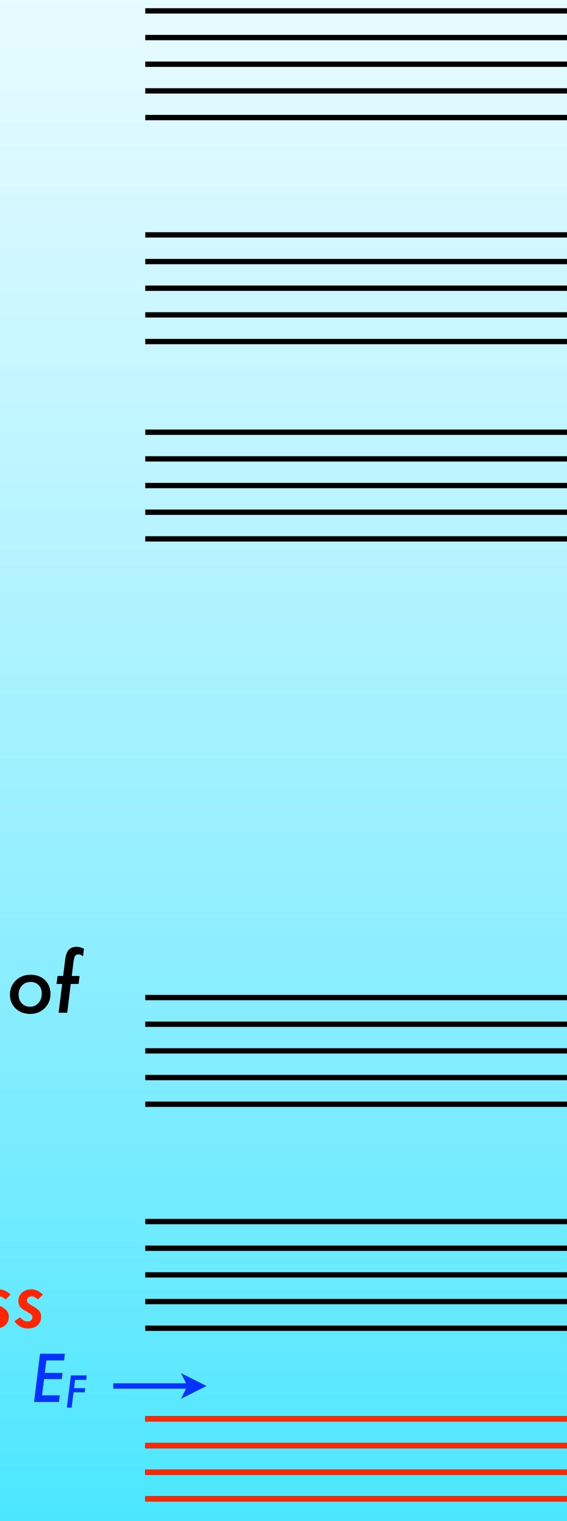
physicists

itinerant electrons



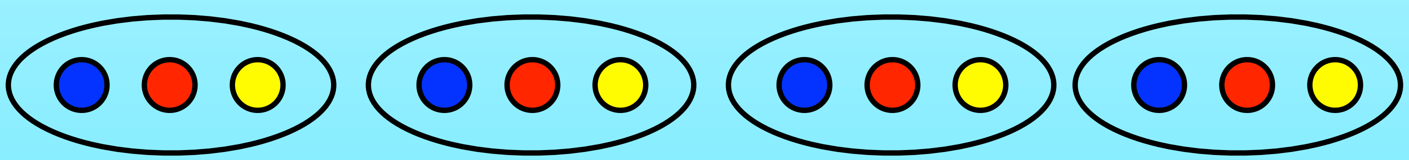
Peierls instability

stabilize



chemists

form molecules first



make bands of molecules

non orthogonality

short range entanglement

✓ Adiabatic process

Insulator

chemist way: BETTER !

✓ Think locally when gapped!

molecules, singlet pairs, bonds

Validity of our general scheme

- Examples in 1D, 2D, 3D and ...
  - Integer spin chains with dimerization
  - Random hopping models
  - Orthogonal dimers in 2D
  - BEC-BCS crossover at half filling
  - Dimerization transition on Kagome & Pyrochlore

1D S=1/2 chains with dimerization

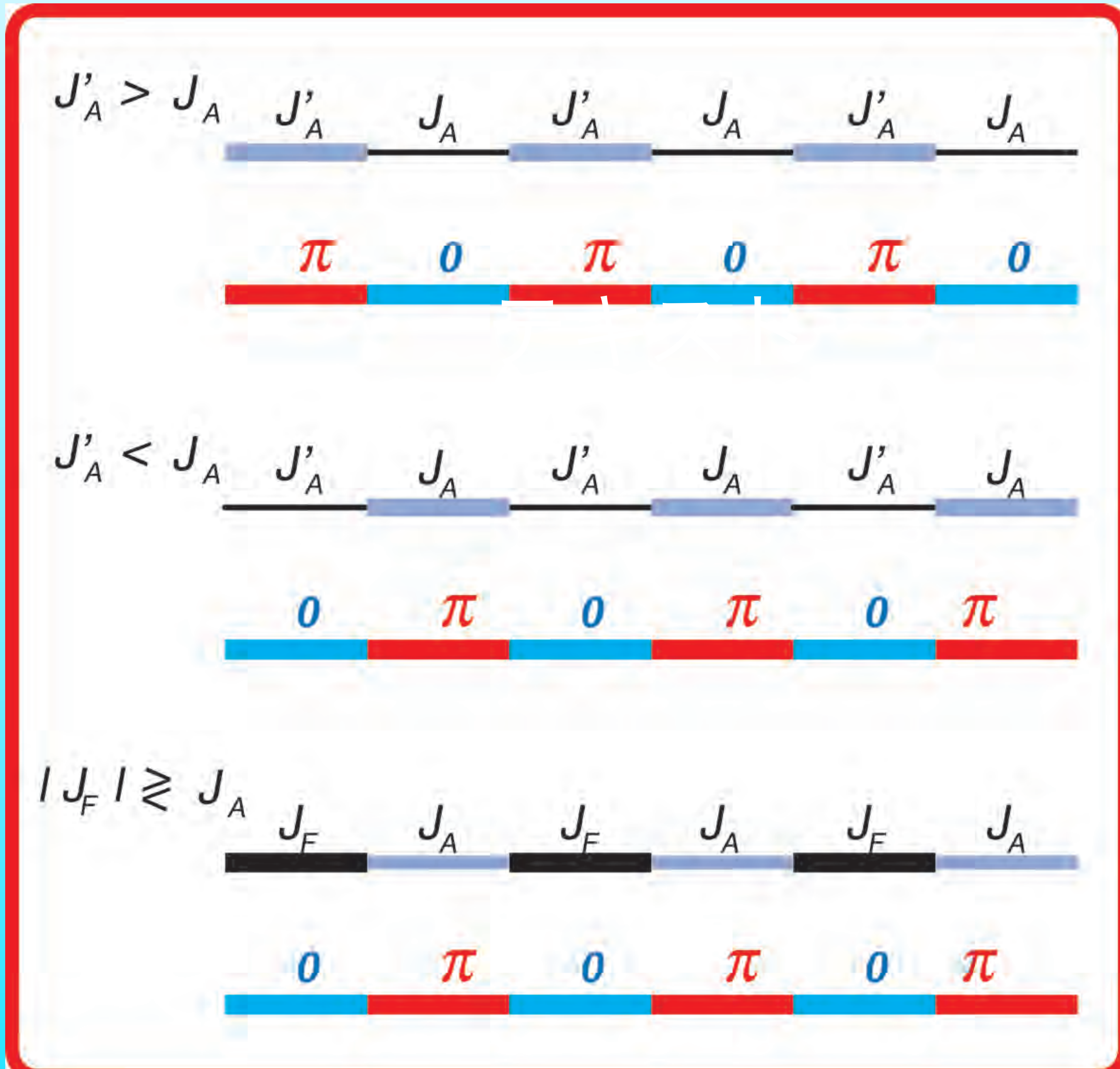
$$H = \sum_{\langle i \rangle} J_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

Y.H., J. Phys. Soc. Jpn. 75 123601 (2006)

AF-AF

Ferro-AF

Hida



AF-AF case

Strong bonds

: π bonds

F-AF case

AF bonds

: π bonds

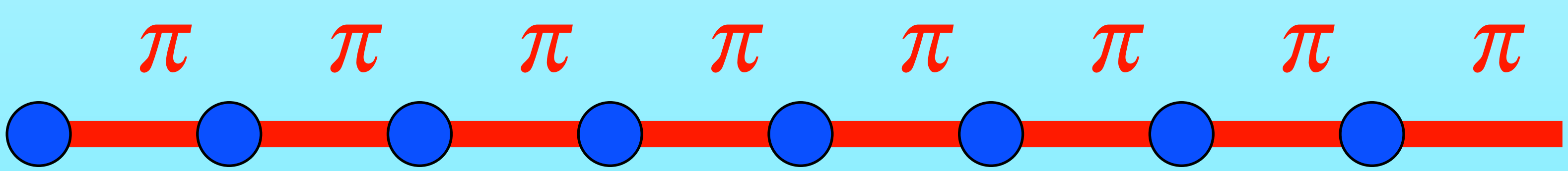
Heisenberg Spin Chains with integer S

S=1 $\quad (\boldsymbol{S}_i)^2 = S(S+1),\ S=1$

$$H = J\sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + D\sum_i (S_i^z)^2$$

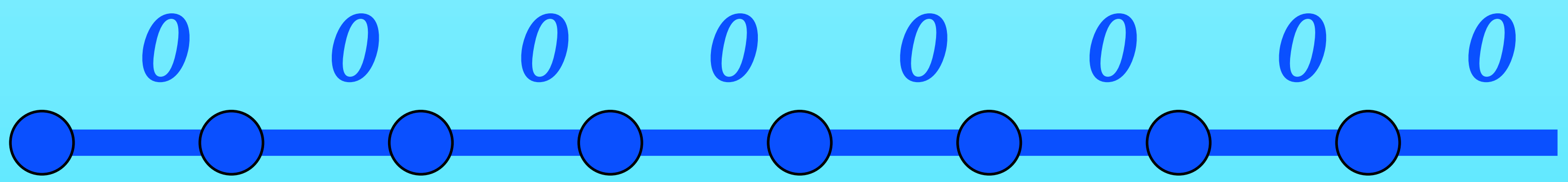
Y.H., J. Phys. Soc. Jpn. 75 123601 (2006)

Haldane phase



$D < D_C$

Large D phase



$D > D_C$

Characterize the Quantum Phase Transition

S=1,2 dimerized Heisenberg model

T.Hirano, H.Katsura &YH, Phys.Rev.B77 094431'08

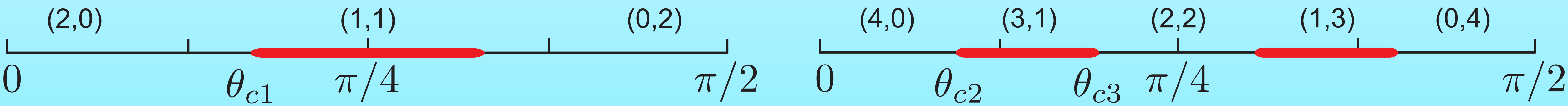
$$H = \sum_{i=1}^{N/2} (J_1 \boldsymbol{S}_{2i} \cdot \boldsymbol{S}_{2i+1} + J_2 \boldsymbol{S}_{2i+1} \cdot \boldsymbol{S}_{2i+2})$$

$$J_1 = \cos\theta, J_2 = \sin\theta$$

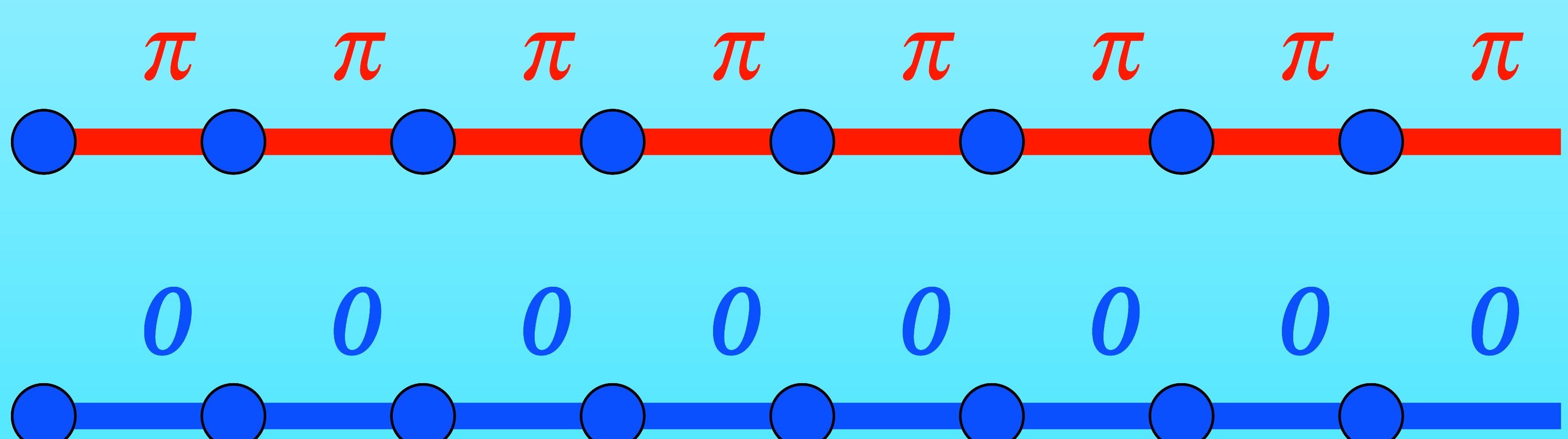
Z_2 Berry phase

Red line :Berry phase π

$S = 1\ N = 14$ | $S = 2\ N = 10$



θ : dimerization strength | θ : dimerization strength



S=1 & 2

Sequential transitions among gapped phases

S=1,2 dimerized Heisenberg model

T.Hirano, H.Katsura &YH, Phys.Rev.B77 094431'08

$$H = \sum_{i=1}^{N/2} (J_1 \boldsymbol{S}_{2i} \cdot \boldsymbol{S}_{2i+1} + J_2 \boldsymbol{S}_{2i+1} \cdot \boldsymbol{S}_{2i+2})$$

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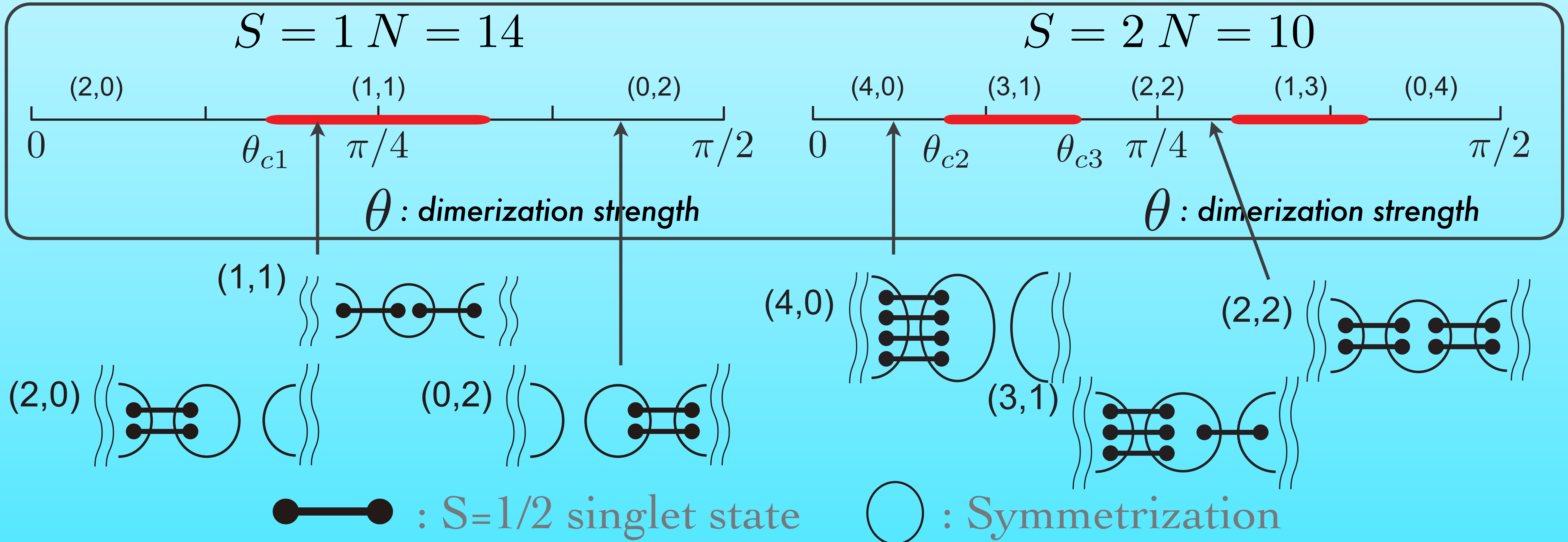


Reconstruction of valence bonds!

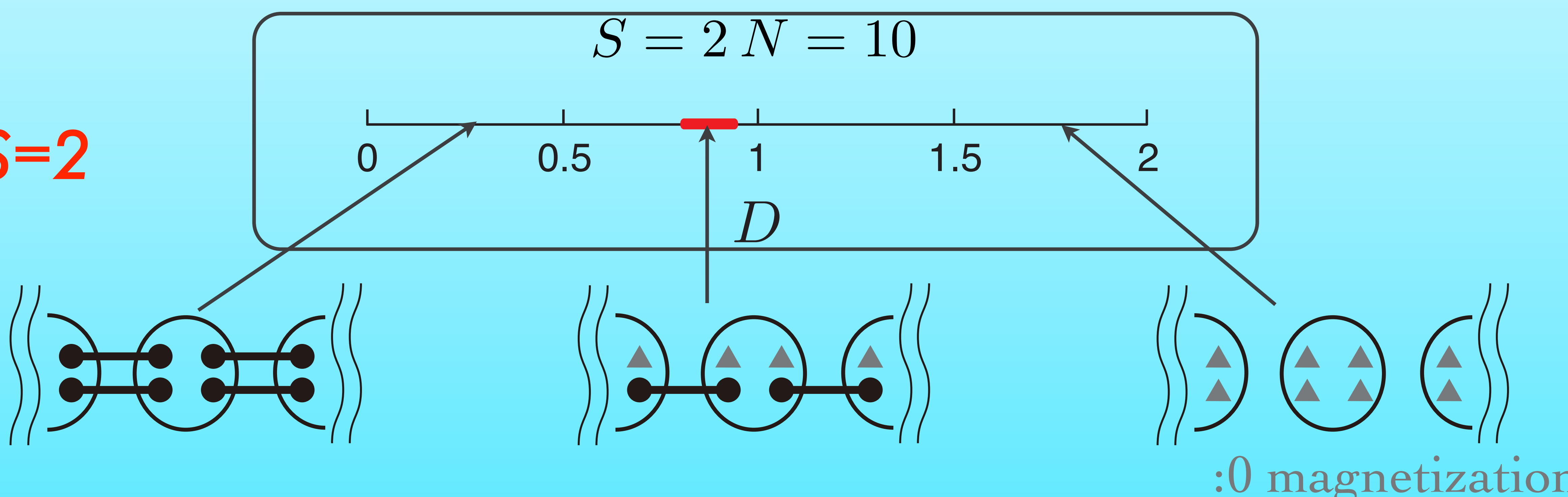
S=2 Heisenberg model with D term

T.Hirano, H.Katsura &YH, Phys.Rev.B77 094431'08

$$H = \sum_{i}^{N} \left[J \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + D \left(S_i^z \right)^2 \right]$$

Red line :Berry phase π

S=2



Reconstruction of valence bonds!

Generic AKLT (VBS) models

T.Hirano, H.Katsura &YH, Phys.Rev.B77 094431'08

Twist the link of the generic AKLT model

$$H(\{\phi_{i,i+1}\}) = \sum_{i=1}^{N} \sum_{J=B_{i,i+1}+1}^{2B_{i,i+1}} A_J P^J_{i,i+1}[\phi_{i,i+1}]$$

VB

$$|\{\phi_{i,j}\}\rangle = \prod_{\langle ij\rangle} \left(e^{i\phi_{ij}/2} a_i^\dagger b_j^\dagger - e^{-i\phi_{ij}/2} b_i^\dagger a_j^\dagger\right)^{B_{ij}} |\text{vac}\rangle$$

Berry phase on a link (ij)

$$\gamma_{ij} = B_{ij}\pi \mod 2\pi$$

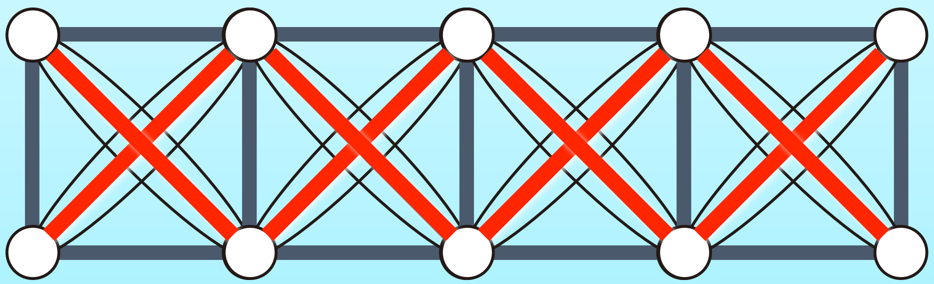
S=1/2

The Berry phase counts the number of the valence bonds!

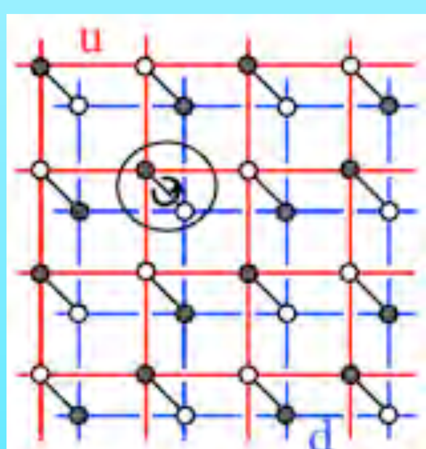
S=1/2 objects are fundamental in integer spin chains

Other systems applied

Spin ladders with ring exchange



I. Maruyama, T. Hirano, and Y. H.,Phys. Rev. B 79, 115107 (2009)
M. Arikawa, S. Tanaya, I. Maruyama, Y. H.,Phys. Rev. B 79, 205107 (2009)

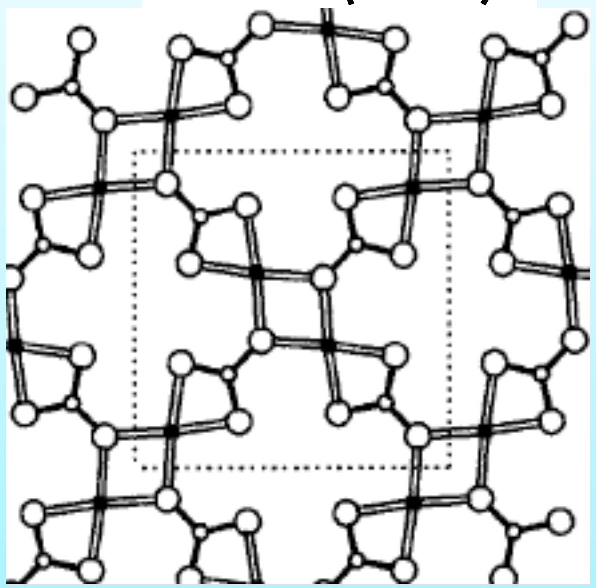


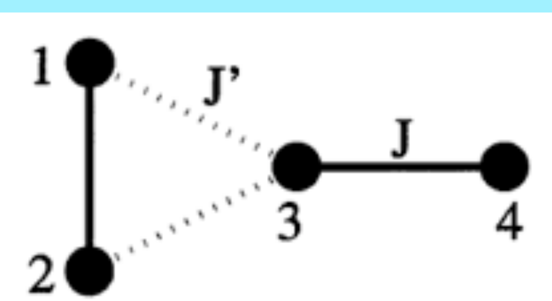
BEC-BCS crossover at half filling

M. Arikawa, I. Maruyama, and Y. H., Phys. Rev. B 82, 073105 (2010)

Orthogonal dimers

Sr $Cu_2(BO_3)_2$



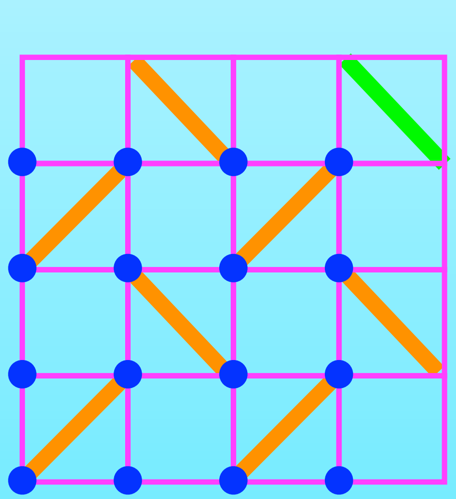


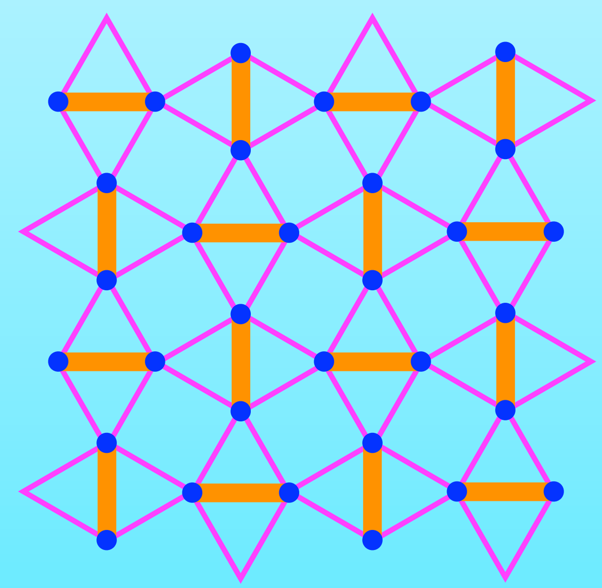
☆ discovery

H. Kageyama et al. , Phys. Rev. Lett. 82, 3168 (1999)

☆ Theory: spin gap & magnetic plateaus

B. S. Shastry and B. Sutherland, Physica, 108B, 1069 (1981).





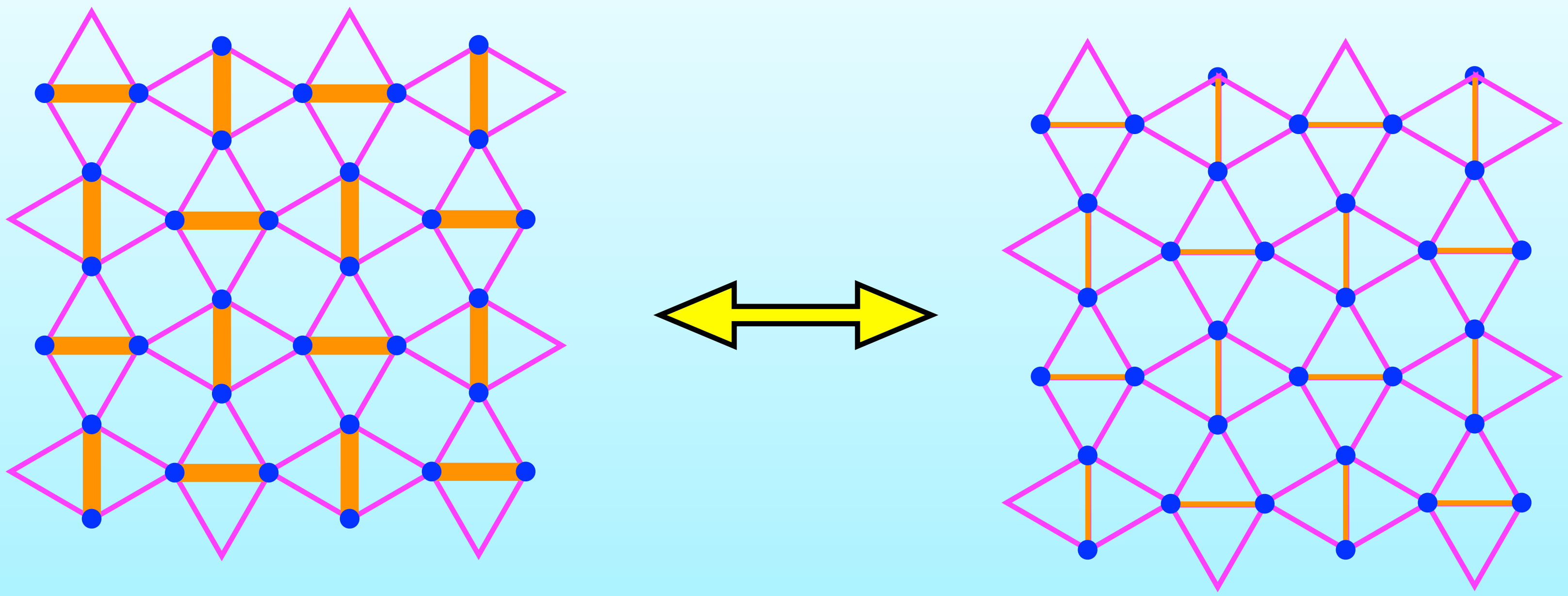
$$H = J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J' \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

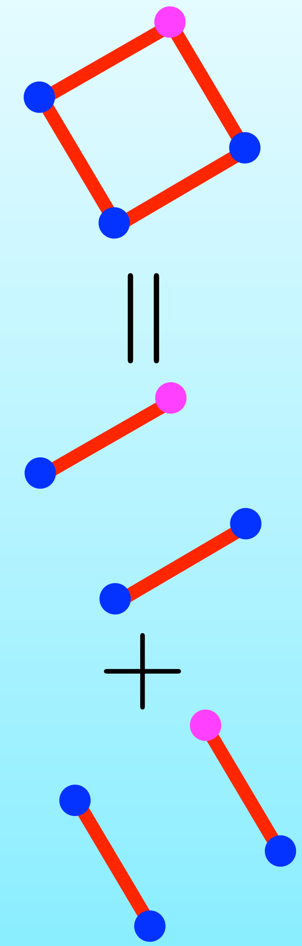
S. Miyahara & K. Ueda , Phys. Rev. Lett. 82, 3701 (1999)

T. Momoi and K. Totsuka, Phys. Rev. B 61, 3231 (2000)

Gapped to gapped transition

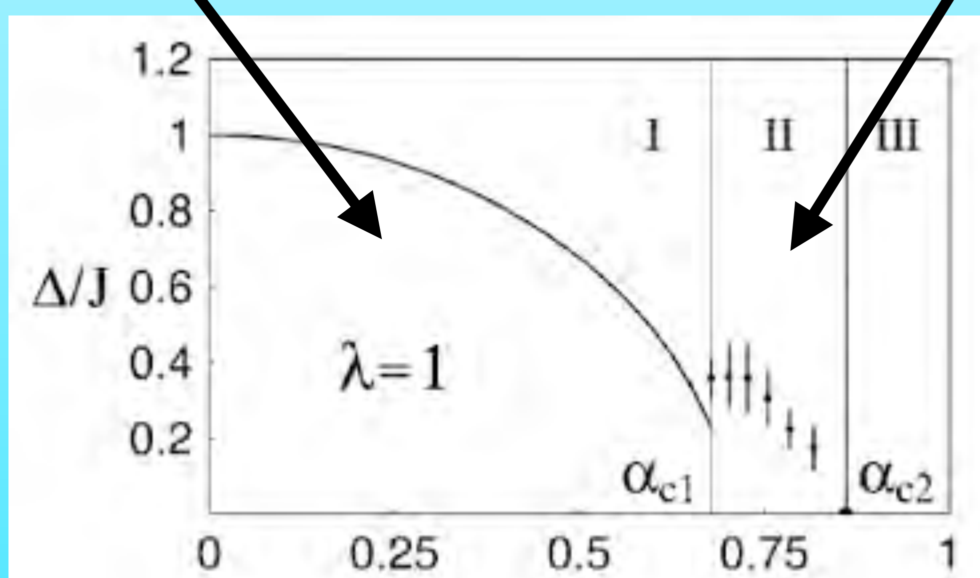
Dimer phase | *Plaquette singlet phase*





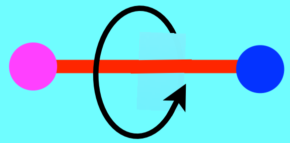
$J >> J'$

$J \approx J'$



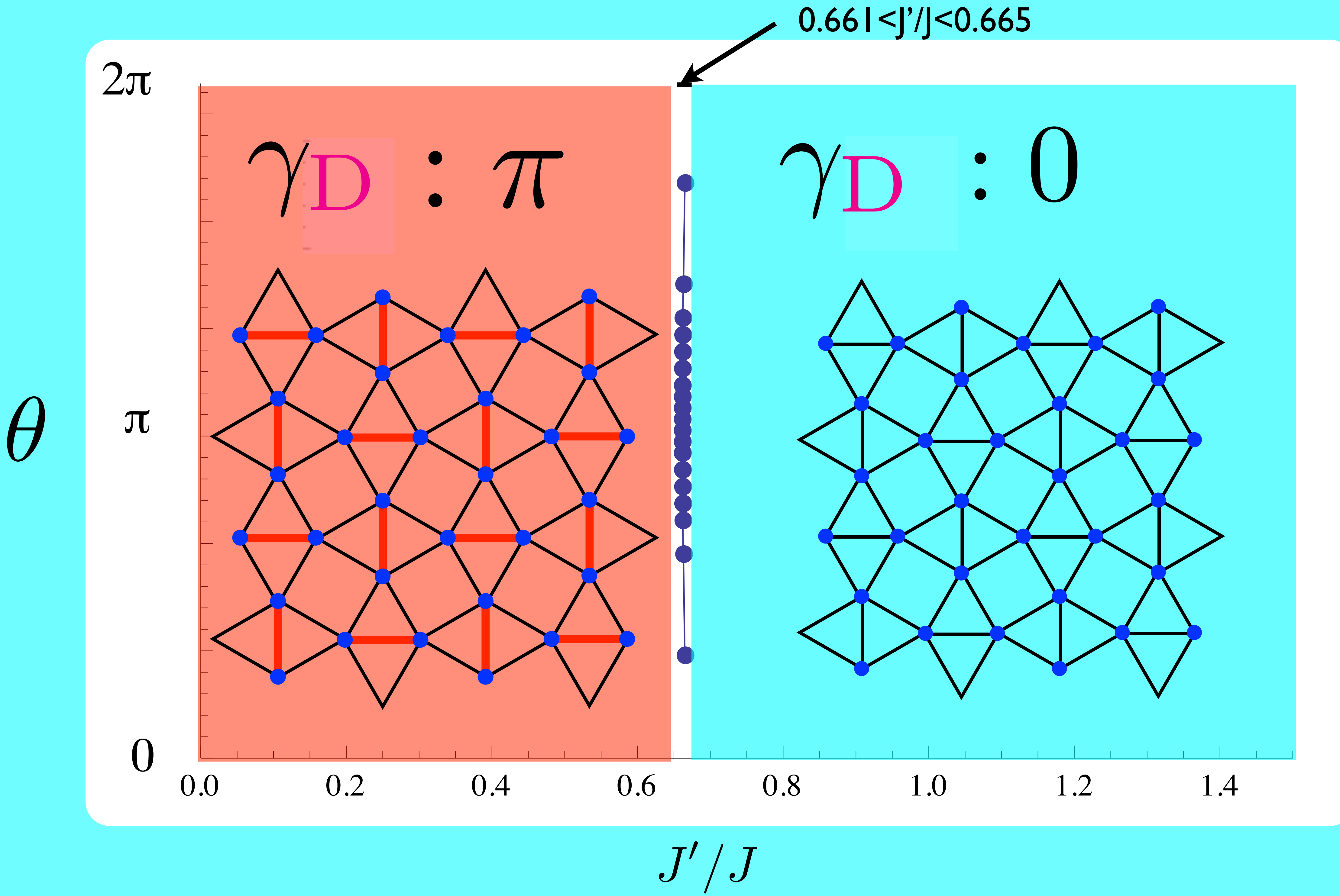
$$H = J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J' \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

A. Koga & N. Kawakami, Phys. Rev. Lett. 84, 4461 (2000)

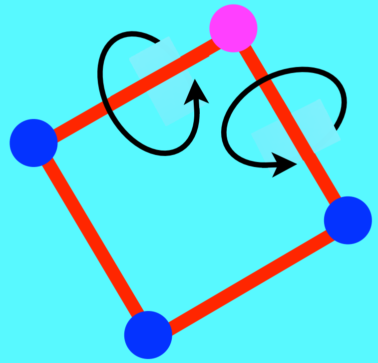


Z_2 Berry phase γ_D

gauge twist for singlet pair

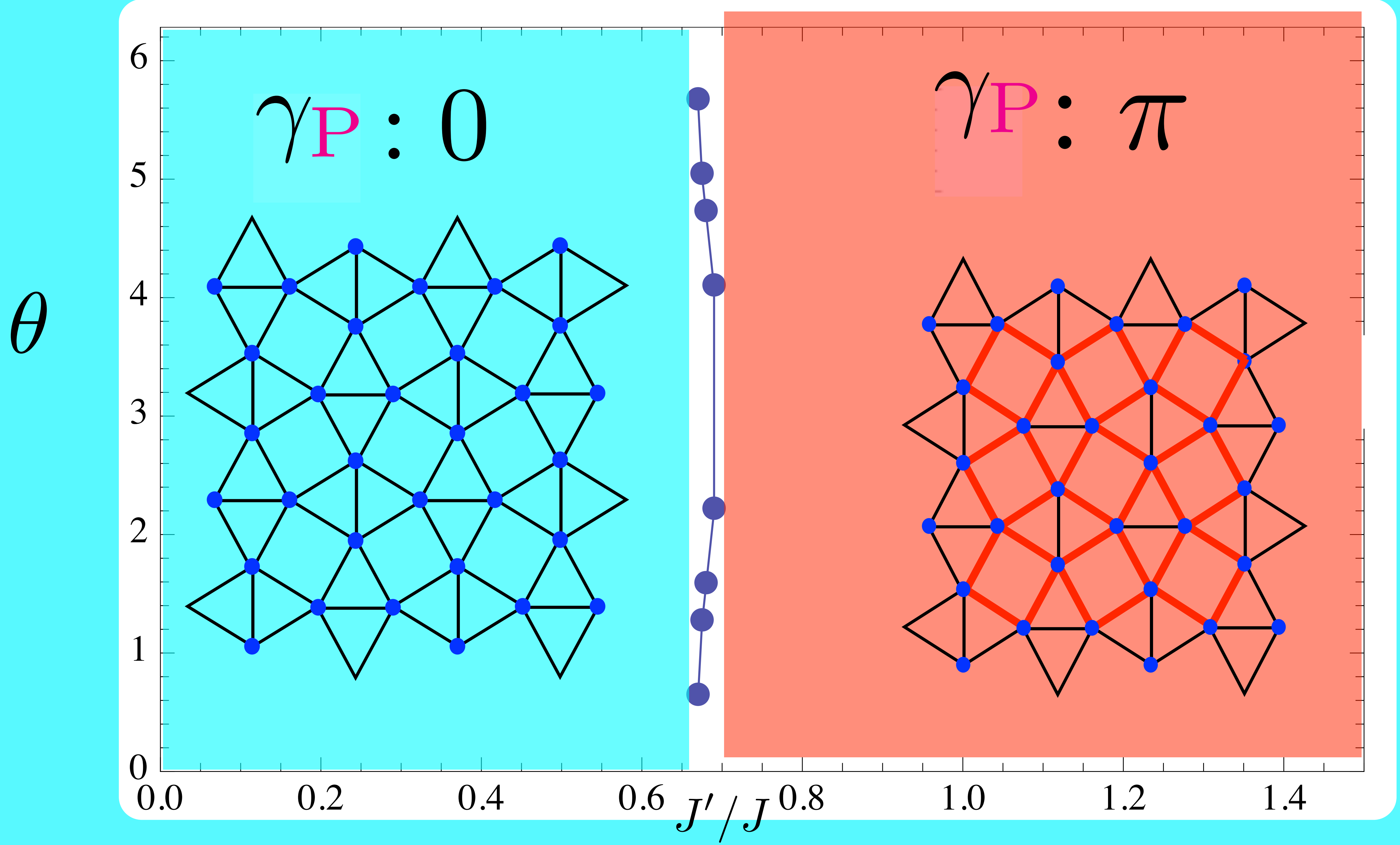


I. Maruyama, S. Tanaya, M.Arikawa & YH. , arXiv:1103.1226



Z_2 Berry phase γ_P

gauge twist for plaquette singlet



I. Maruyama, S. Tanaya, M.Arikawa & YH. , arXiv:1103.1226

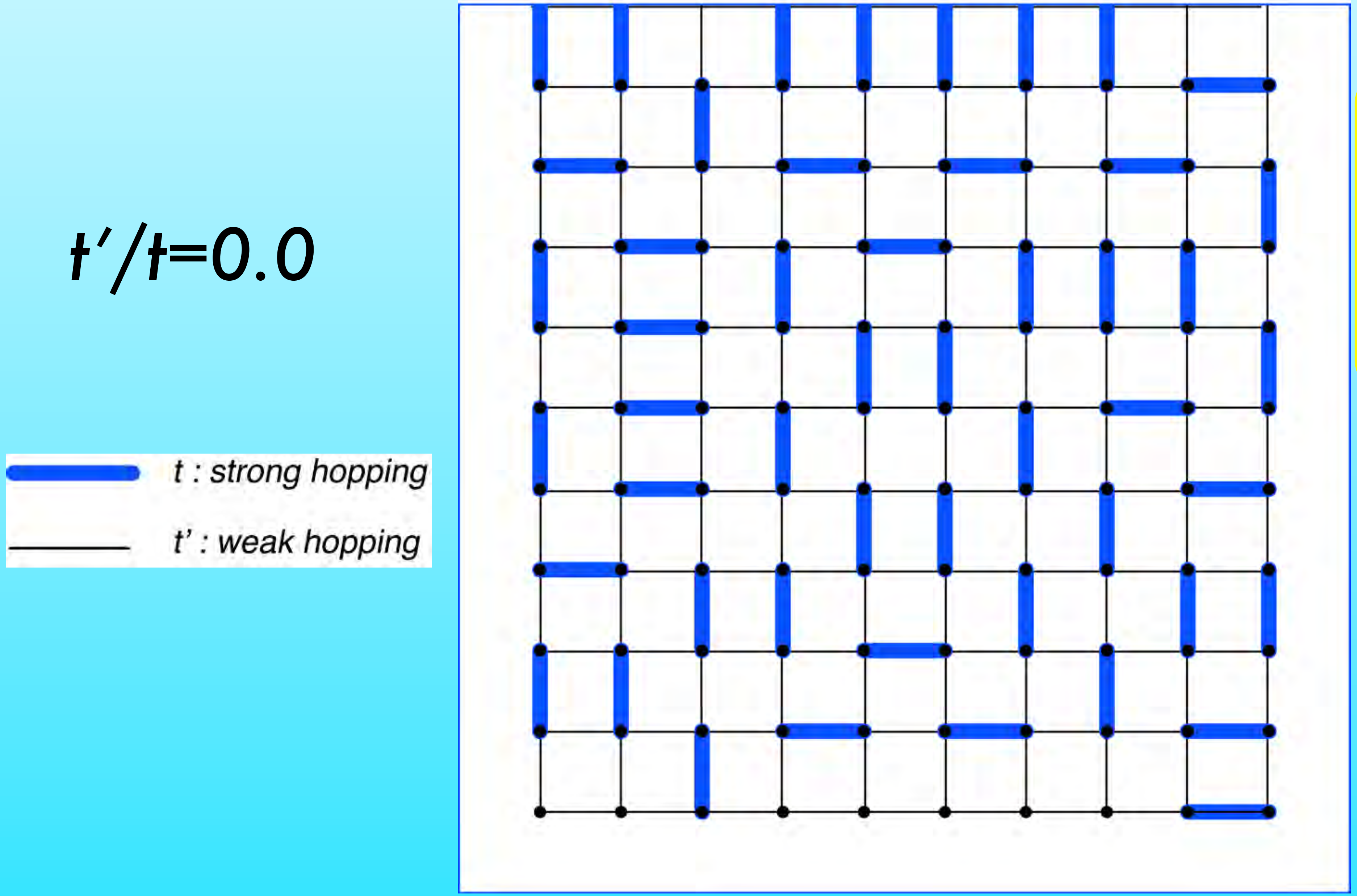
Local Order Parameters of Bonds

- 2D Extended SSH (Su-Schrieffer-Heeger) Model
  - Strong Coupling Limit has a gapped unique ground state.

Distribution of hoppings

$$V_{ij} = 0$$

t'/t=0.0



Non-Abelian Connection for the Fermi-Sea

Large System is available

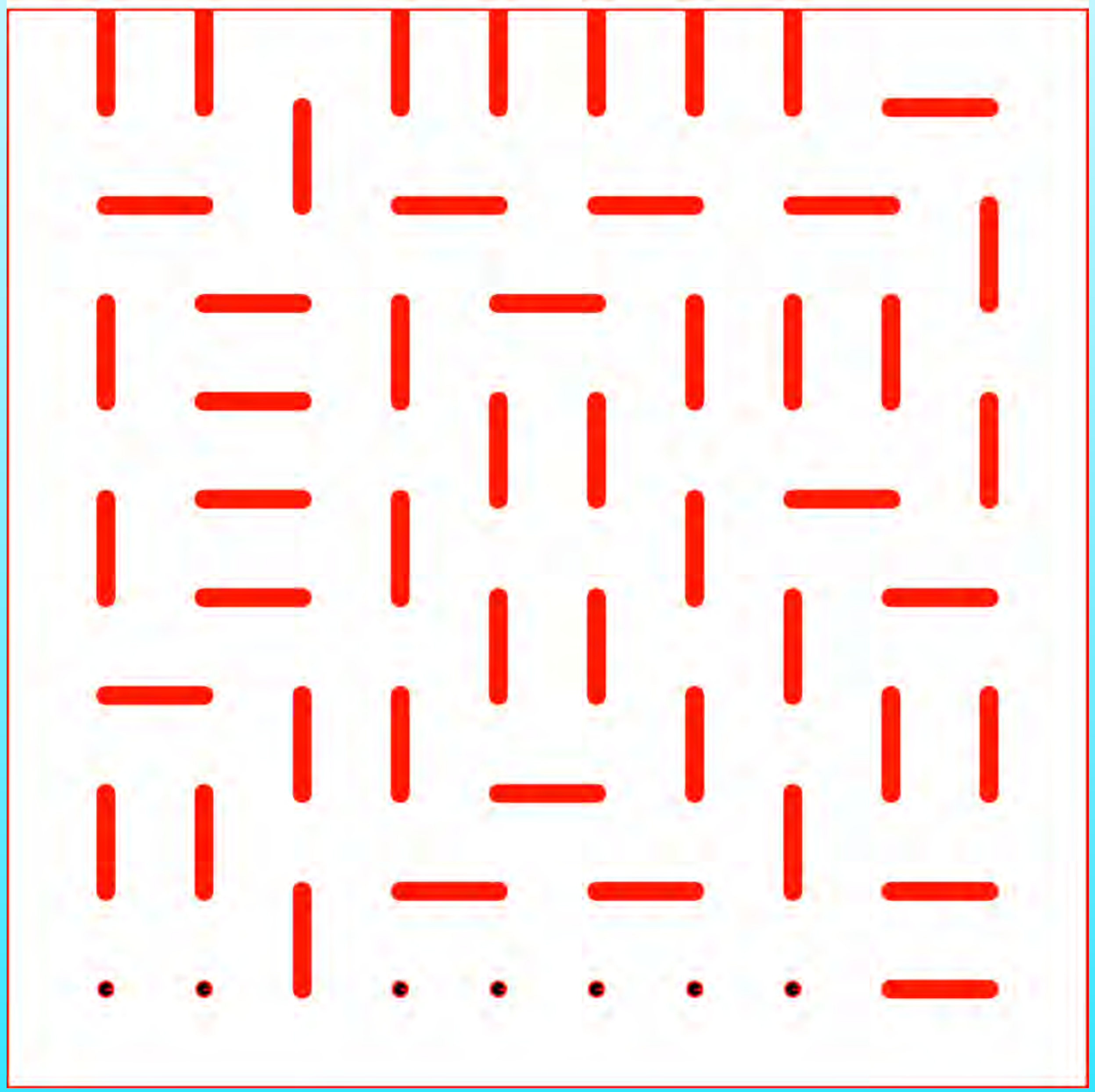
YH, JPSJ. 73, 2604 (2004), 74, 1374 (2005)

Local Order Parameters of Dimer Pairs

- 2D Extended SSH (Su-Schrieffer-Heeger) Model
  - Strong Coupling Limit has a gapped unique ground state.

Distribution of the Quantized Berry Phases

$t'/t=0.6$



$$\gamma_C = \pi$$

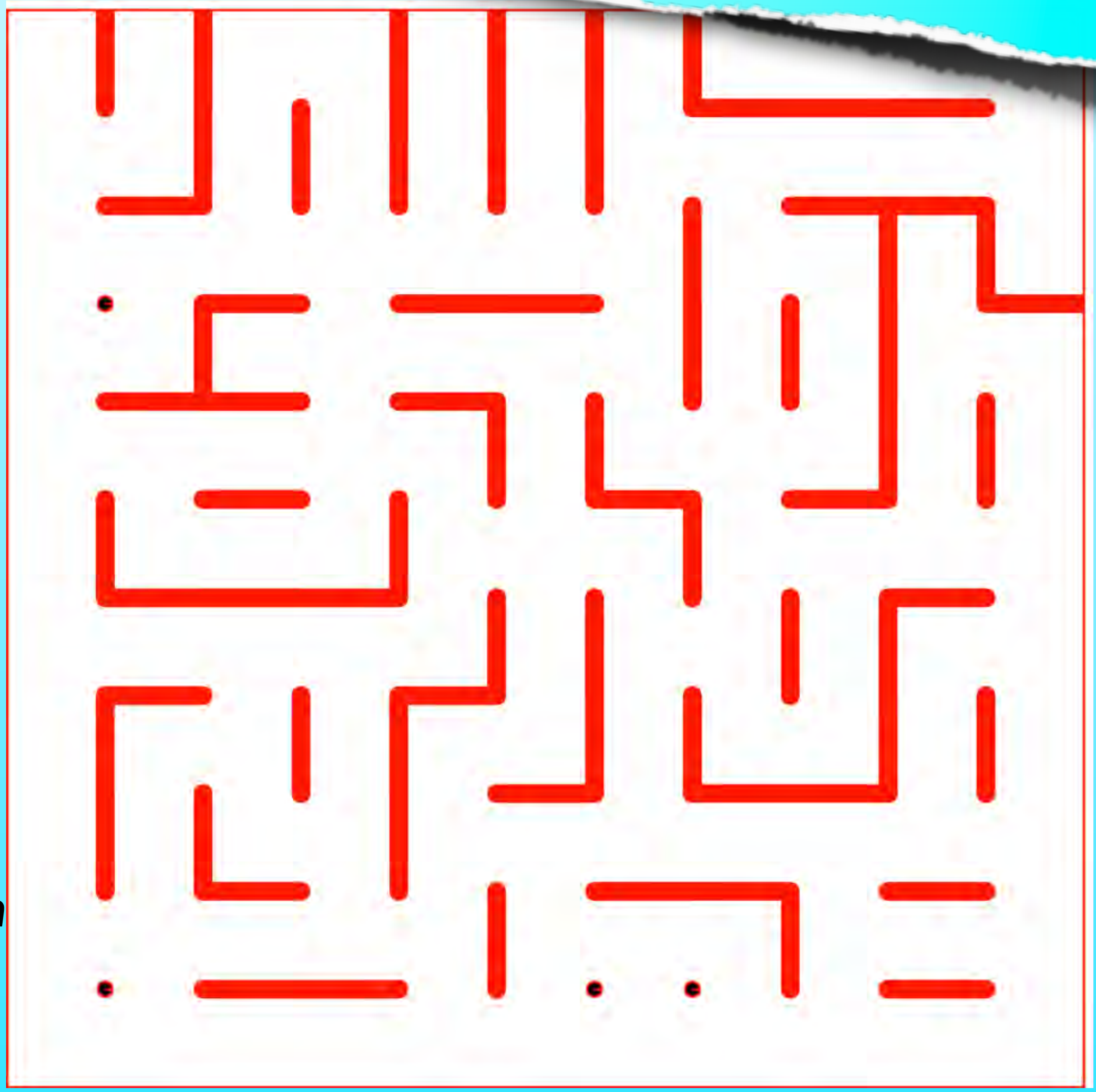
Local Order Parameters of Dimer Pairs

- 2D Exte[…]ded SSH (Su-Schrieffer-Heeger) Model
  - Strong C[…] […]nique ground state.

Reconstruction of bonds

Distributio[…] […]hases

t′/t=0.7

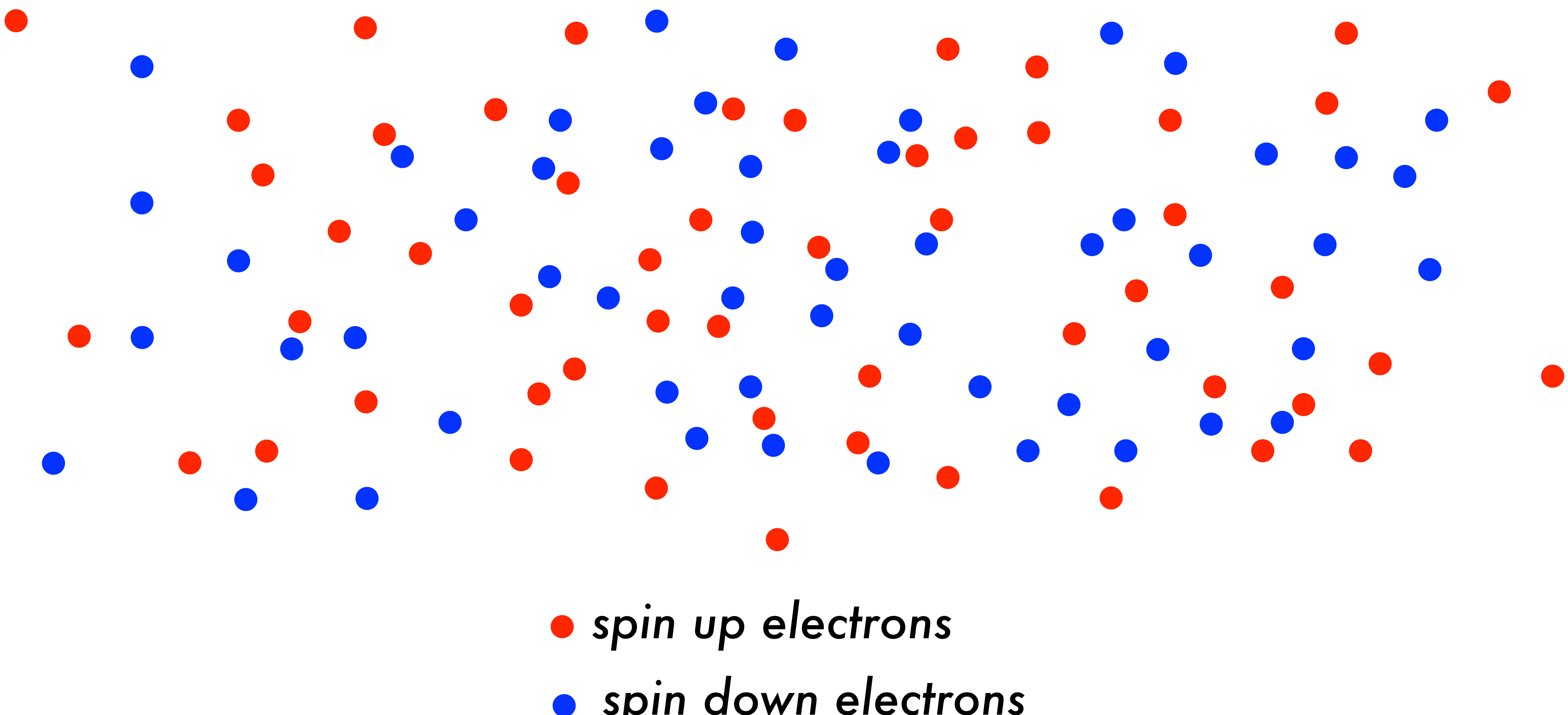


$\gamma_C = \pi$

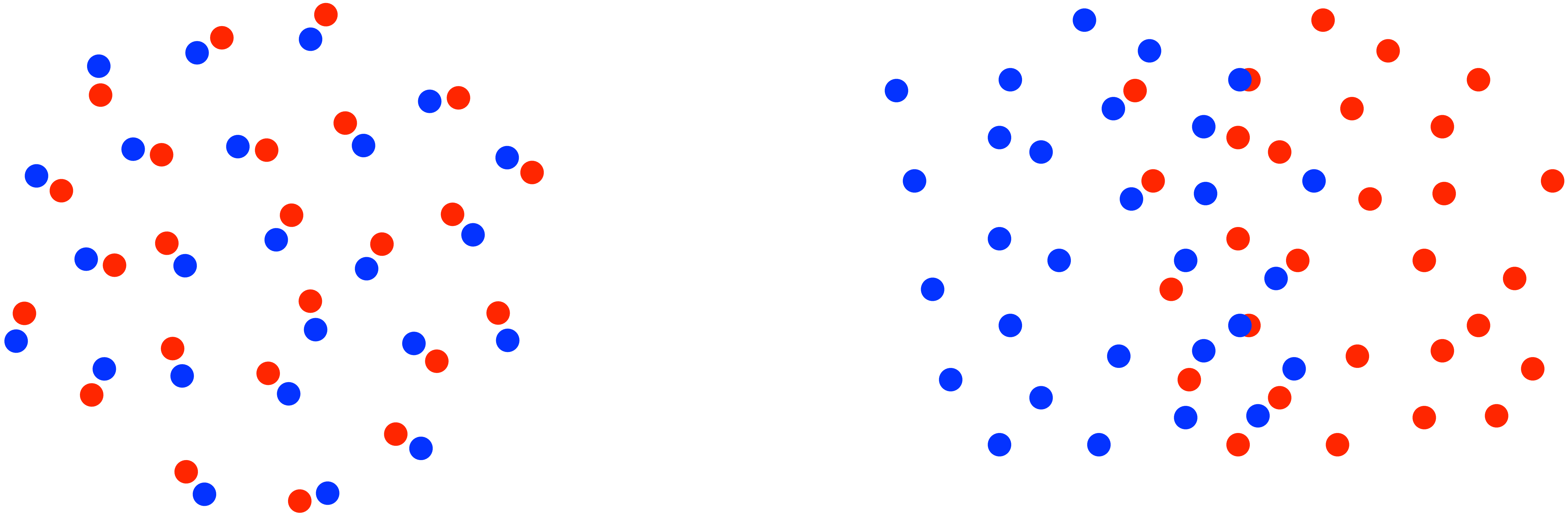
Quantum Phase Transition with (local) Gap Closing

BEC-BCS crossover as a local quantum phase transition

Switching on attractive interaction among particles



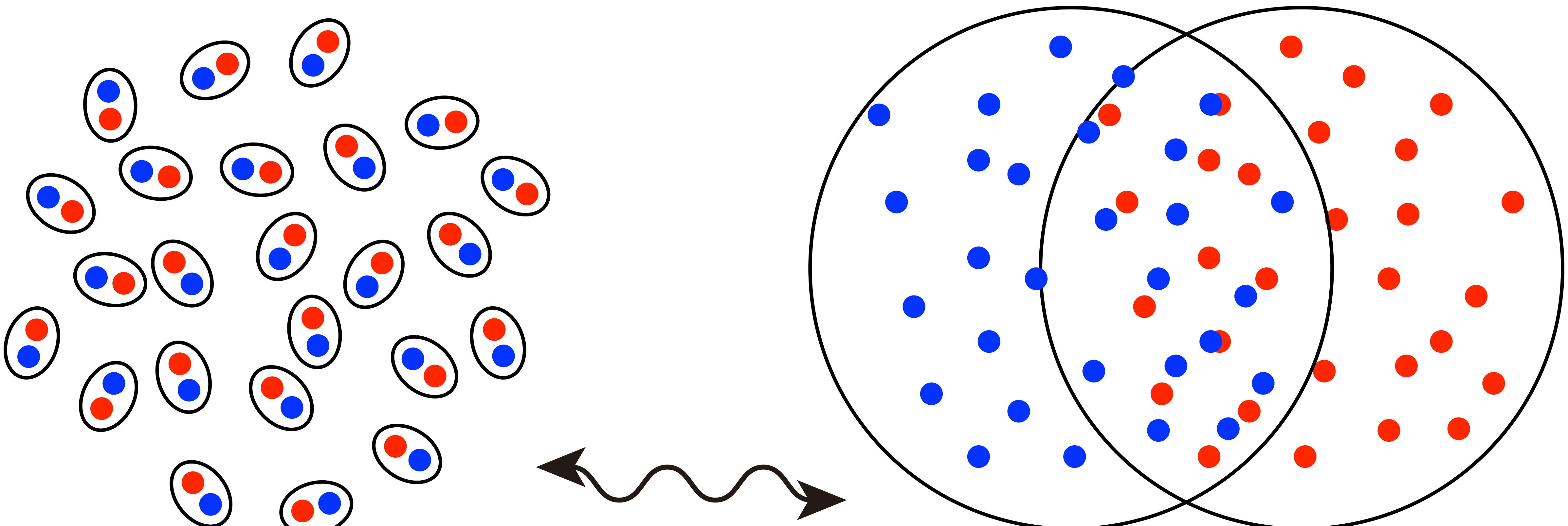
BEC-BCS crossover as a local quantum phase transition



BEC-BCS crossover
as a local quantum phase transition

BEC : strong coupling

BCS : weak coupling



Crossover
adiabatically connected

Making bosons in real space
then condense

Cooper pairing
in momentum space

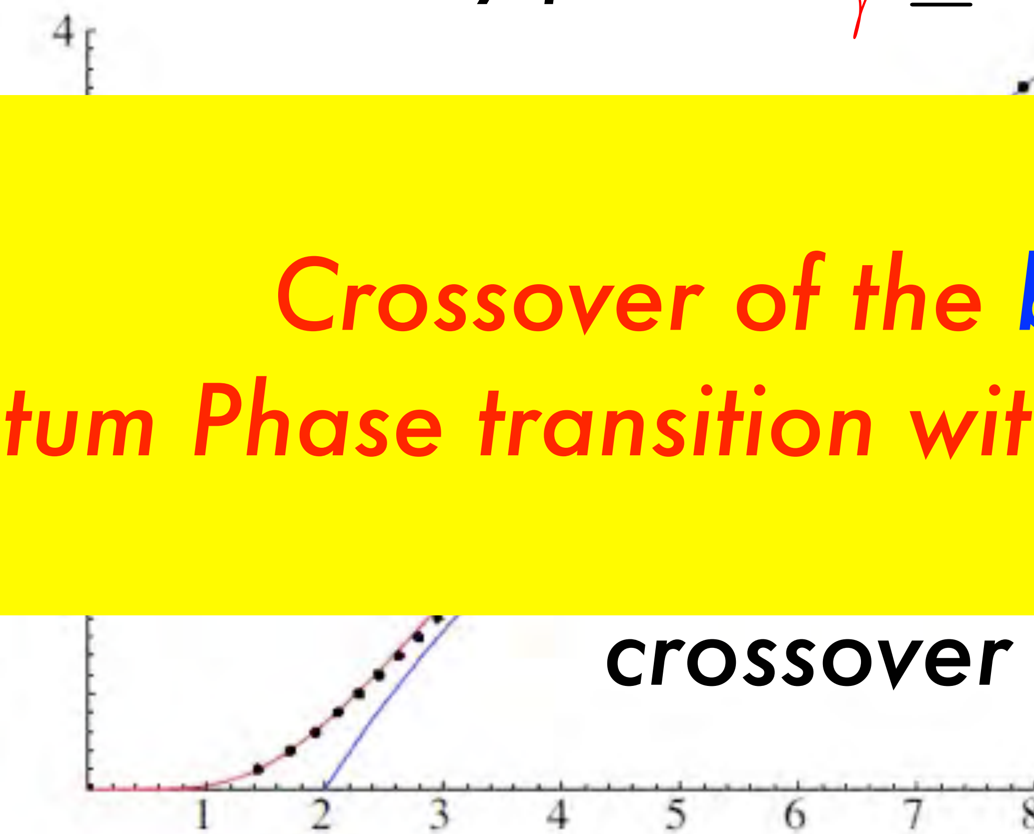
BCS Model at half filling

Arikawa-Maruyama-YH, 2010

$$H = -t \sum_{\sigma,i,j} c^\dagger_{i\sigma} c_{j\sigma} - |U| \sum_{ij} \Delta_{ij} c_{i\uparrow} c_{i\downarrow}$$

modify only at special (local) order parameter $\Delta_{ij} \to \Delta_{ij} e^{i\theta}$ $(\theta : 0 \to 2\pi)$

to calculate the Berry phase $\gamma = -i \int_0^{2\pi} d\theta \, \langle \psi | \partial_\theta \psi \rangle$



Crossover of the bulk
by Quantum Phase transition with local gauge twist

crossover : gapped always

weak coupling (BCS) strong coupling (BEC)

$\gamma = 0$ $\gamma = \pi$

$|U_C|/t = 2/\sqrt{3} = 1.15, \; 1.25, \; 1.6$

1D 2D 3D