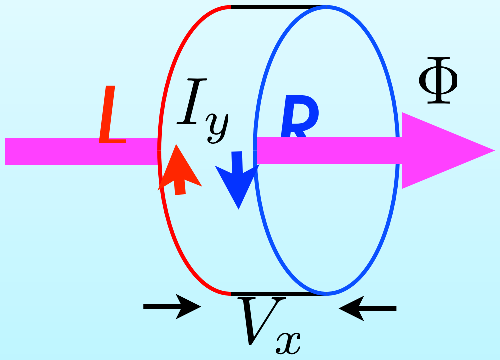
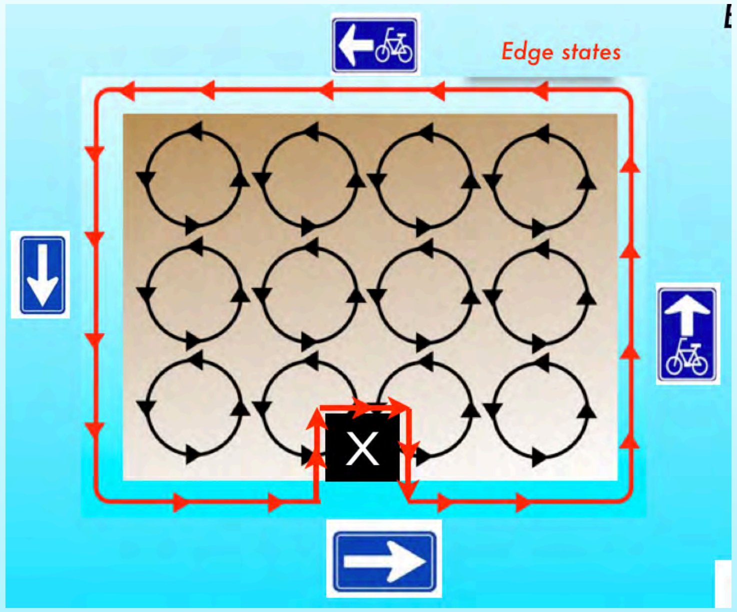
バルクエッジ対応の物理の多様性と普遍性





量子ホール効果・Laughlinの議論

Mother of topological phases

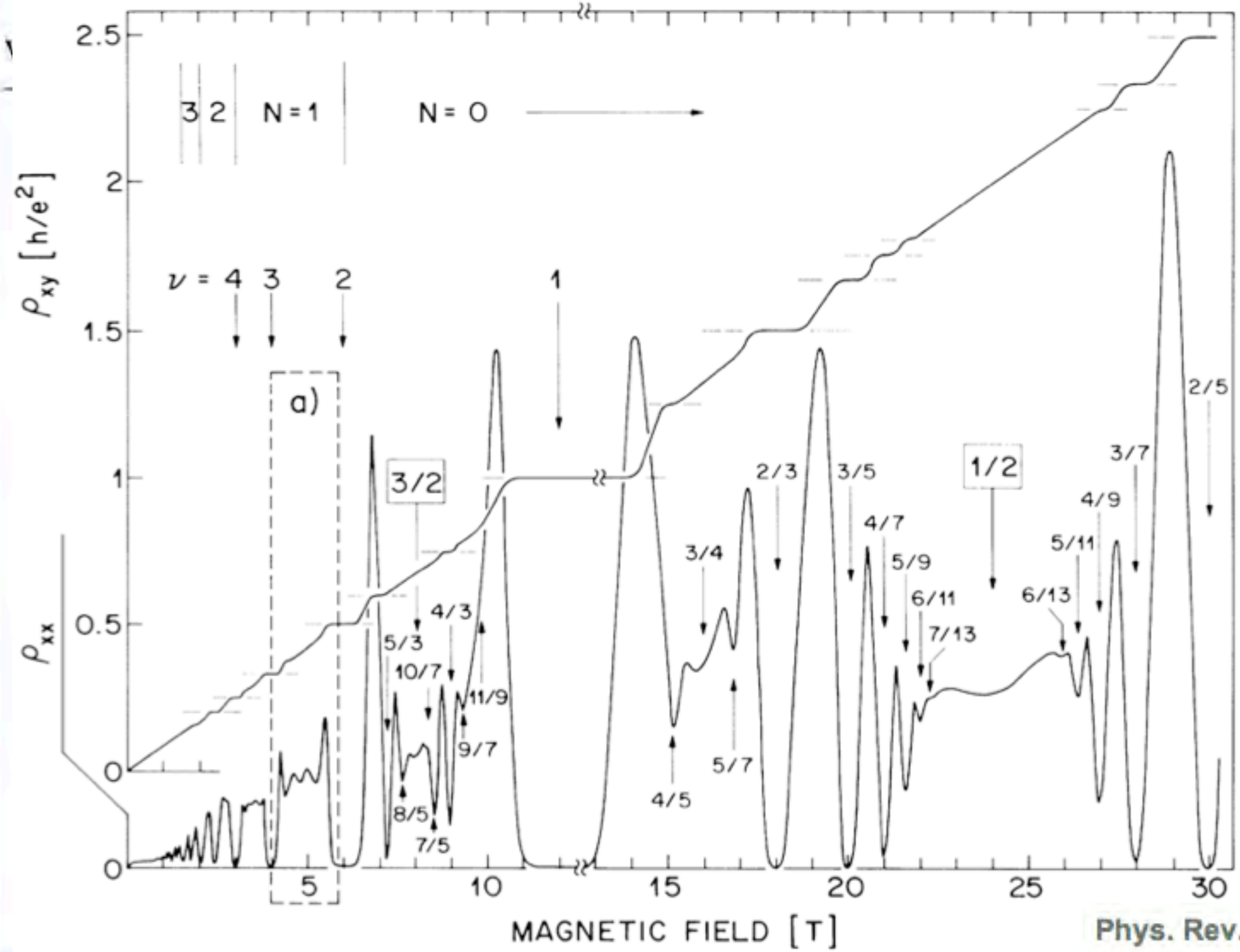
TKNN# & Chern #

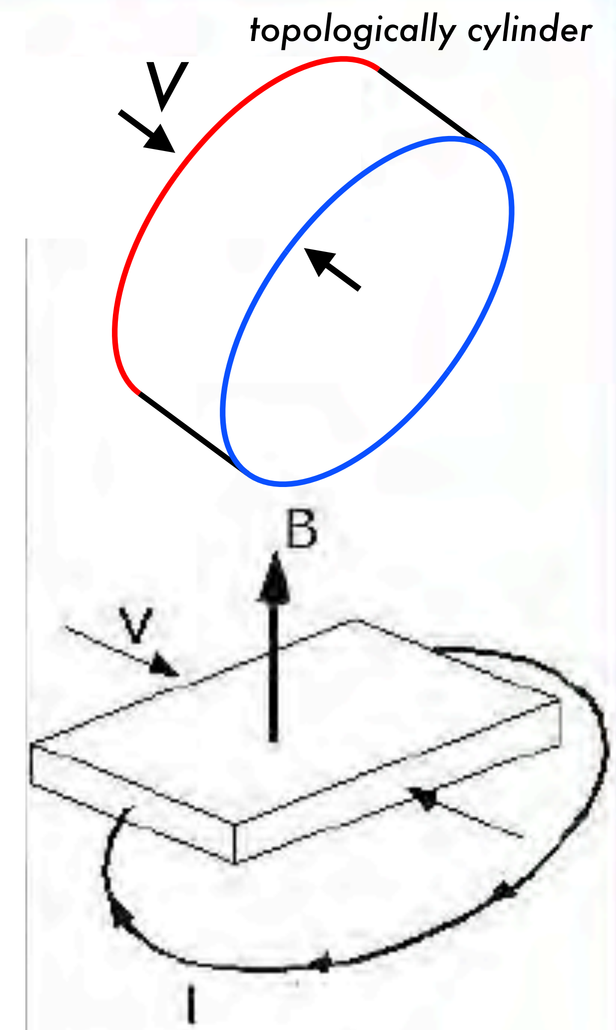
筑波大学数理物質系 物理学域
初貝　安弘

Topol. char. by edges

Quantum Hall Effect '80, K.v.Klitzing et al.

Quantization of the Hall conductance σ_{xy} with anomalous accuracy: $I = \sigma_{xy} V$



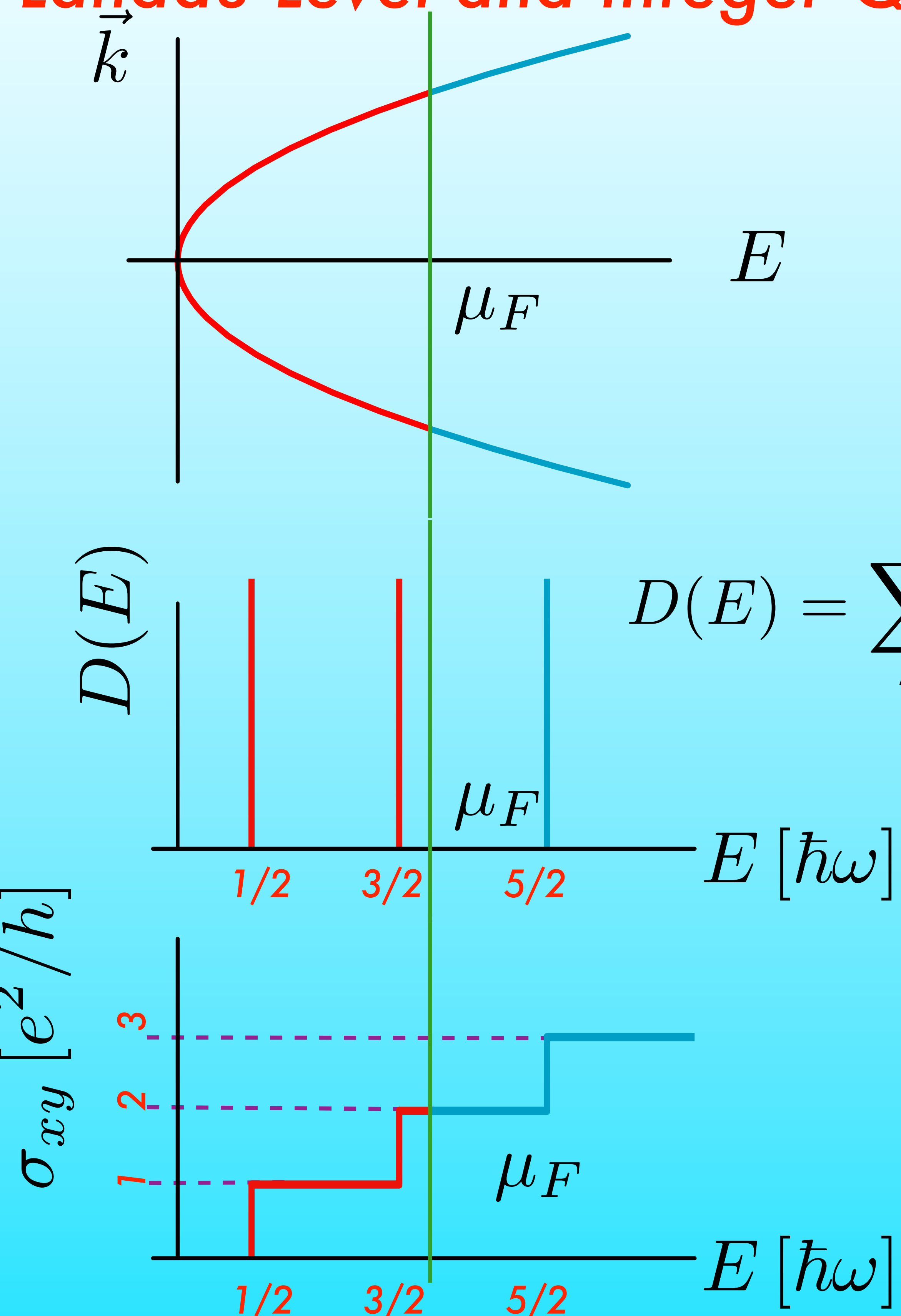


Phys. Rev. Lett. 59, 1776–1779 (1987)

R. Willett et al.

Conventional QHE

☆ Landau Level and Integer QHE



$$E(B=0) = \frac{\hbar^2}{2m} k^2$$

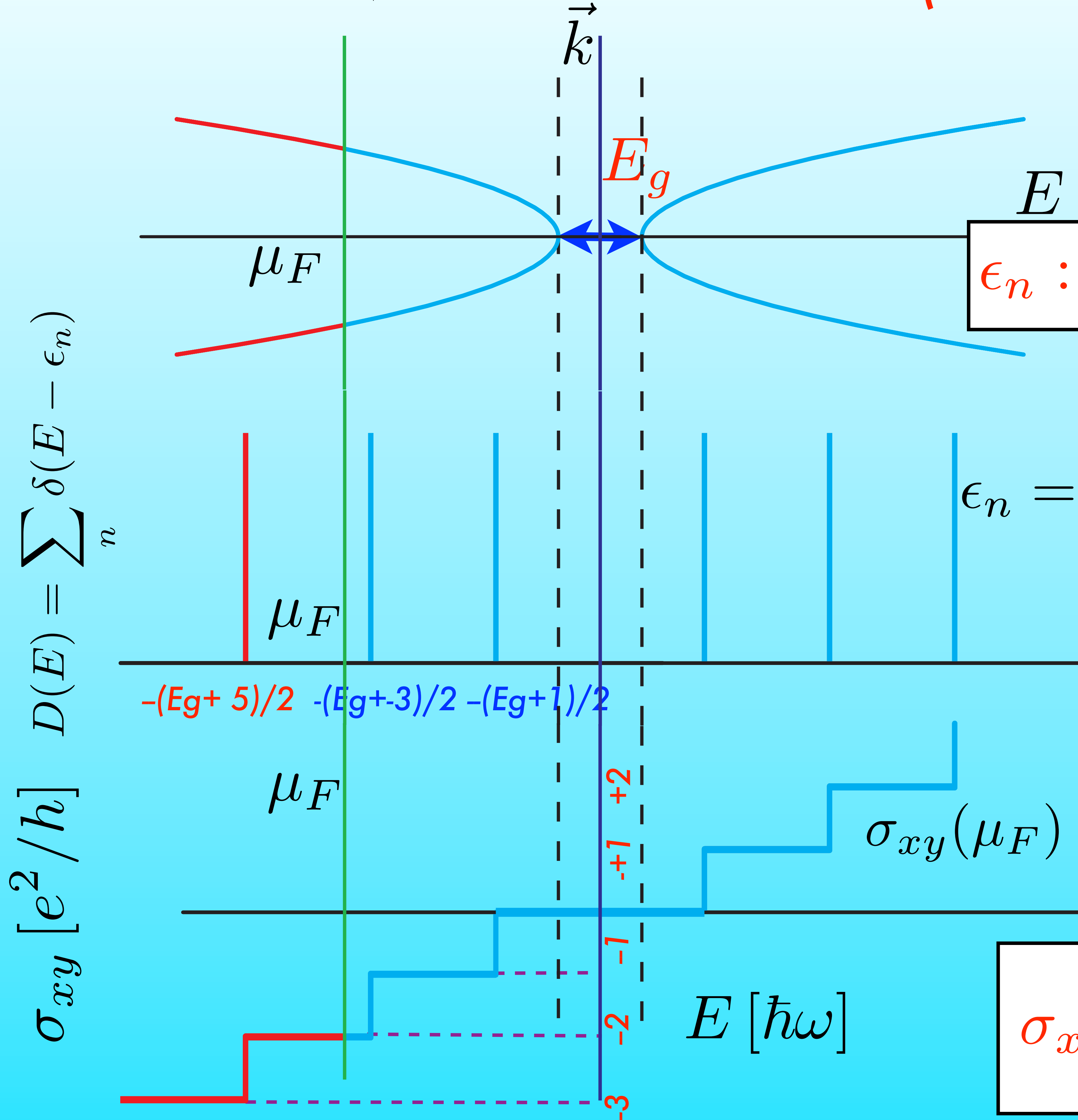
$$D(E) = \sum_n \delta(E - \epsilon_n) \qquad \epsilon_n = \hbar\omega(n + \frac{1}{2})$$

$$\sigma_{xy}(\mu_F) = \frac{e^2}{\hbar} n, \quad n = 1, 2, 3, \cdots$$

$$\epsilon_{n-1} < \mu_F < \epsilon_n$$

QHE of Semiconductors

☆ Landau Level of Valence band (Holes)



$$E(B=0) = \pm \frac{\hbar^2}{2m} k^2$$

$$\boxed{\epsilon_n \text{ : zero point energy } 1/2}$$

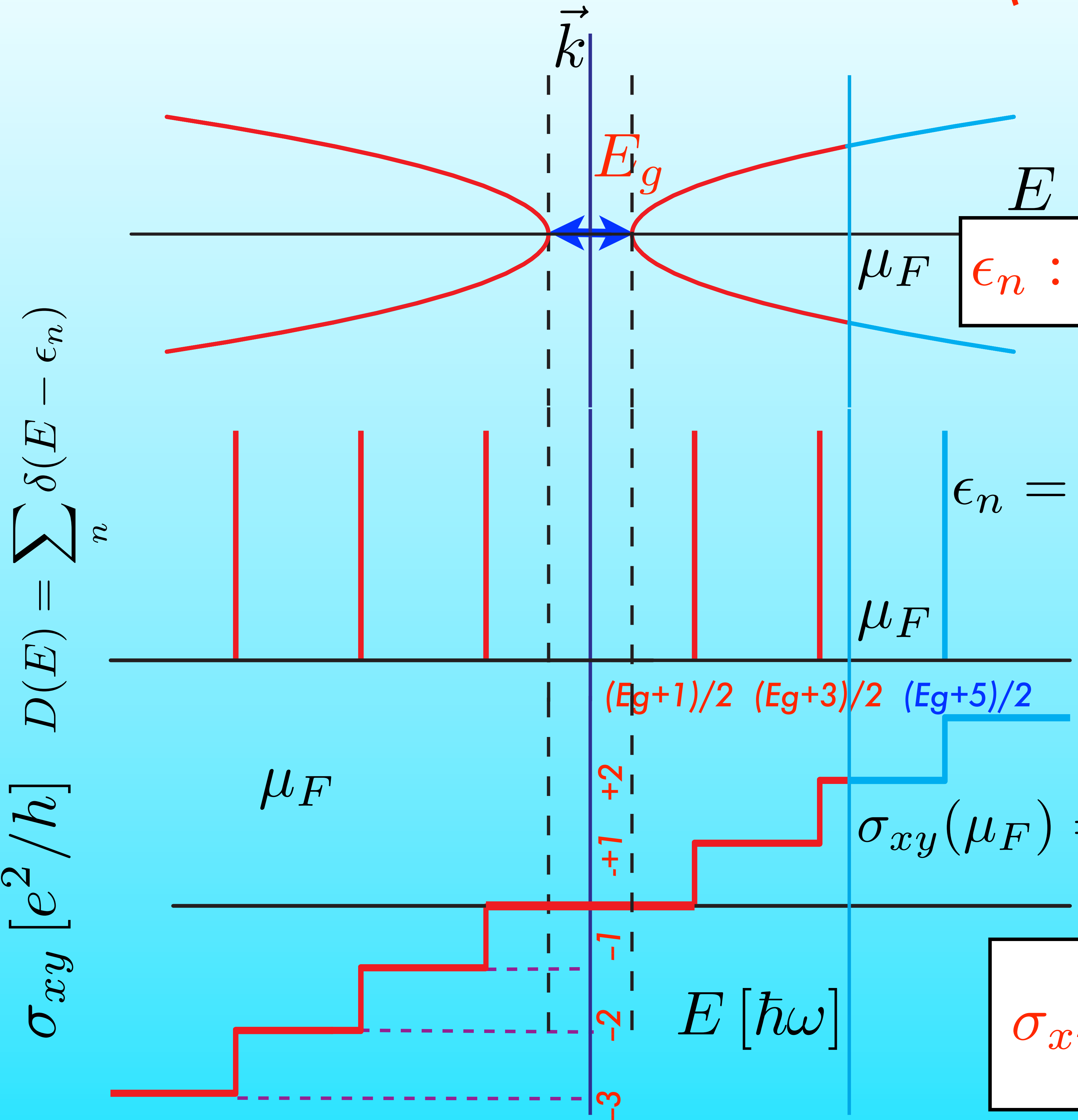
$$\epsilon_n = \begin{cases} E_g/2 + \hbar\omega(n + \frac{1}{2}) & n \geq 0 \\ -E_g/2 + \hbar\omega(n + \frac{1}{2}) & n < 0 \end{cases}$$

$$\sigma_{xy}(\mu_F) = \frac{e^2}{\hbar} \times \begin{cases} n & (n > 0) \\ n+1 & (n \leq 0) \end{cases}$$

$$\boxed{\sigma_{xy} : \frac{e^2}{h} \times \text{integer}}$$

QHE of Semiconductors

☆ Landau Level of Conduction band (Electrons)



$$E(B=0) = \pm\frac{\hbar^2}{2m}k^2$$

$$\boxed{\epsilon_n \text{ : zero point energy } 1/2}$$

$$\epsilon_n = \begin{cases} E_g/2 + \hbar\omega(n+\frac{1}{2}) & n \geq 0 \\ -E_g/2 + \hbar\omega(n+\frac{1}{2}) & n < 0 \end{cases}$$

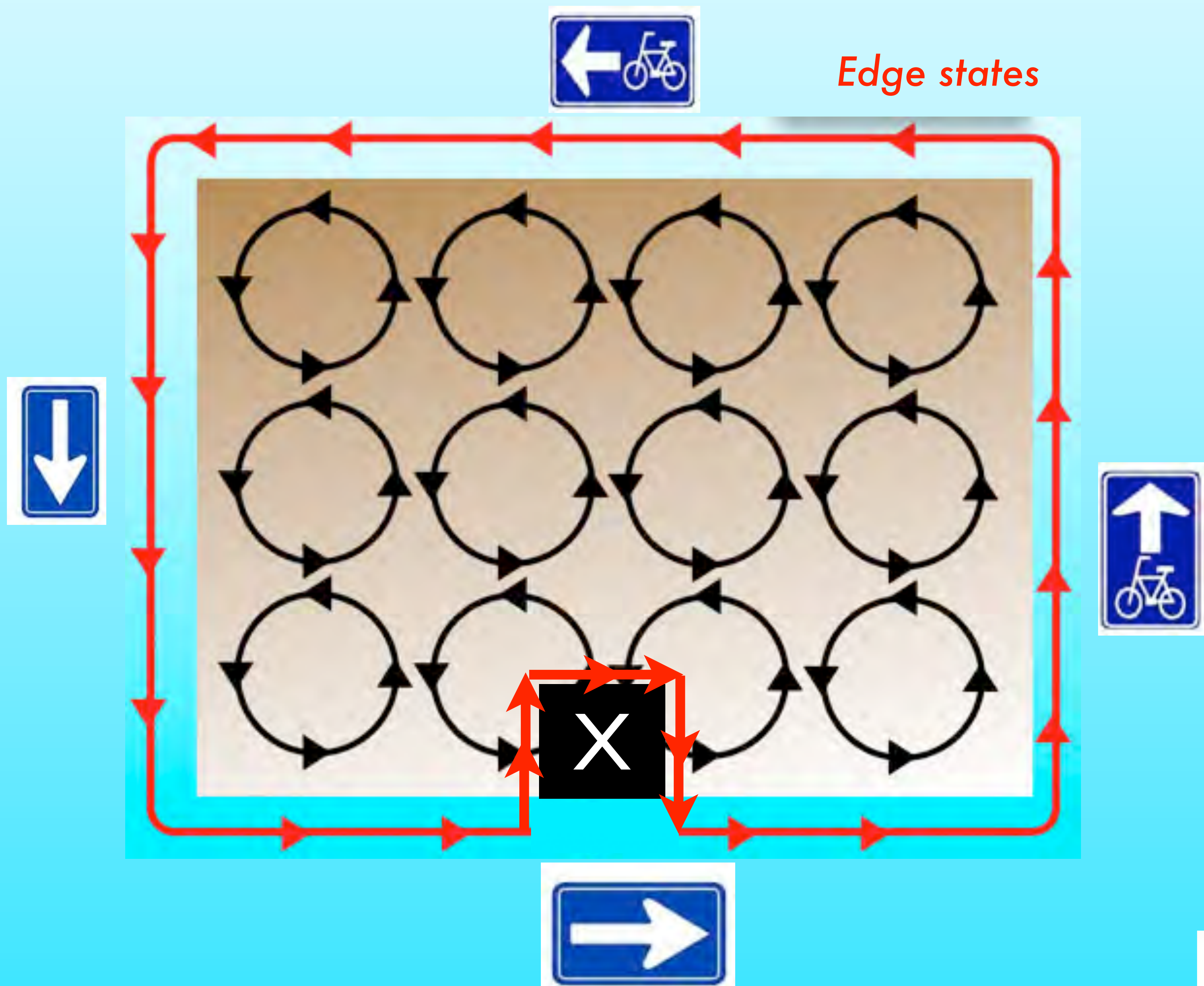
$$\sigma_{xy}(\mu_F) = \frac{e^2}{\hbar} \times \begin{cases} n & (n > 0) \\ n+1 & (n \leq 0) \end{cases}$$

$$\boxed{\sigma_{xy} : \frac{e^2}{h} \times \text{integer}}$$

Edge states are topologically stable

Cyclotron motion by Lorentz force $\boldsymbol{F} = -e\boldsymbol{v} \times \boldsymbol{B}$

Currents are canceled in the bulk but induces a boundary current



Edge states are chiral

One way going !!

Cannot stop !

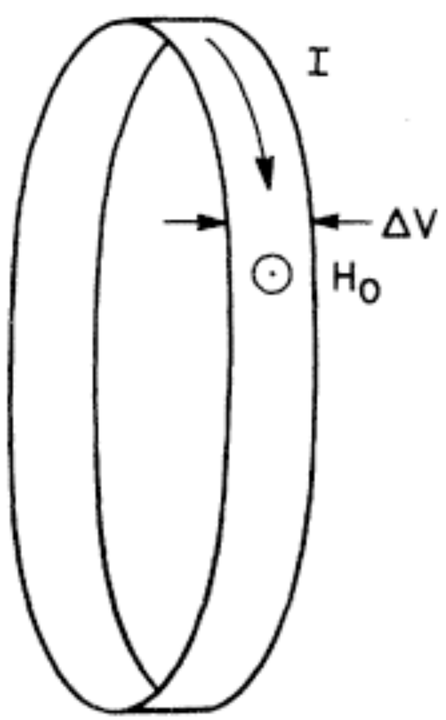
No back scattering

Stable for impurities !!

Topological stability
of
Chiral edge states

☆ Edge states in quantum Hall effects

PHYSICAL REVIEW B VOLUME 23, NUMBER 10 15 MAY 1981



Quantized Hall conductivity in two dimensions

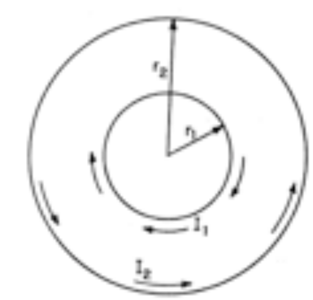
R. B. Laughlin
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 20 January 1981)

Everything started from here
discuss later

PHYSICAL REVIEW B VOLUME 25, NUMBER 4 15 FEBRUARY 1982

Quantized Hall conductance, current-carrying edge states, and the existence of extended states in a two-dimensional disordered potential

B. I. Halperin



PHYSICAL REVIEW B VOLUME 48, NUMBER 16 15 OCTOBER 1993-II

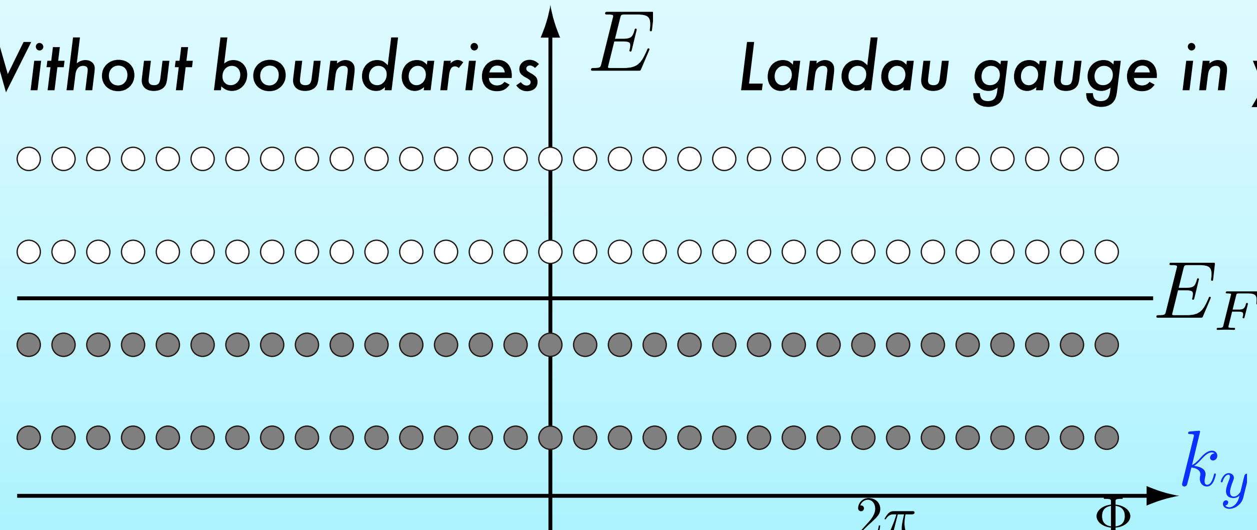
Edge states in the integer quantum Hall effect and the Riemann surface of the Bloch function

Yasuhiro Hatsugai*

Stability of the quantized Hall Conductance

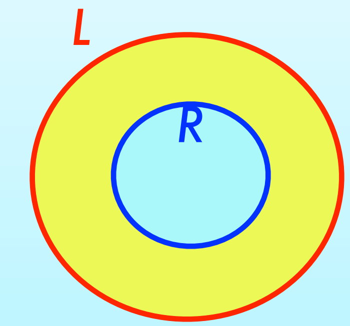
☆ **Edge states and Hall conductance σ_{xy}** Halperin '82

Without boundaries



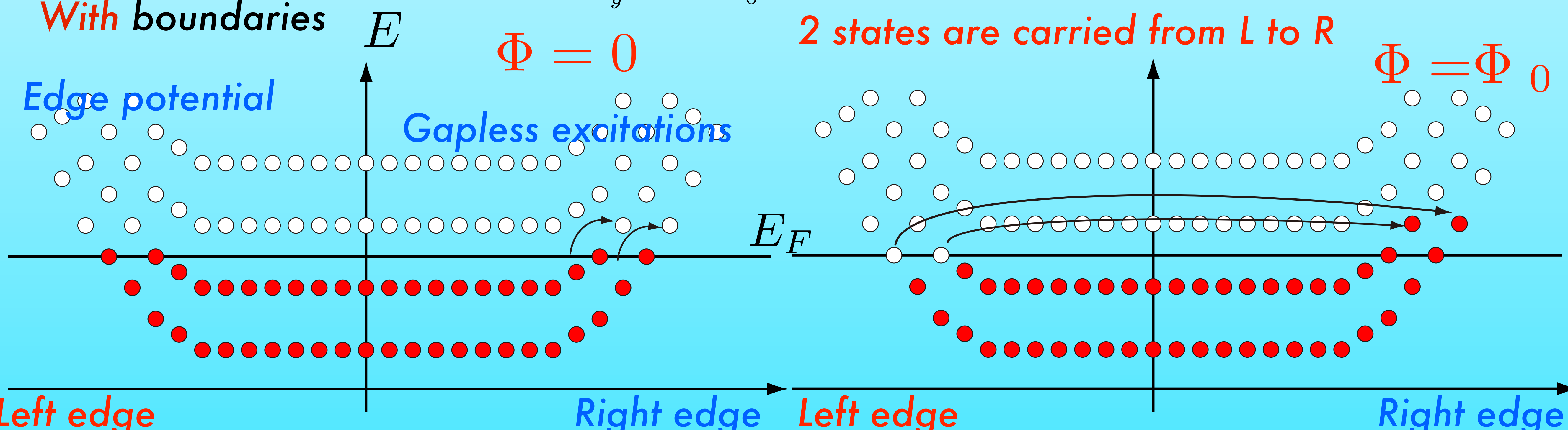
$$k_y = \frac{2\pi}{L_y}(n_y + \frac{\Phi}{\Phi_0}) \quad n_y = 0, \pm 1, \pm 2, \cdots$$

$$H_{2D} = \sum_{k_y} H_{1D}(k_y)$$



$H_{1D}(k_y)$: harmonic osc. centered at $\langle x \rangle \sim \ell_B^2 k_y$

With boundaries



Laughlin's undetermined n : # of Landau Levels below E_F

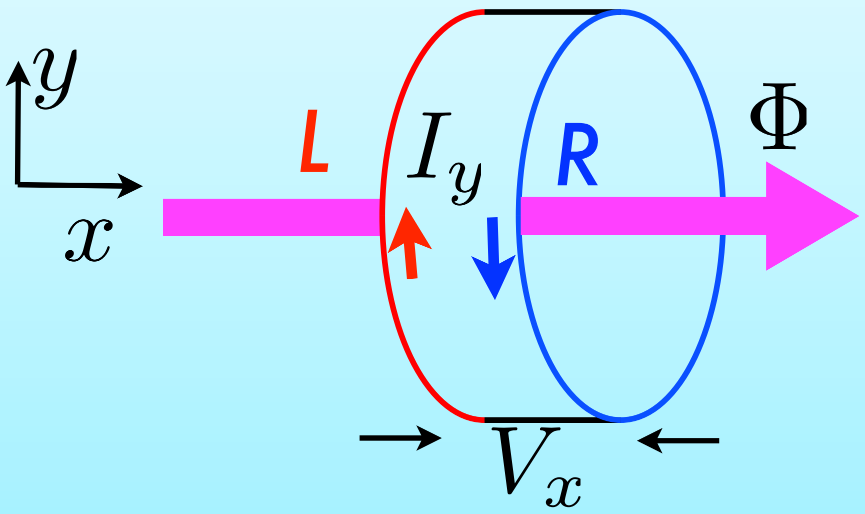
Edge states are essential in the QHE !

Stability of the quantized Hall Conductance

☆ Gauge invariance and quantization of σ_{xy}

Laughlin '81

adiabatic process to increase Φ



$$A \to A' = A + \nabla\Phi, \quad \delta\Phi = \int_{\bigcirc} (A' - A)$$

$$\psi \to \psi' = e^{i2\pi\delta\Phi/\Phi_0}\,\psi$$ one particle state

$$\delta\Phi = \Phi_0 = \frac{h}{e} \longrightarrow \psi' = \psi$$

Gauge transformation

flux quantum

Byers-Yang formula

$$I_y = \frac{\Delta E}{\Delta\Phi} = \frac{neV_x}{h/e} = \underbrace{\boxed{n(\frac{e^2}{h})}}_{\sigma_{yx}} V_x = \sigma_{yx} V_x$$

$$\Delta\Phi = \Phi_0 = \frac{h}{e}, \quad \Delta E = n \cdot eV_x$$

All states are invariant up to phase after the process:

Some n states are carried from L to the R

n : generic integer (but undetermined)

Emerging Dirac Fermions in condensed matter

- ☆ Bloch Electrons in a magnetic field (Hofstadter problem)
- ☆ Graphene & 2D gapless superconductors

Non Abelian gauge structures in condensed matter

- ☆ Quantum Hall effects, especially of Graphene
- ☆ Lattice gauge fields in a parameter space

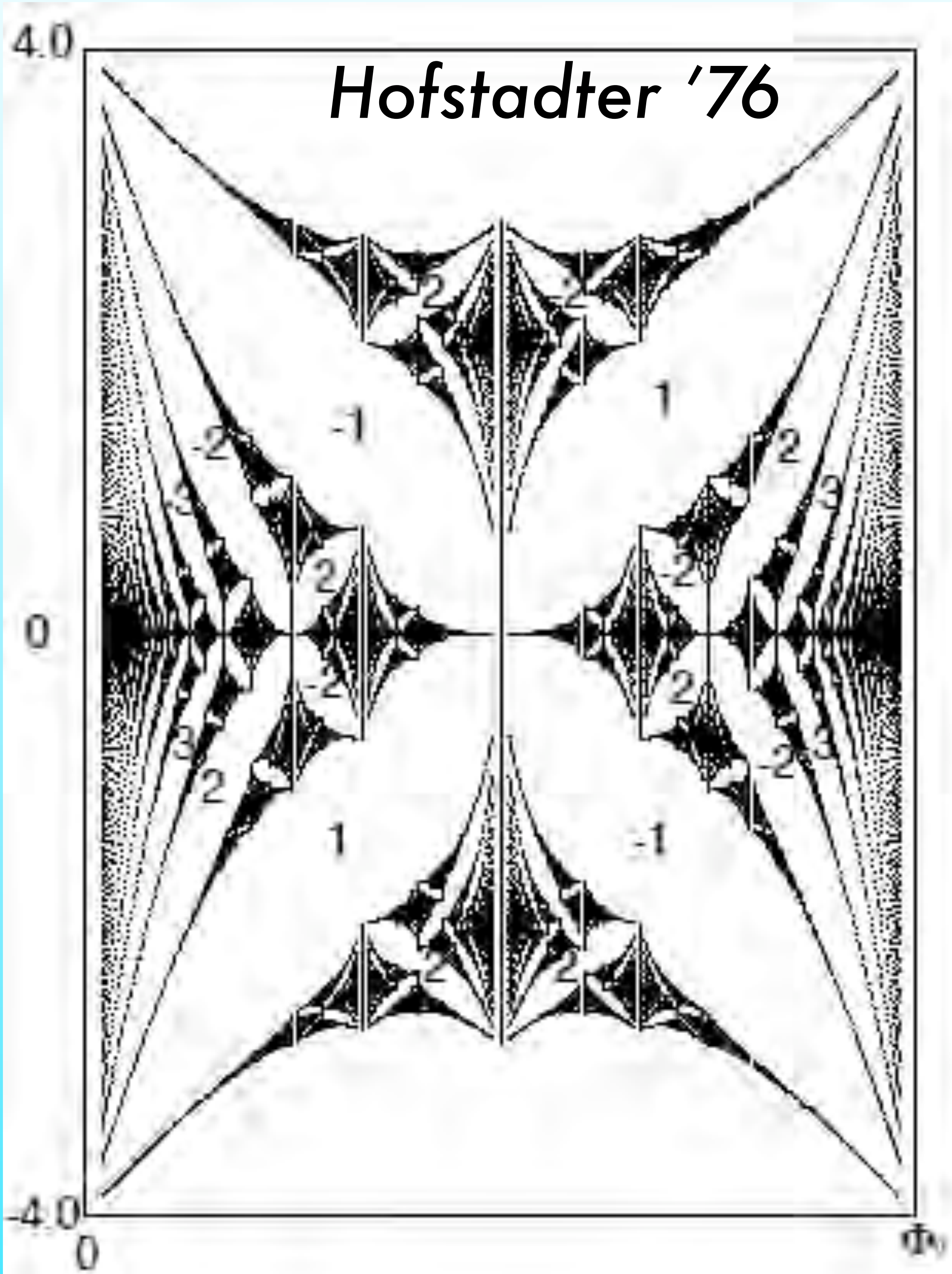
Dirac Fermion & Zero modes

- ☆ Universality for the Zero modes of Dirac fermions
- ☆ Bulk-Edge correspondence of topological ordered states

Random Dirac ferimion & "real" Gauge field

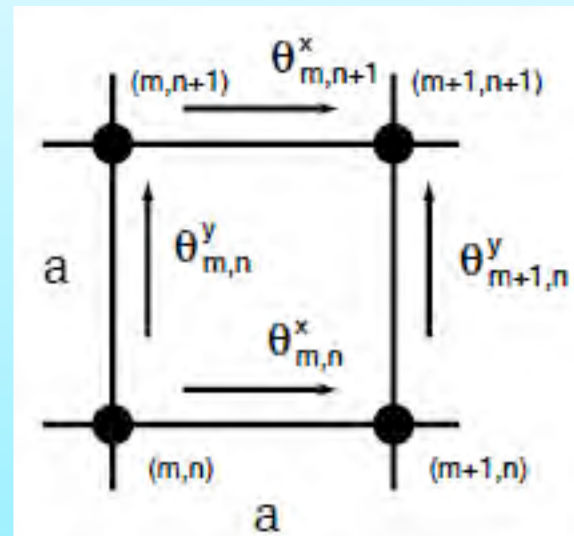
- ☆ Ripples as Random gauge field in graphene
- ☆ Realization of exact fix point of random Dirac fermions

Energy spectrum of Bloch Electrons in a magnetic field



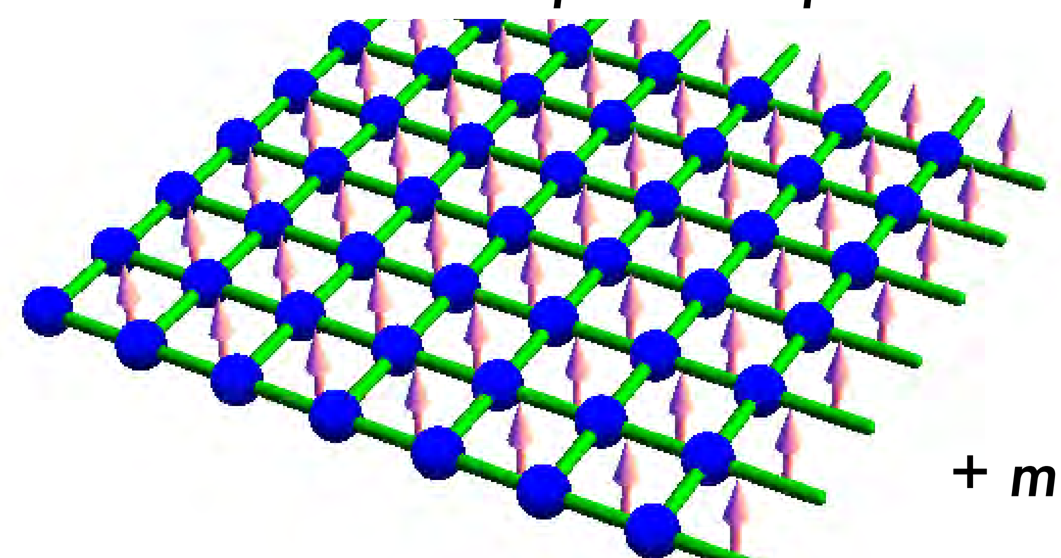
$$H = \sum_{\langle ij \rangle} c_i^\dagger e^{i\theta_{ij}} c_j + h.c.$$

c_i : electron annihilation operator



$$\sum_{j \in \langle ij \rangle} \theta_{ij} = 2\pi\phi_i = 2\pi\phi$$

Electrons in a 2D periodic potential

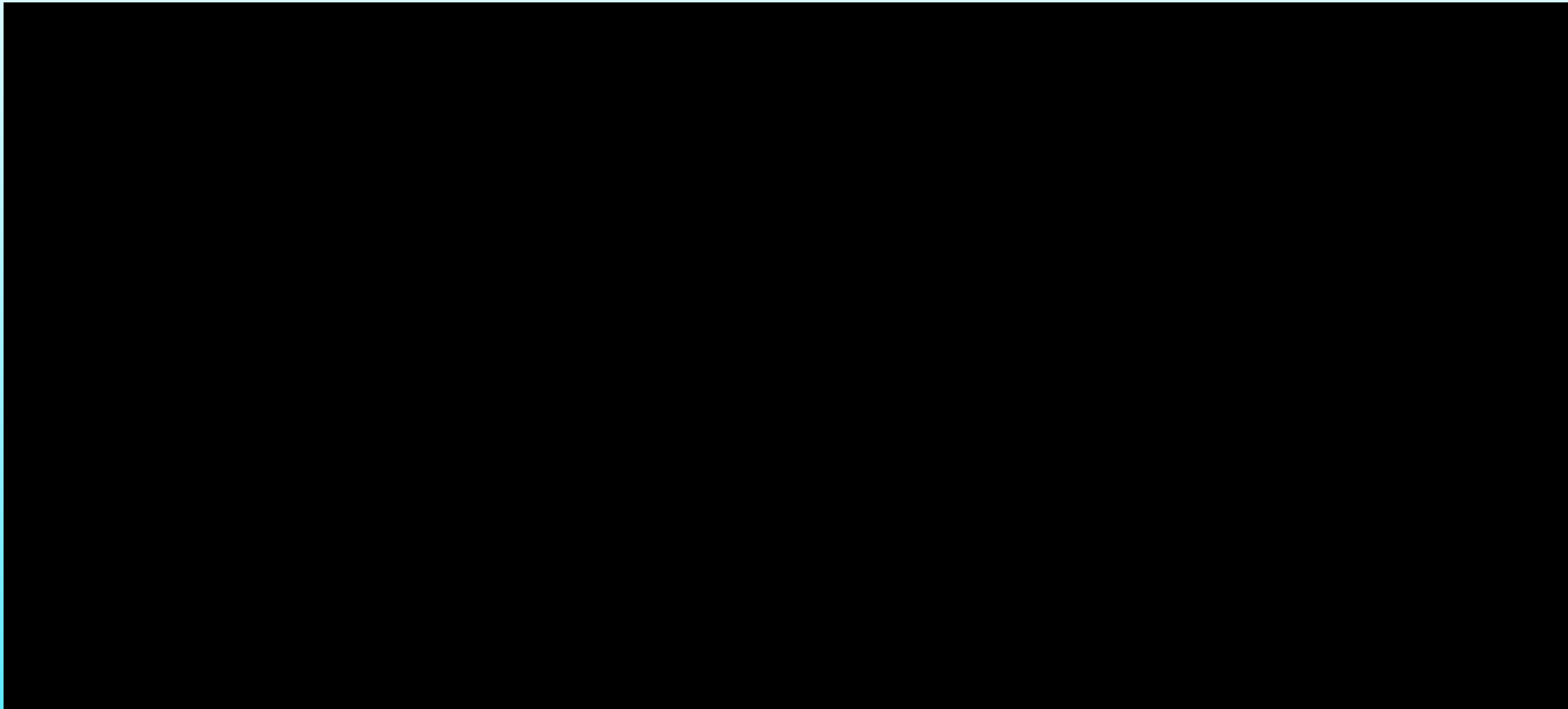


+ magnetic field

It might be realized by cold atoms?

The spectrum is fractal as a function of magnetic flux per plaquette

Hofstadter's Butterfly
Self-similar



Fractal in condensed matter

C. Albrecht, J.H. Smet, K. von Klitzing, D. Weiss, V. Umansky, H. Schweizer:
Evidence of Hofstadter's Fractal Energy Spectrum in the Quantized Hall Conductance.
Phys. Rev. Lett. 86(1), 147-150 (2001).