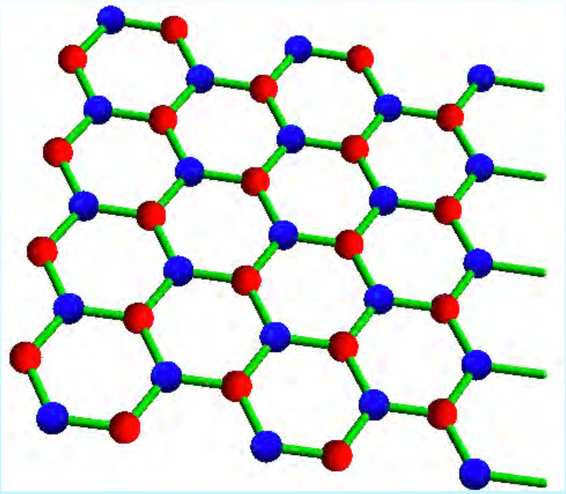
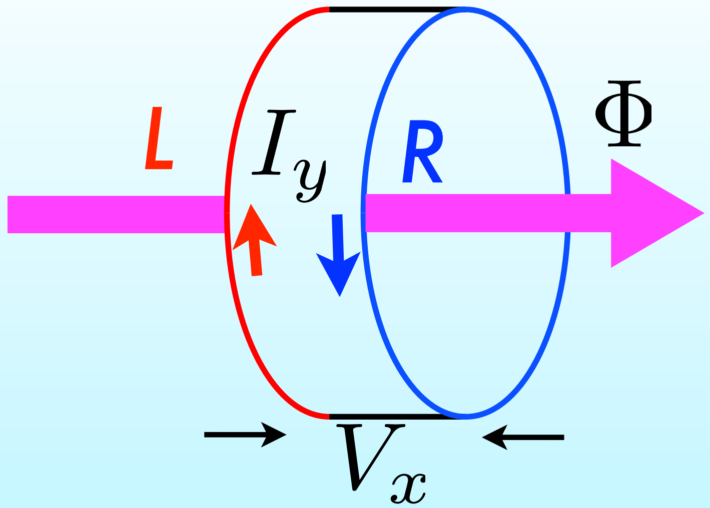
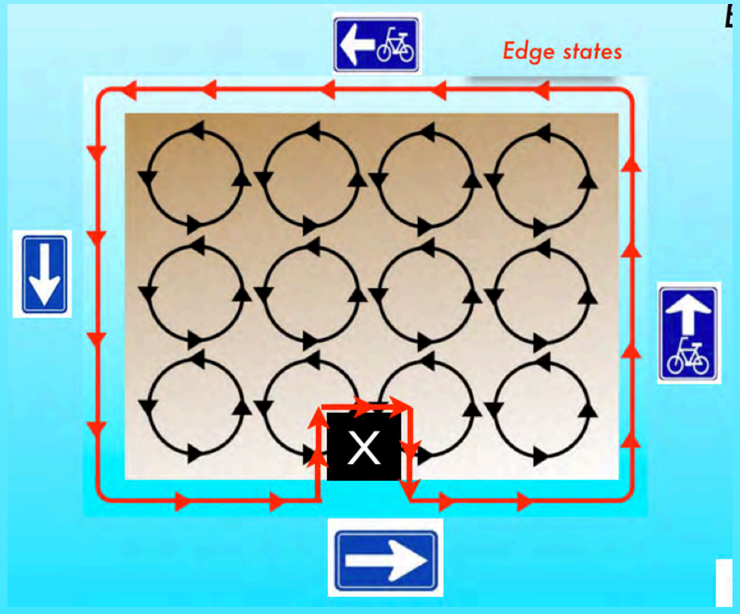
千葉大学理学部集中講義　2015年7月9日-10日





バルクエッジ対応の物理の多様性と普遍性

筑波大学数理物質系 物理学域
初貝　安弘



科研費
KAKENHI

筑波大学
University of Tsukuba

イントロダクション

今なぜトポロジカル相なのか

- 物理学の大事にするもの
  - 自然科学のなかの物理学
  - 多様性と普遍性
- 物理学における対称性とその破れ
  - 連続対称性
  - 対称性の破れと物性物理学
- 量子的な物質としてのグラフェン
  - 物質中の相対論的粒子
  - 量子力学とゲージ対称性
- トポロジカル相とは
  - 量子ホール効果と量子スピンホール効果
  - 端をみて中身を考える：バルク・エッジ対応

エッジ状態はどこにでもある！

講義予定

- 物理学の大事にするもの
  - 自然科学のなかの物理学
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  - 相対論的粒子とエッジ状態
- トポロジカル相とは
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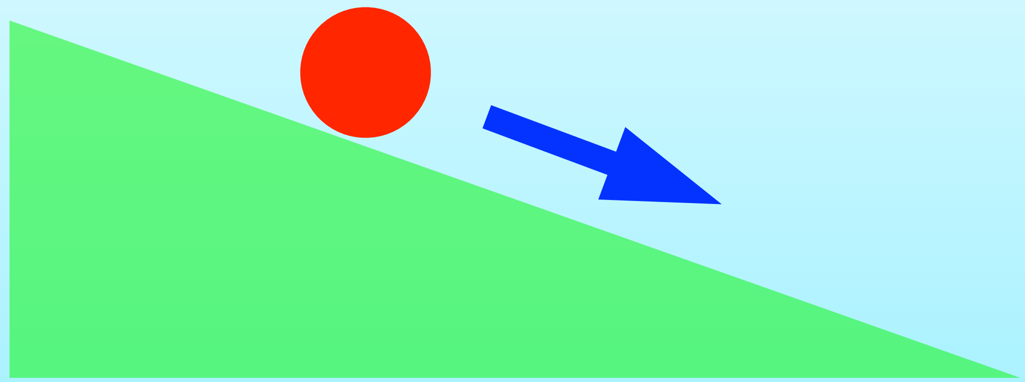
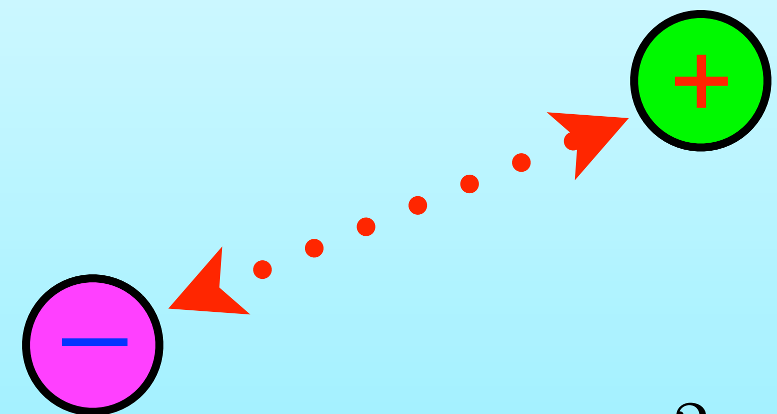
物理学とは

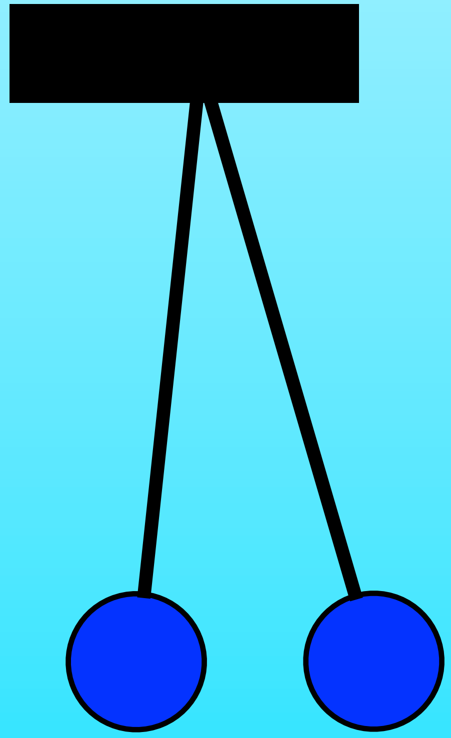
- 物理学とはなにか

$$F = mg\sin\theta$$





$$F = k\frac{Q^2}{r^2}$$



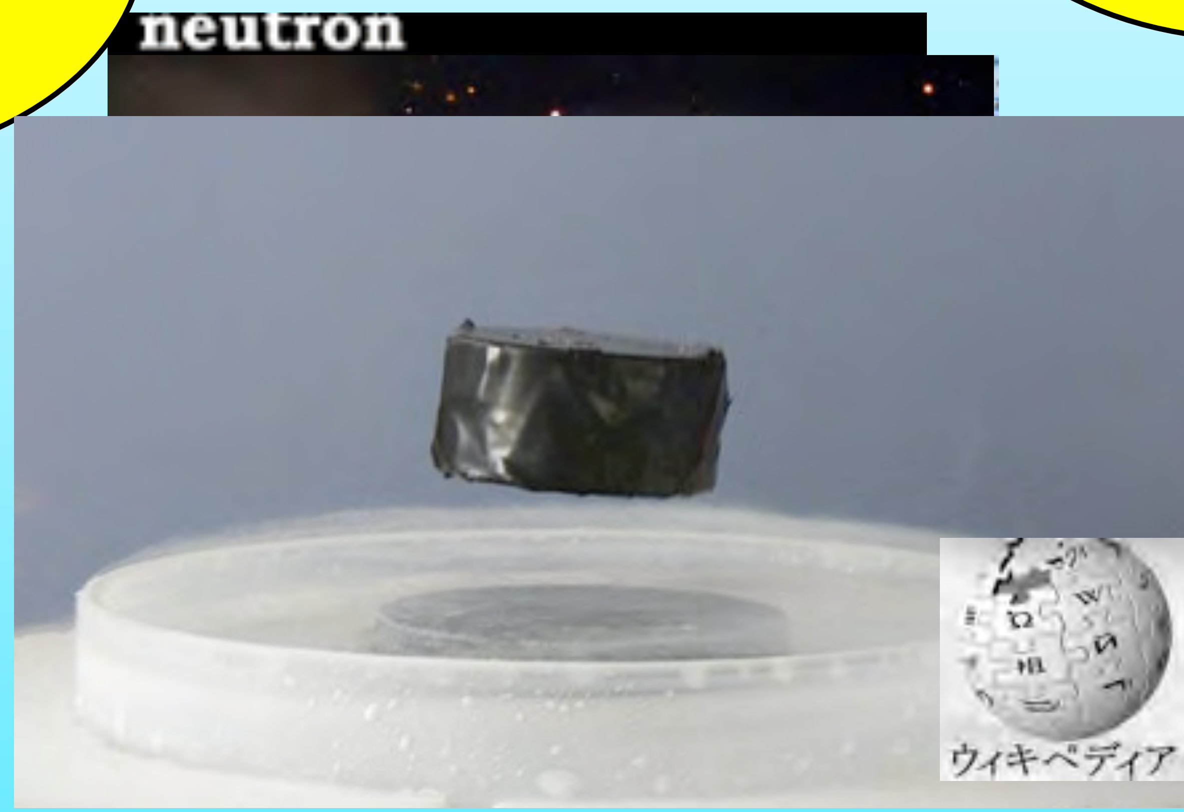
$$\ddot{\theta} = -\omega^2\theta$$

物理学とは

- 物理学とはなにか

素粒子

物性物理学



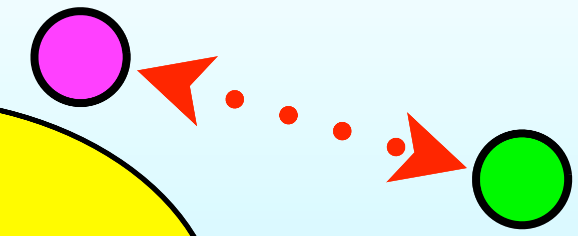
原子核

プラズマ

宇宙

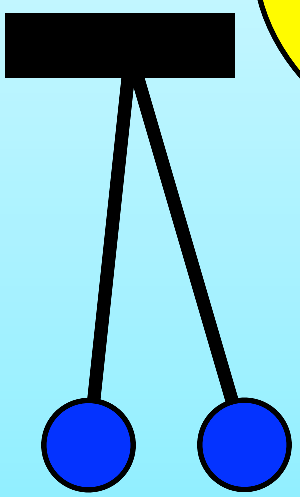
物理学とは

☆ 物理学とはなにか



物性物理学

素粒子



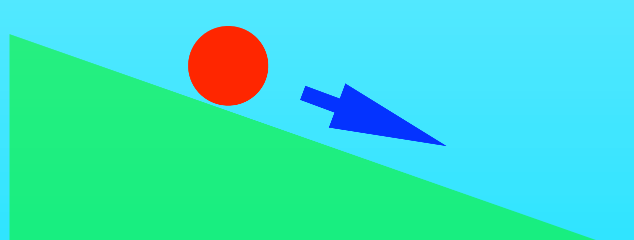
普遍性

Universality

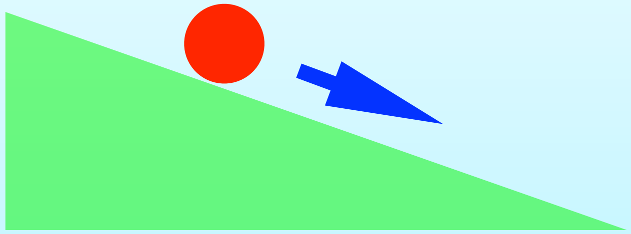
原子核

プラズマ

宇宙



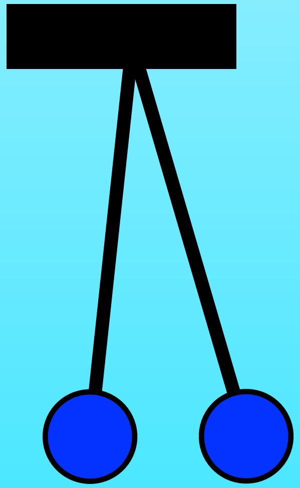
Universality (普遍性)とは？



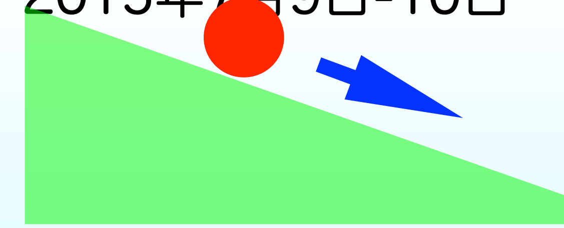
各論を超えた

統一的理解

http://



Universality (普遍性)とは？



法則

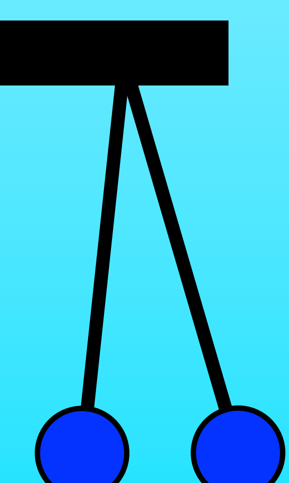
質点の運動 として統一的に理解

$$F = ma$$ ：押しただけスピードアップ

質点：大きさの無い質量(*m*)のみを持つ点

理想化

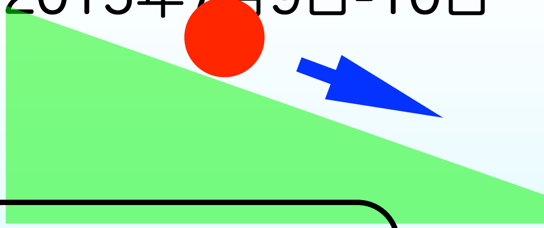
近似？



数学的厳密さを失った？

http://

Universality (普遍性)とは？



法則

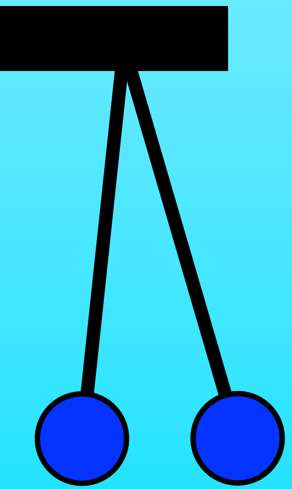
$F = ma$ ：押しただけスピードアップ

理想化　質点：大きさの無い質量(*m*)のみを持つ点

複雑な対象 $\xrightarrow{\text{情報の縮約}}$ m : 唯一の特性として抽出

ある時間スケール、空間スケールで
成立する階層的基本法則の発見

Universality



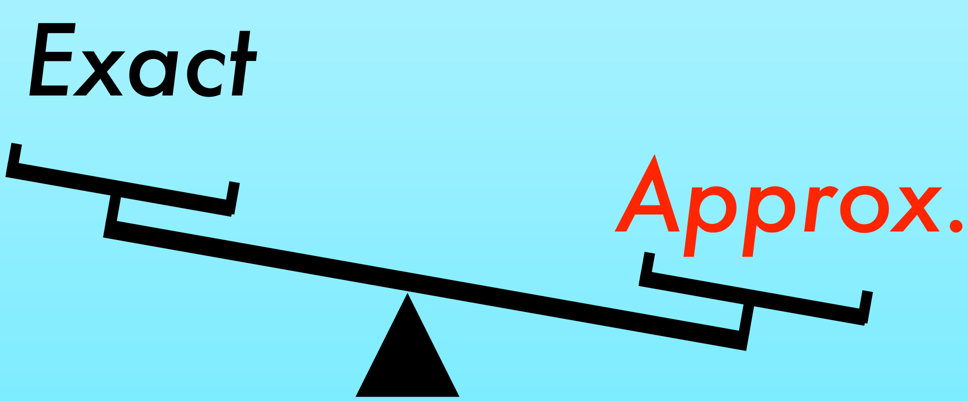
http://

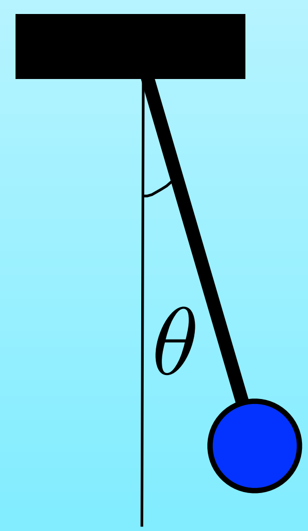
Universality (普遍性)とは？

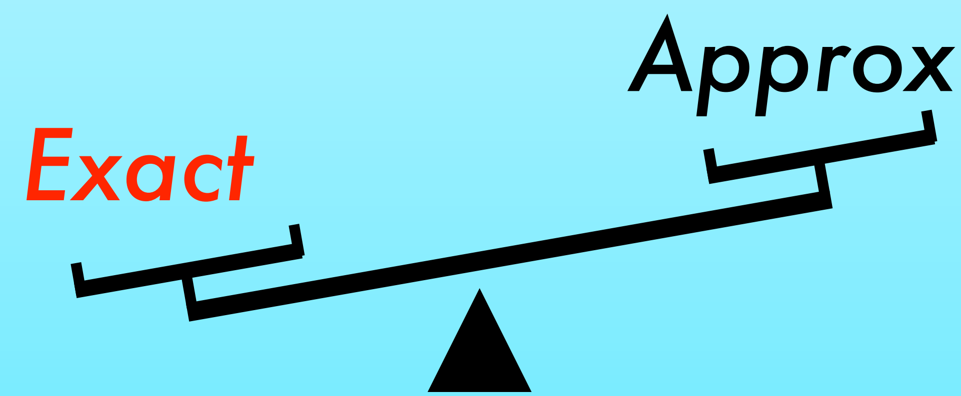
ある時間スケール、空間スケールで
成立する階層的基本法則の発見

Universality

階層間のクロスオーバー
すべての物理法則は破綻する







厳密(exact)

$$\ell\ddot{\theta} = -mg\sin\theta$$

physics

mathematics

$$\theta \ll 1$$

$$\ddot{\theta} \approx -(mg/\ell)\theta$$

近似(approx.)

ある時間スケールにおける
単振動

自然科学としての物理学

普遍性　Universality

自然科学

多様性　Diversity

Physics

各論の基礎付け

質的転回の基礎

化学　chemistry

工学　engineering

生物学　biology

有用な各論

人間の幸福

文化的価値

異なるアプローチ（相補的）

Spontaneous Broken Symmetry

Nobelprize.org

NOBEL PRIZES | ALFRED NOBEL | PRIZE AWARDERS | NOMINATION | PRIZE ANNOUNCEMENTS

By Year | Nobel Prize in Physics | Nobel Prize in Chemistry | Nobel Prize in Medicine | Nobel Prize in L

The Nobel Prize in Physics 2008

"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"

Photo: University of Chicago

Yoichiro Nambu

© The Nobel Foundation Photo: U. Montan

Makoto Kobayashi

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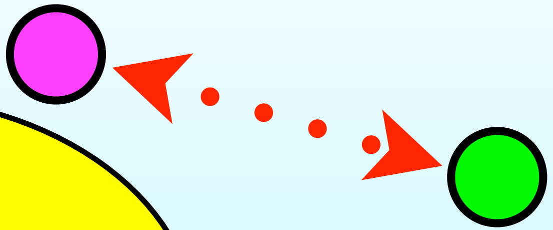
Toshihide Maskawa

Spontaneous
Broken
Symmetry

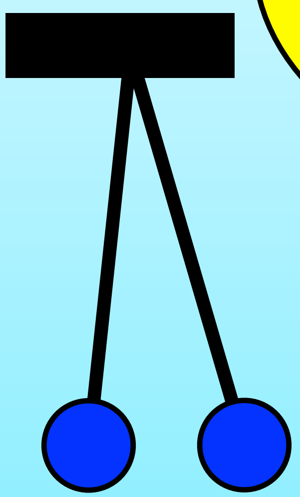
Use it for particle physics : Y. Nambu

Universal & Basics
Concept
in the whole physics



物性物理学

素粒子



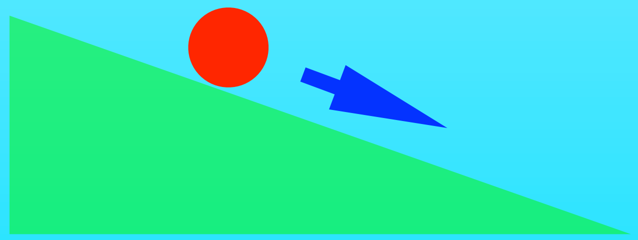
普遍性

Universality

原子核

プラズマ

宇宙



講義予定

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物理学における対称性

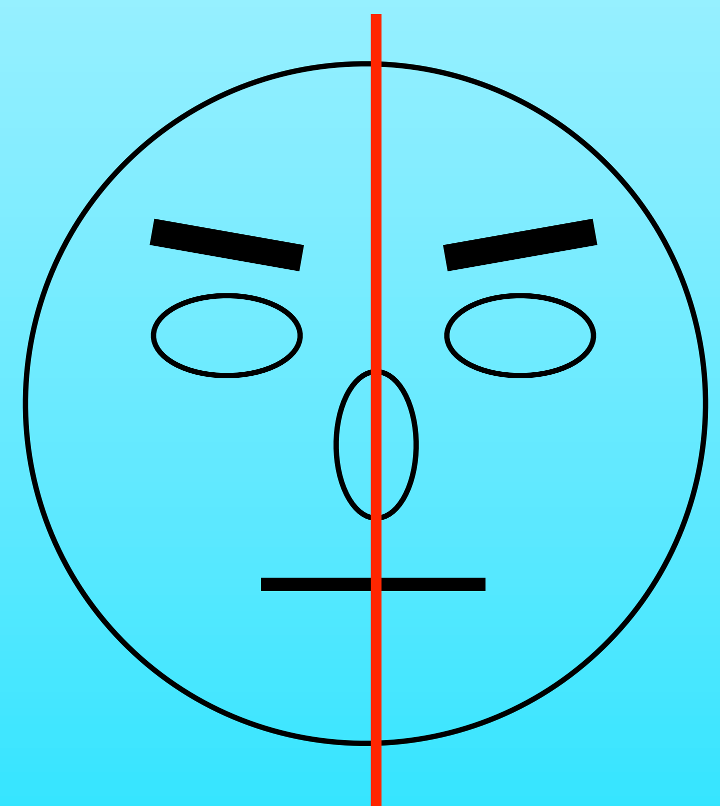
物理学における“タイショウセイ”

対称性　**Symmetry**

~~対照~~　対称　~~対象~~

対称？

美人は顔のつくりが左右対称　？

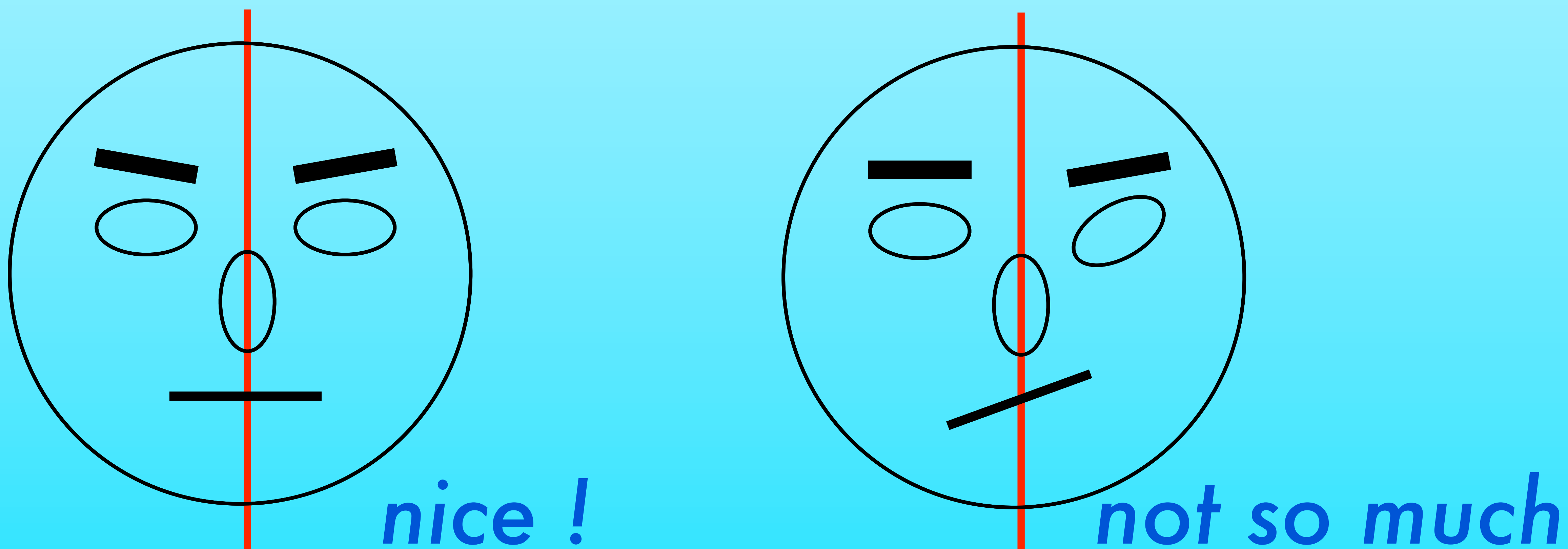


物理学における対称性

対称性　*Symmetry*

対称？

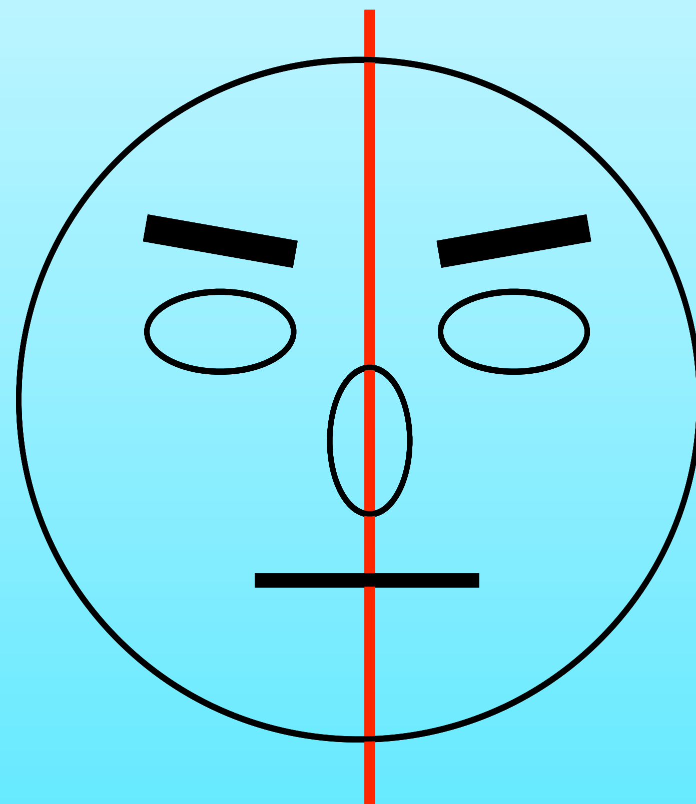
美人は顔のつくりが左右対称　？



物理学における対称性

対称性　*Symmetry*

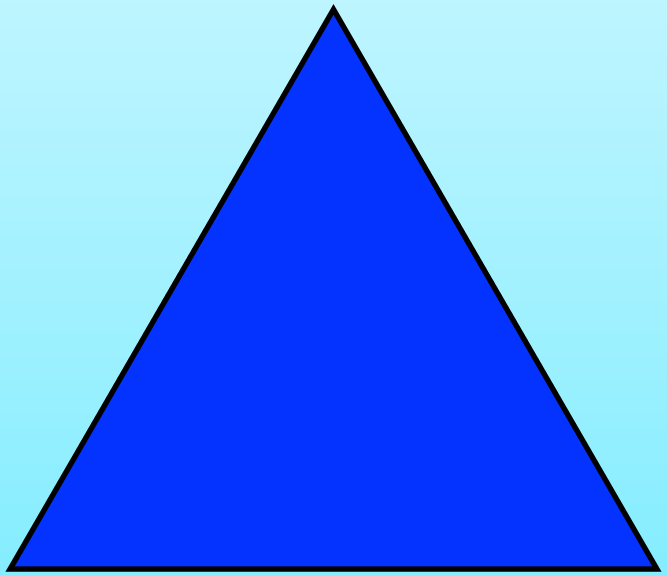
左右対称



ひっくり返す　　「対称操作」

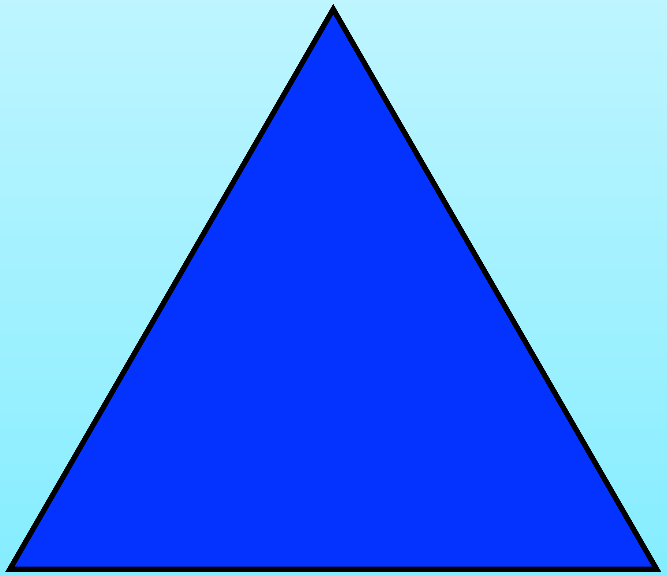
Symmetry

「いろいろな対称操作」



Symmetry

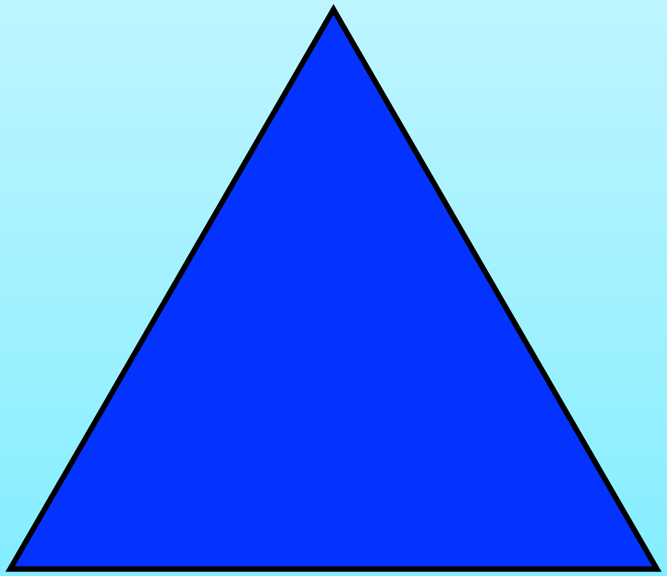
「いろいろな対称操作」



120度回転

Symmetry

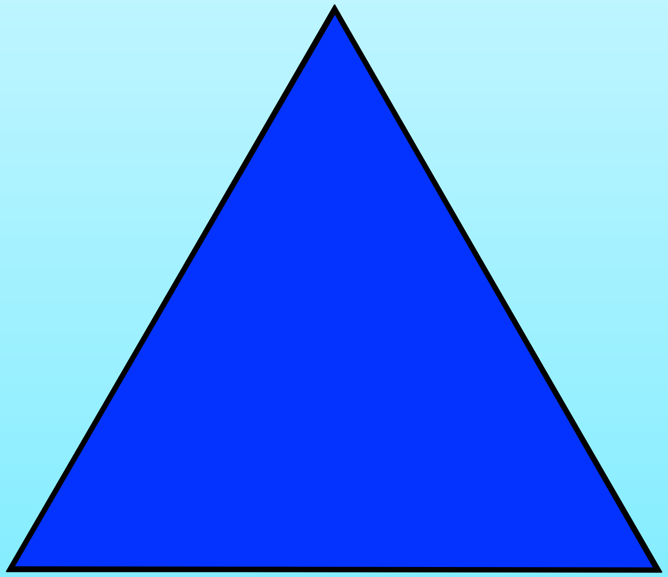
「いろいろな対称操作」



240度回転

Symmetry

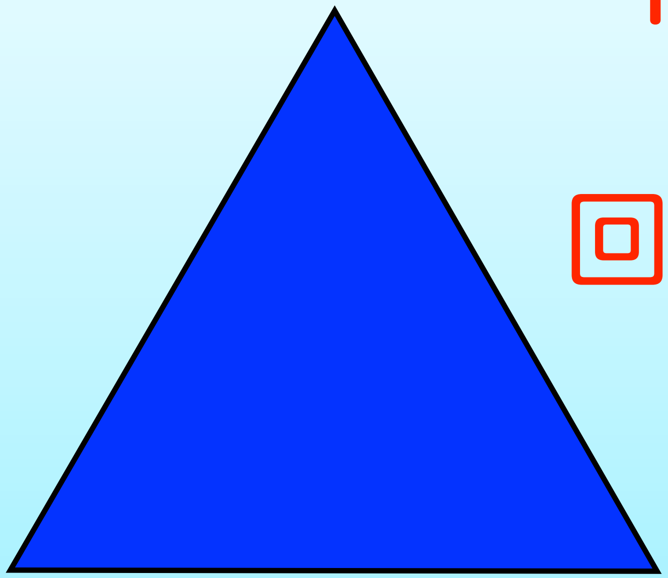
「いろいろな対称操作」



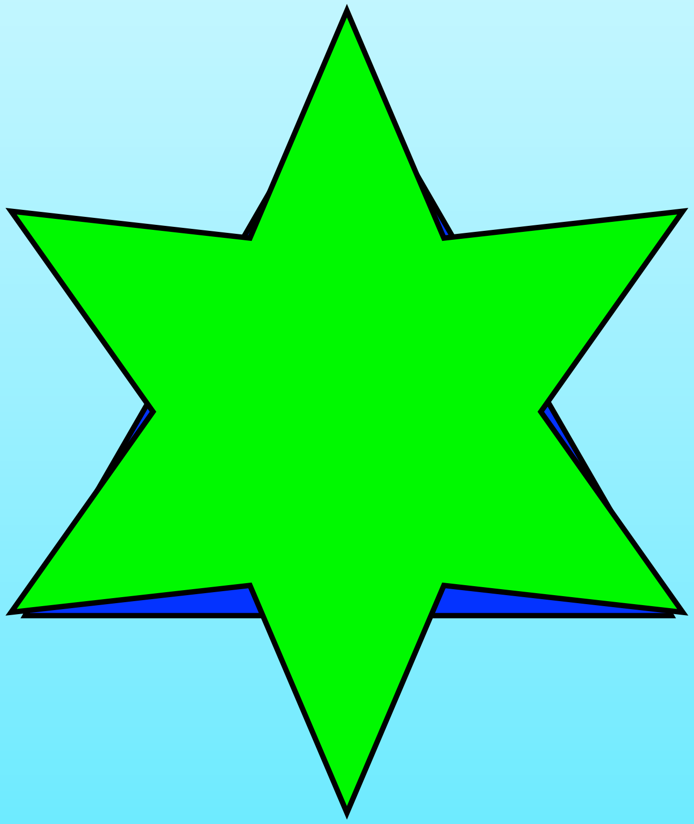
360度回転

Symmetry

「いろいろな対称操作」

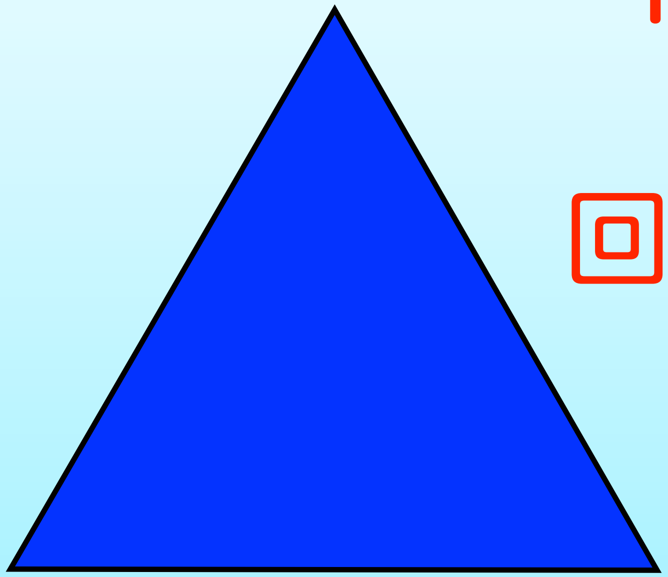


回転角：120度、240度、360度

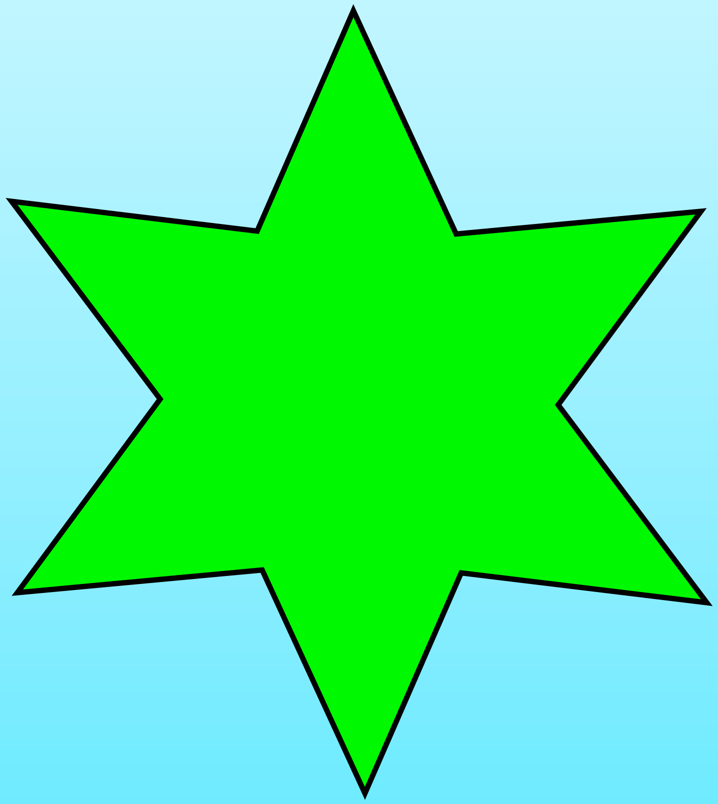


Symmetry

「いろいろな対称操作」



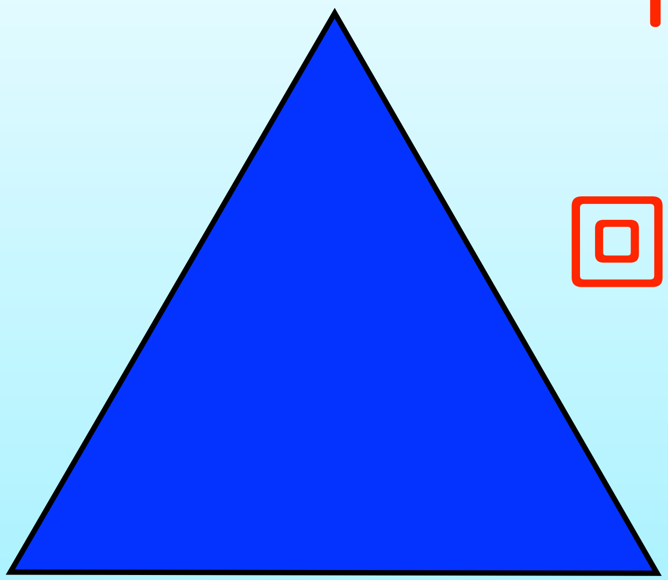
回転角：120度、240度、360度



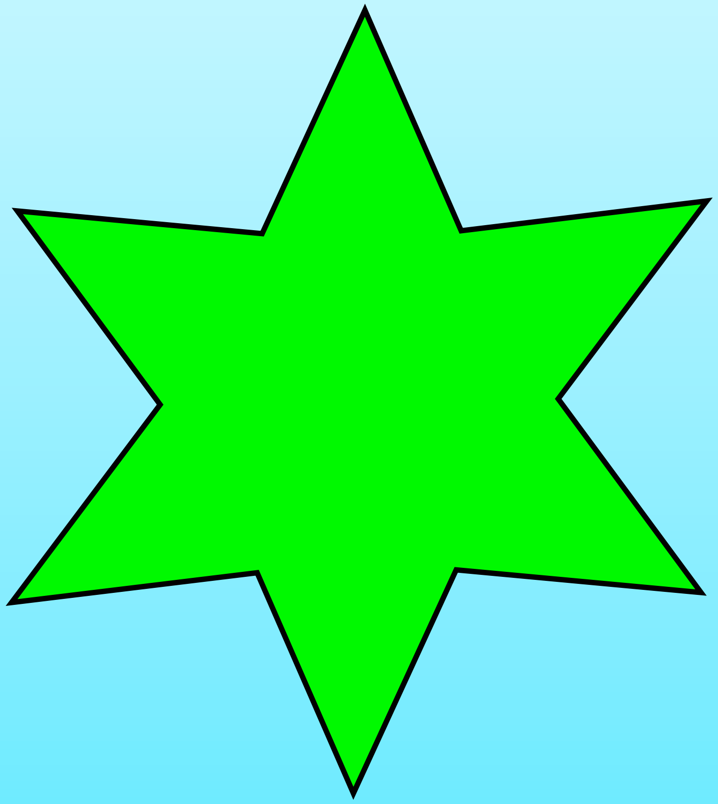
60度回転

Symmetry

「いろいろな対称操作」



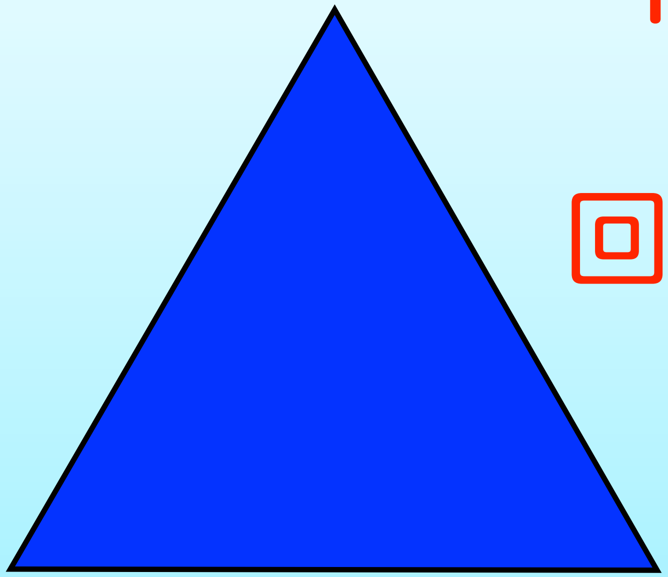
回転角：120度、240度、360度



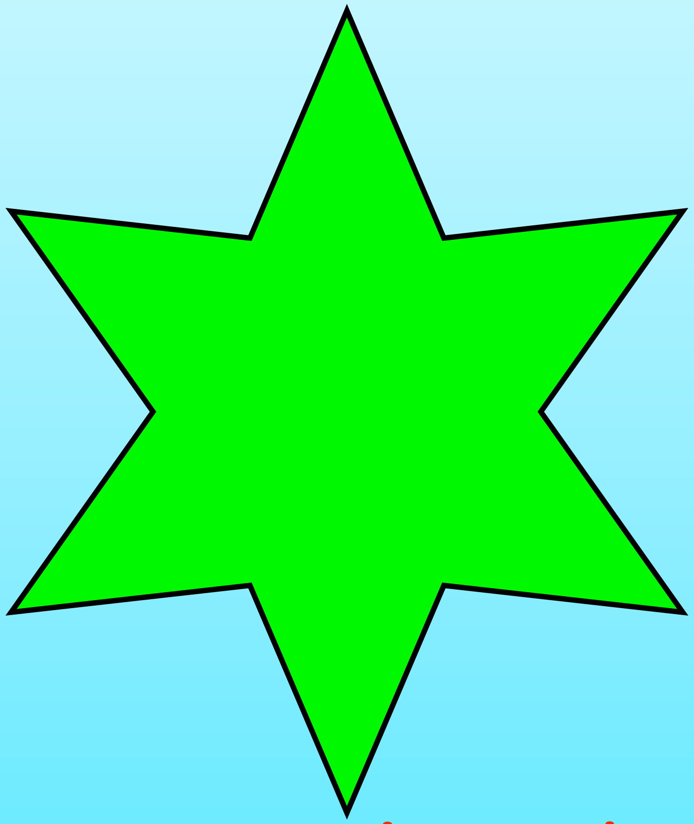
120度回転

Symmetry

「いろいろな対称操作」



回転角：120度、240度、360度

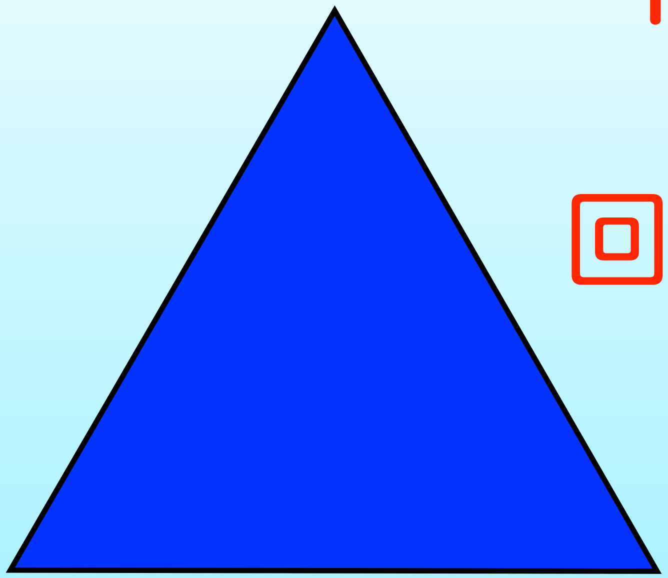


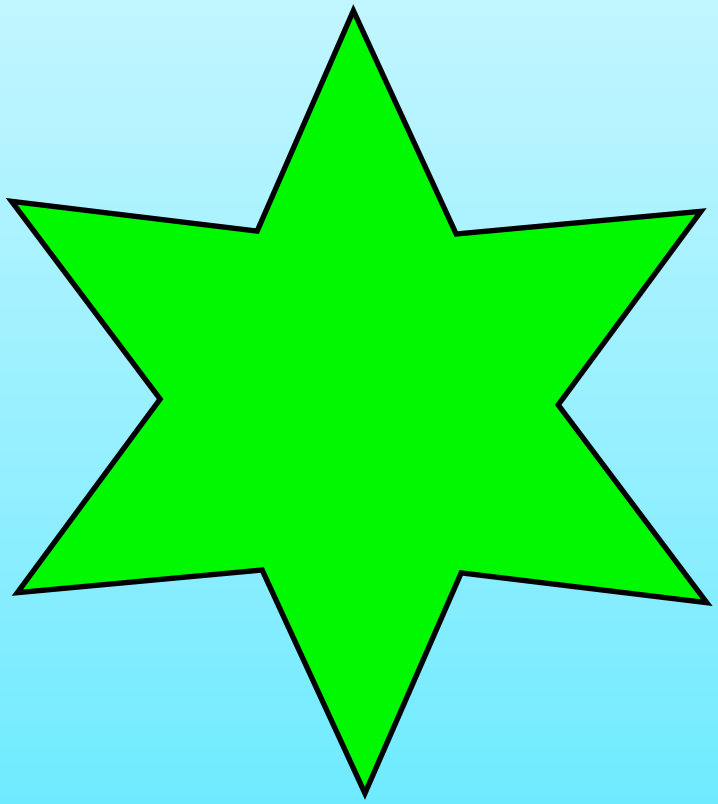
180度回転

Symmetry

「いろいろな対称操作」

回転角：120度、240度、360度

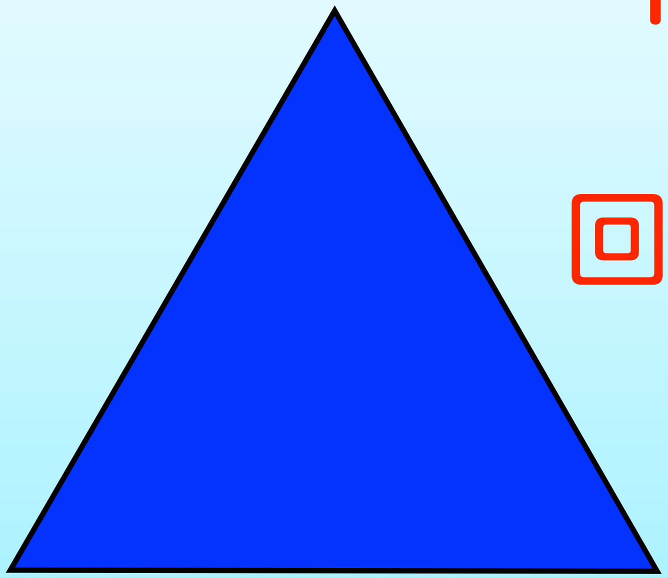




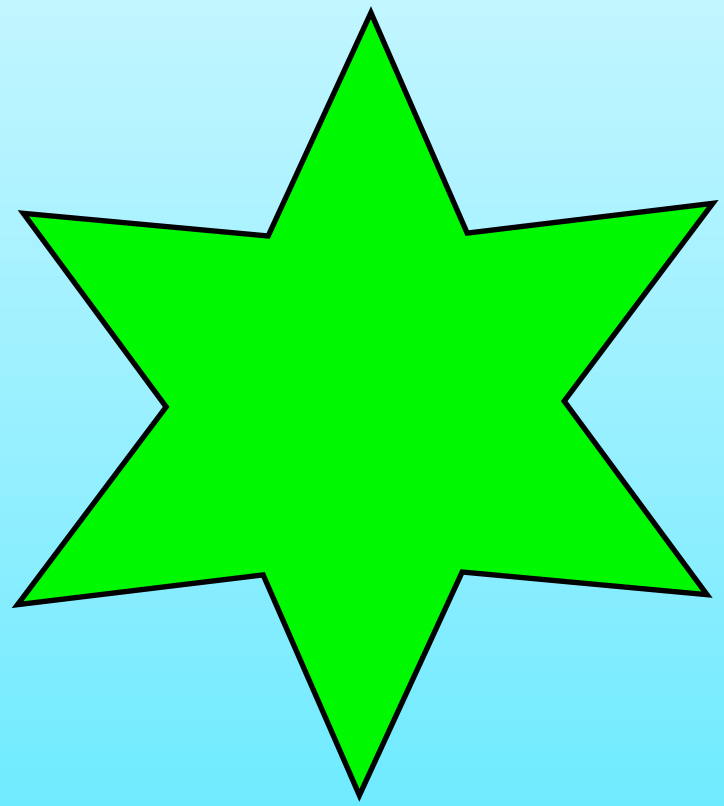
240度回転

Symmetry

「いろいろな対称操作」



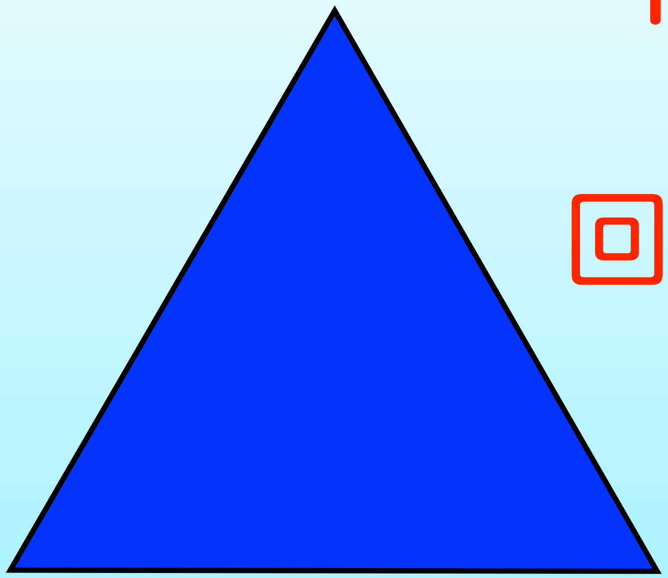
回転角：120度、240度、360度



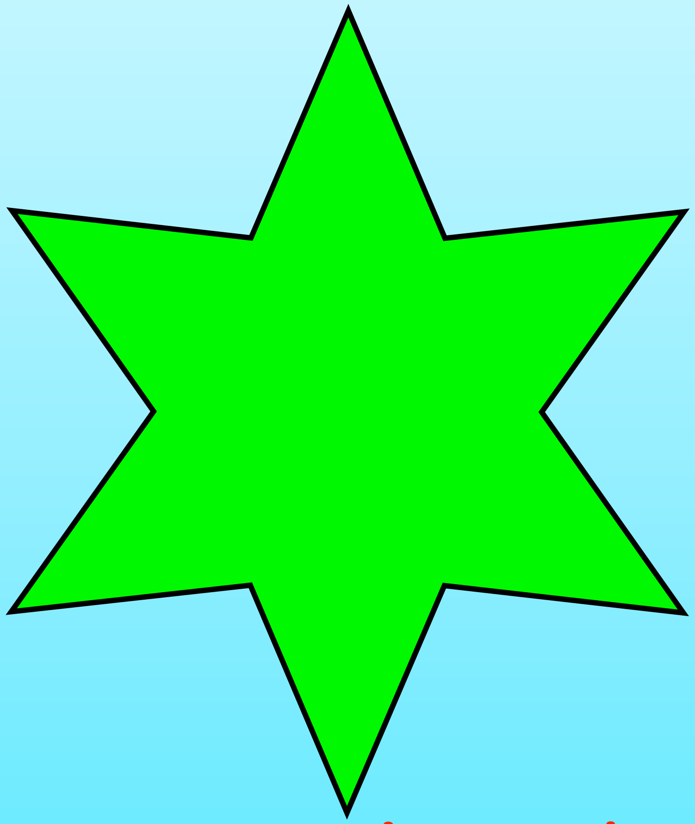
300度回転

Symmetry

「いろいろな対称操作」



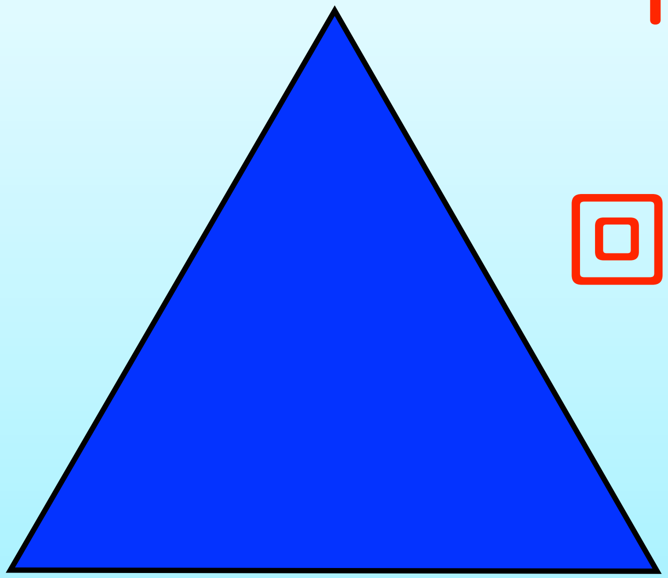
回転角：120度、240度、360度



360度回転

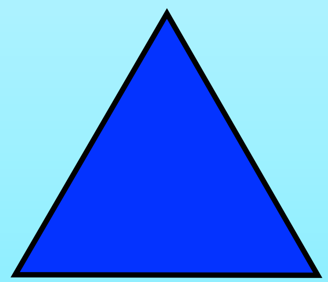
Symmetry

「いろいろな対称操作」



回転角：120度、240度、360度

low symmetry　　*high* symmetry



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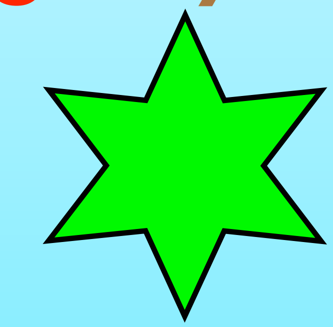
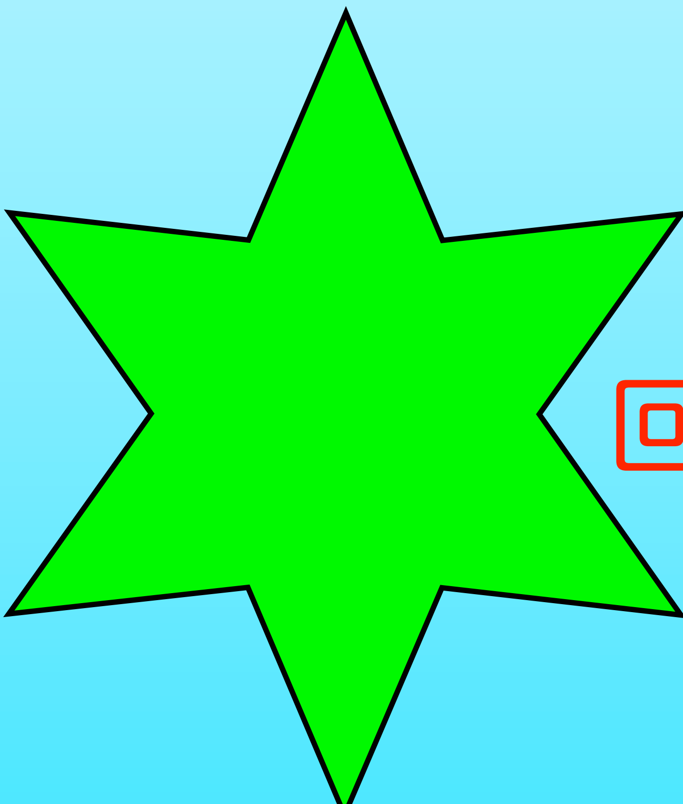


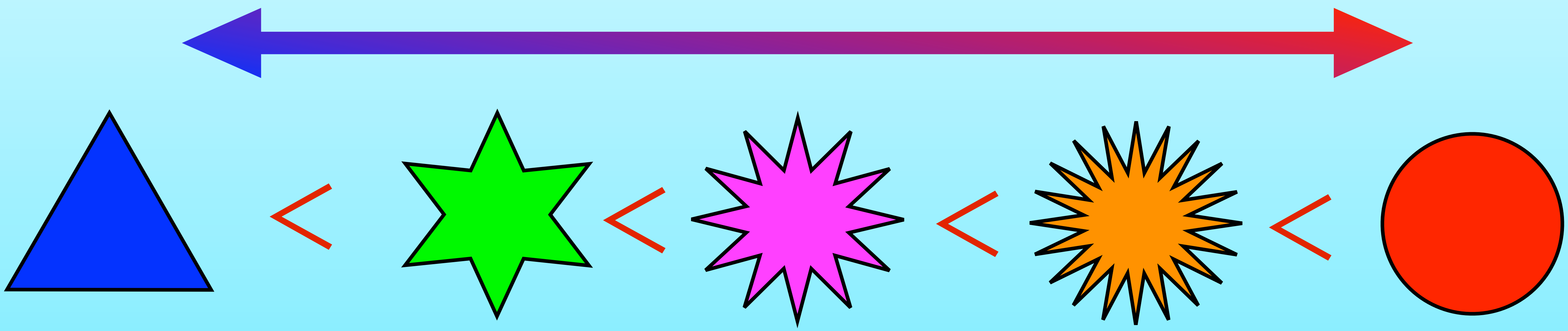


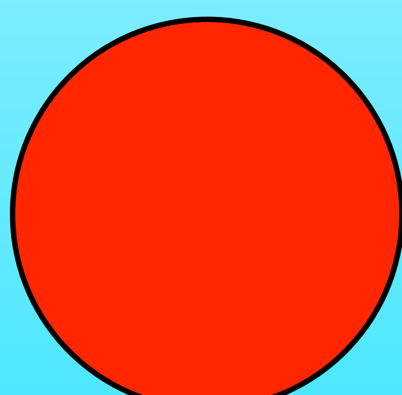
回転角：60度、120度、180度、
240度、300度、360度

対称性の定量化

「いろいろな対称操作」

low symmetry　　　*high symmetry*

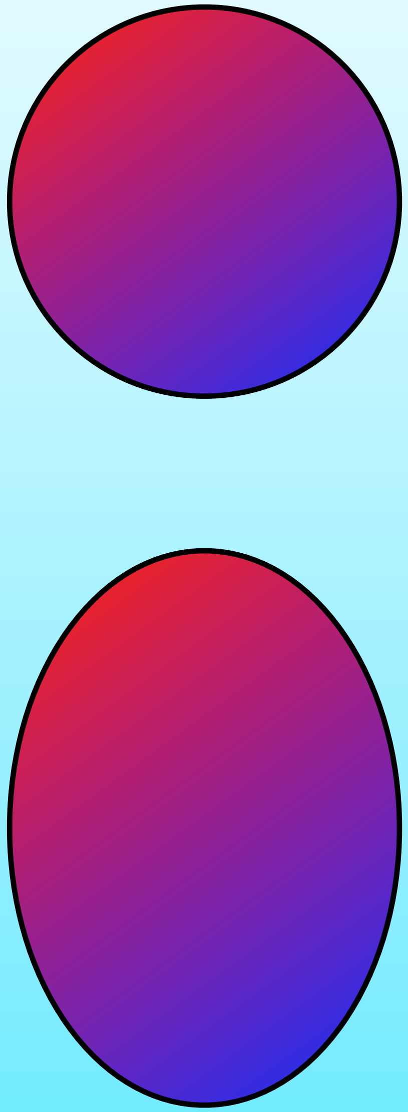




「連続対称性」

Nambu-Goldstone Boson

対称性の低下と対称性の破れ



high symmetry

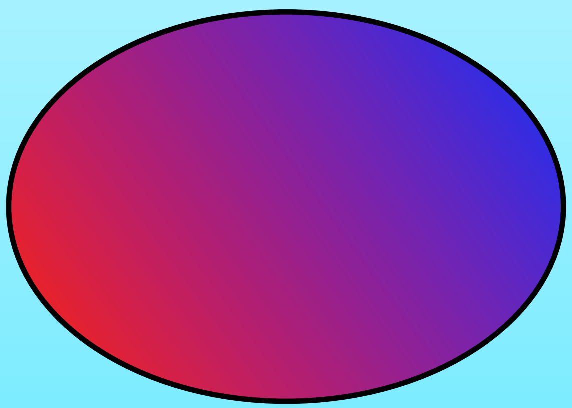
low symmetry

対称性の低下と対称性の破れ

OK!

high symmetry

No good!



low symmetry

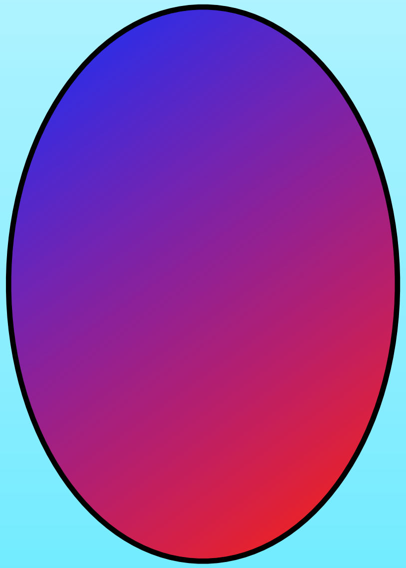
90度回転

対称性の低下と対称性の破れ

OK!

high symmetry

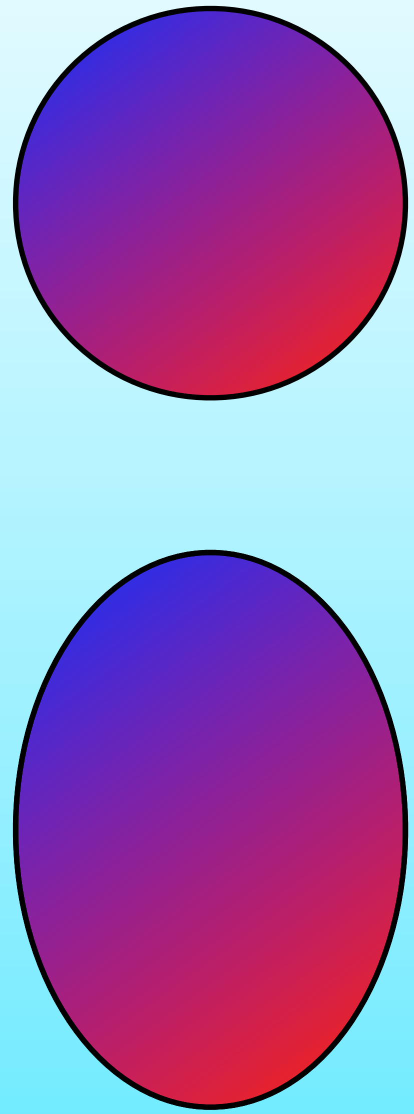
OK!



low symmetry

*180*度回転

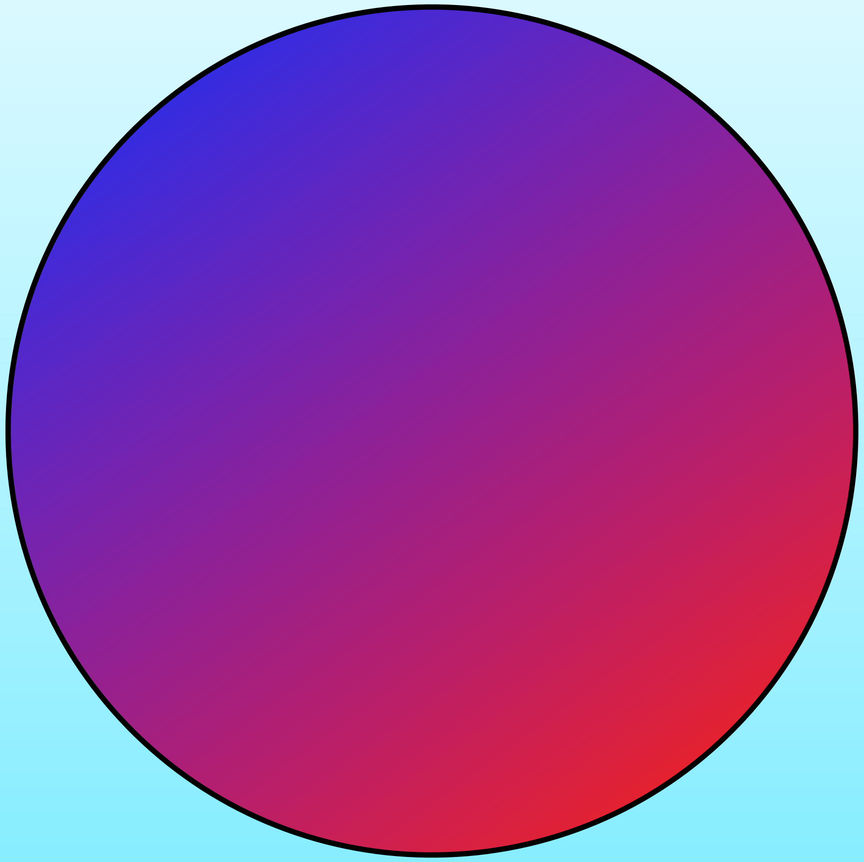
対称性の低下と対称性の破れ



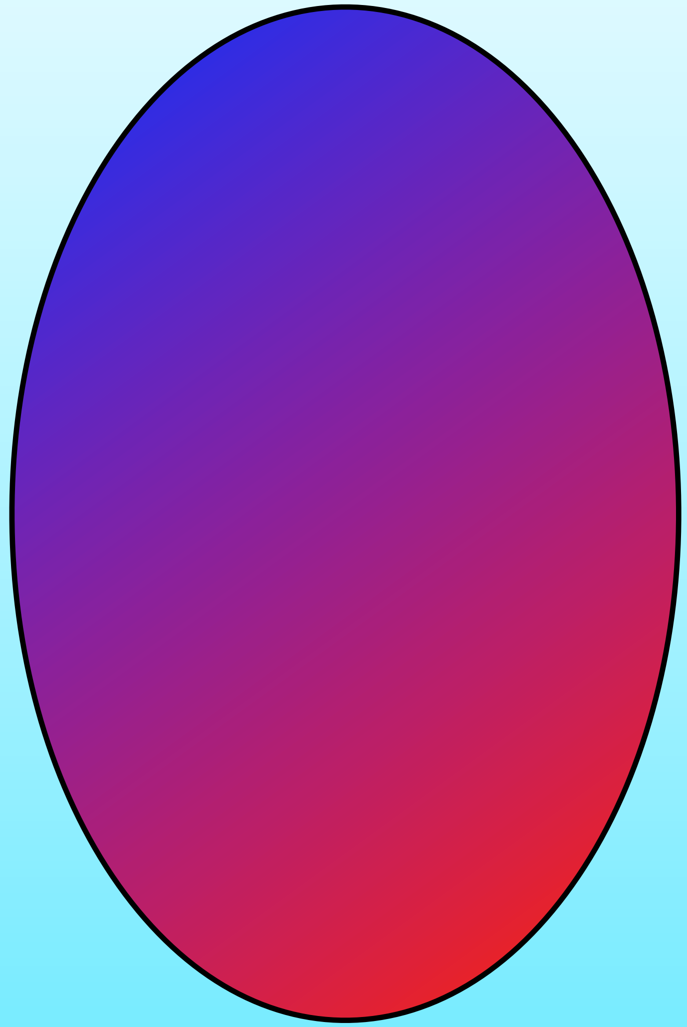
high symmetry

low symmetry

対称性の低下と対称性の破れ



対称性の低下と対称性の破れ



対称性の低下

対称性の破れ

Broken Symmetry

Break down of Continuous Symmetry

連続対称性の破れ

自発的?　*Spontaneous ?*

磁石：秩序形成＝対称性の破れの例

- 電子は電荷の他に固有の角運動量を持つ（スピン）
- スピン＝$\hbar/2$: $\hbar$と同程度：量子論による記述
- 磁石：電子のスピンが一つの方向にそろったもの
  - スピンとは何だろうか (Dirac)
  - なぜスピンがそろうのか？
  - そろいかたにもいろいろある
    - 多様な磁気秩序相
    - ↑↓ ↑↓ ↑↓ ↑↓ ↑↓ (反強磁性：ネール状態)
    - ↑↑ ↑↑ ↑↑ ↑↑ ↑↑ (強磁性)
  - どうやって磁気秩序を区別するか？

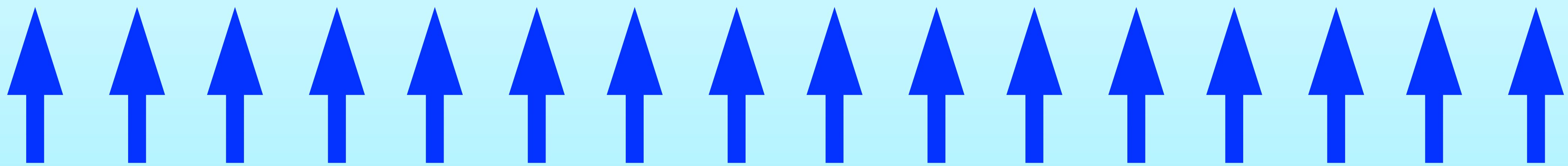
秩序と秩序変数

☆ 磁気秩序（磁石）

室温の磁石

（秩序正しい）

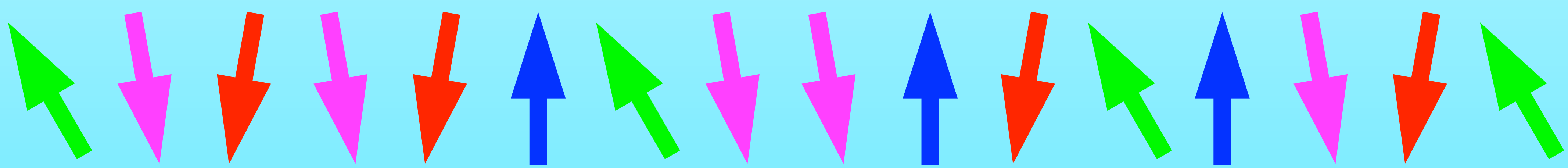
秩序相



磁石をバーナーで熱すると

（でたらめ）

無秩序相

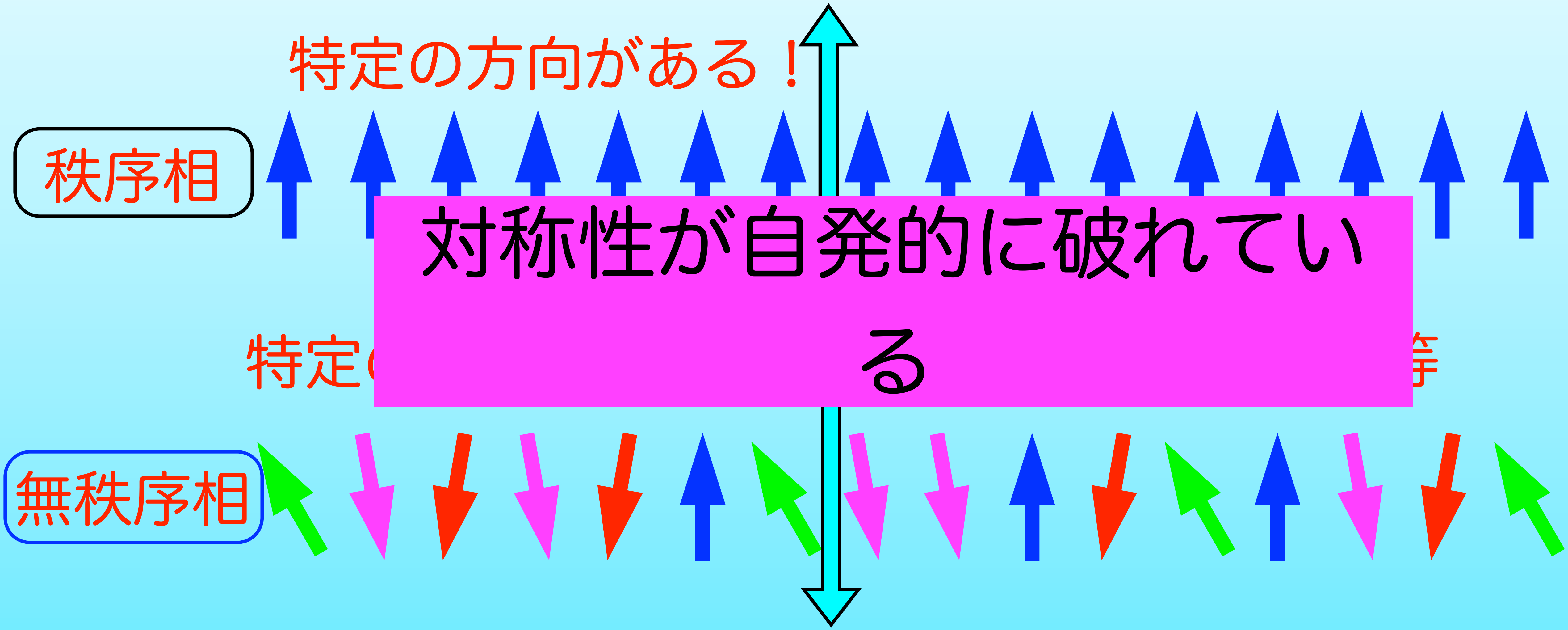


秩序変数：矢印！“vector”

$\vec{m}(\vec{r})$ 場所 $\vec{r}$ での平均の磁化の方向 $\vec{m}$

秩序変数と対称性の自発的破れ

☆対称性の観点から区別しよう！！



秩序変数 $m(\vec{r}) : \begin{cases} \neq 0 & \text{対称性が破れている} \\ = 0 & \text{対称性が破れていない} \end{cases}$

対称性の自発的破れ
Spontaneous Symmetry Breaking

物質はすべての方向は同等のはず！

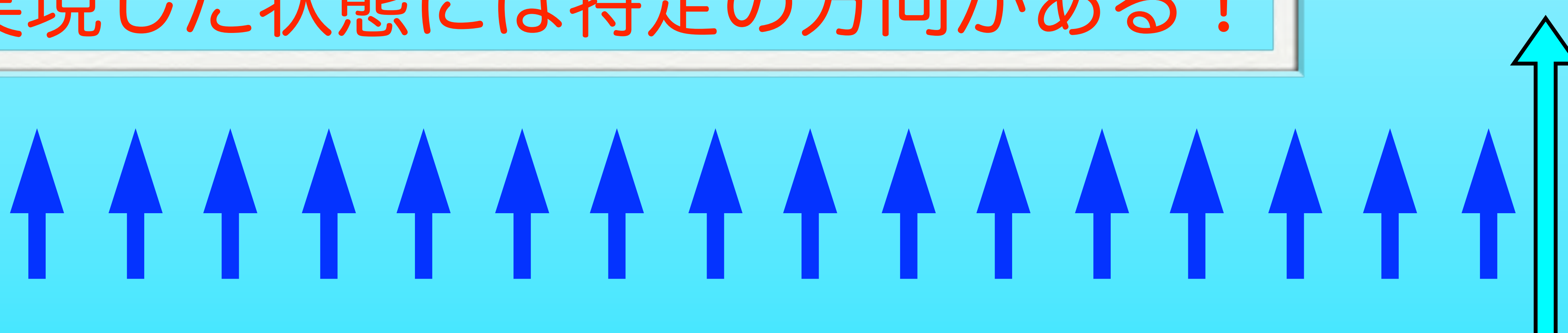
特定の方向はない

すべての方向が同等

物質の形態を定める法則
は完全に等方的

実現した状態には特定の方向がある！

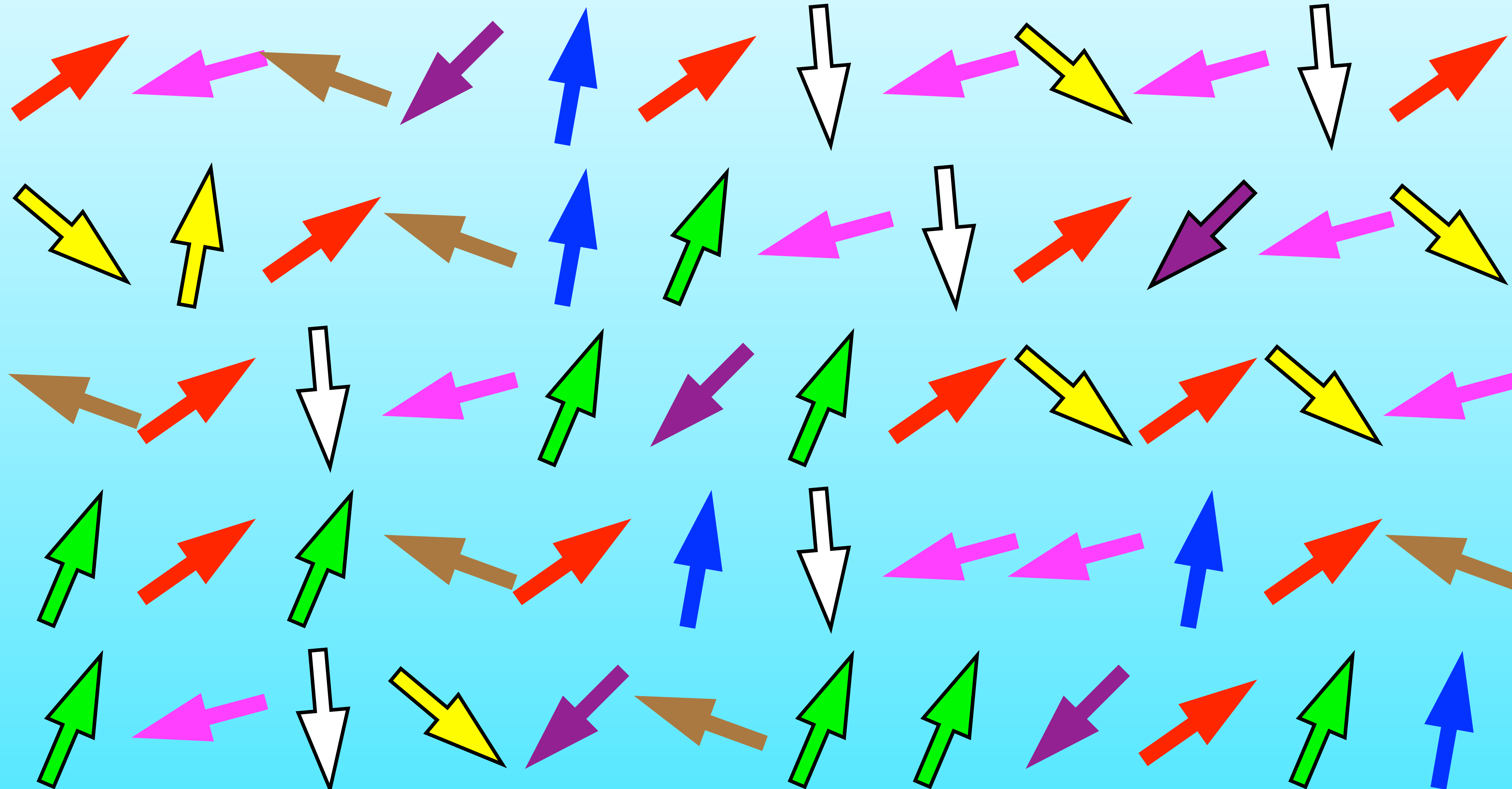
秩序相



対称性が自発的に破れてい

温度低下による自発的対称性の破れ

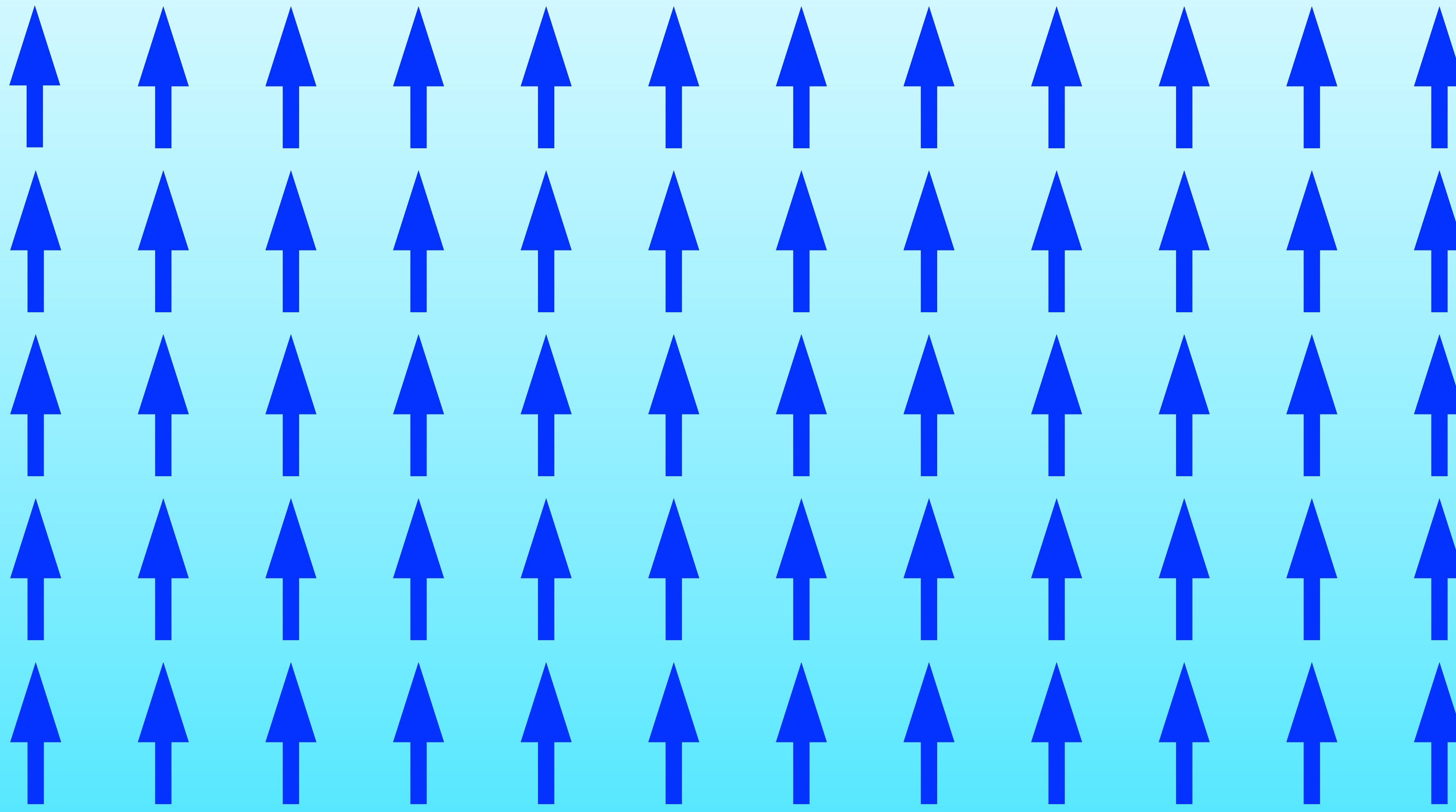
Disordered　　高温相　　*Symmetric*



特定の方向が無い：平均的にすべての方向が同等

温度低下による自発的対称性の破れ

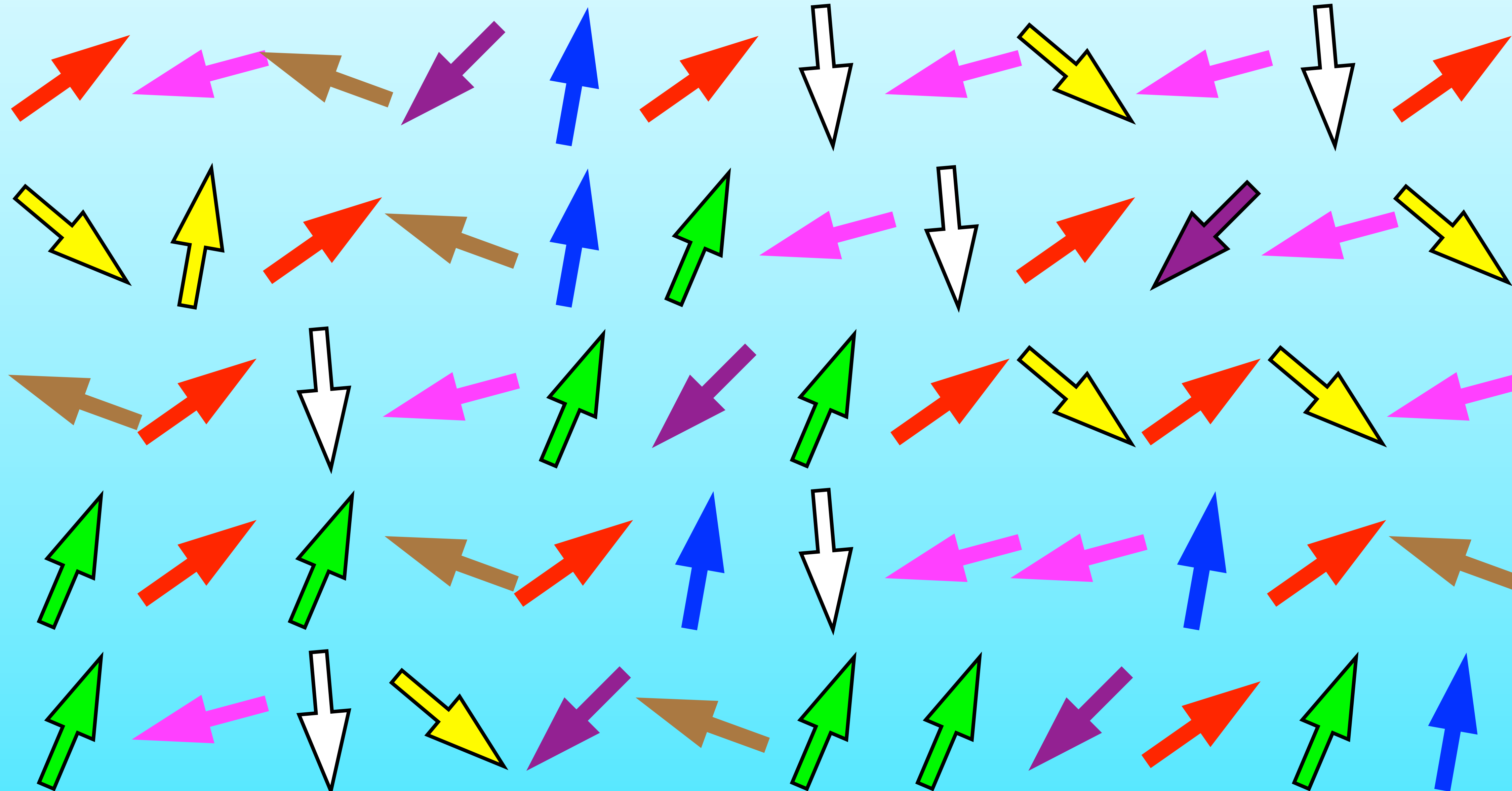
Ordered　　低温相　　*Broken Symmetry*



特定の方向を自発的に物質が選び出す！

温度低下による自発的対称性の破れ

Disordered　　高温相　　*Symmetric*



特定の方向が無い：平均的にすべての方向が同等

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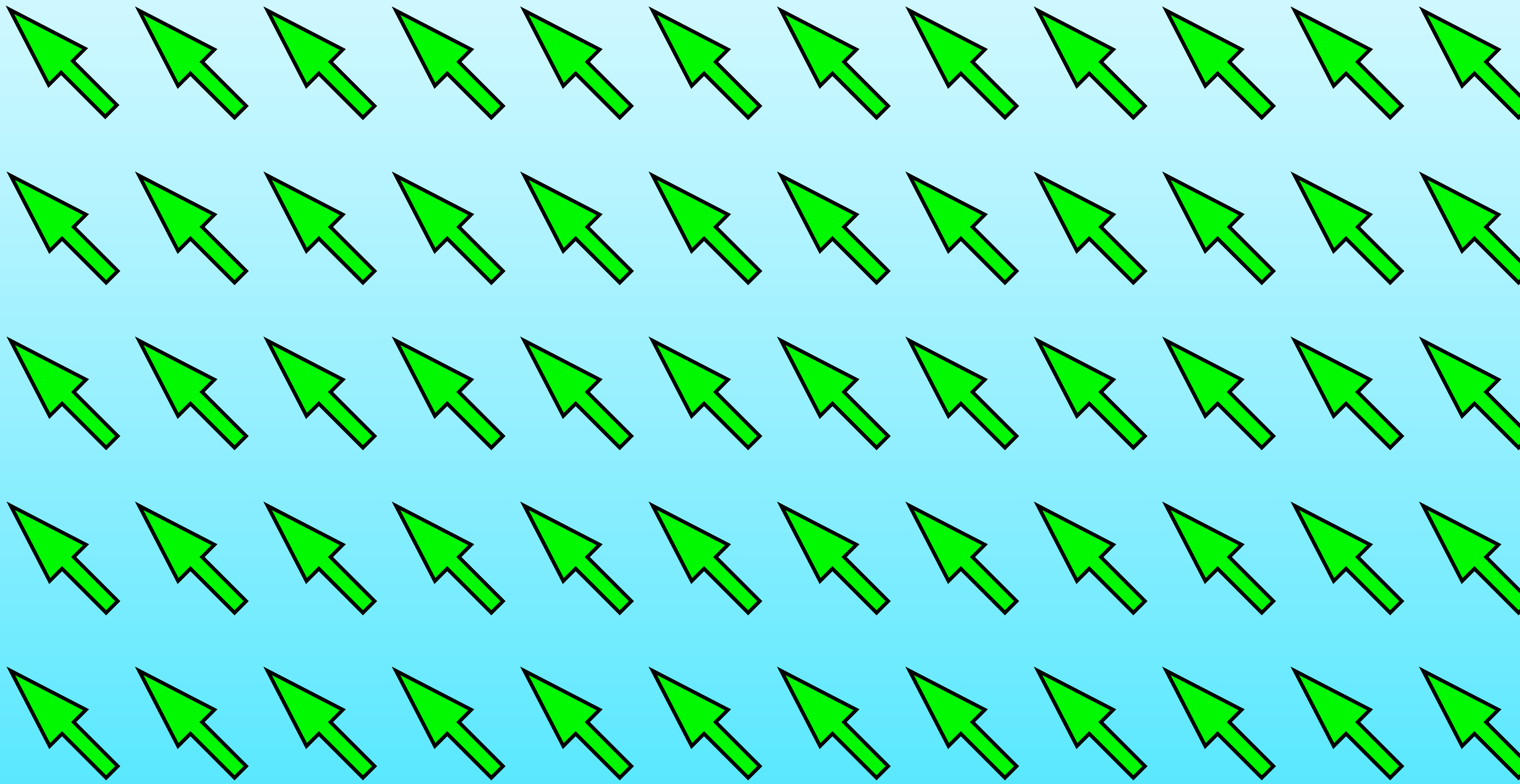
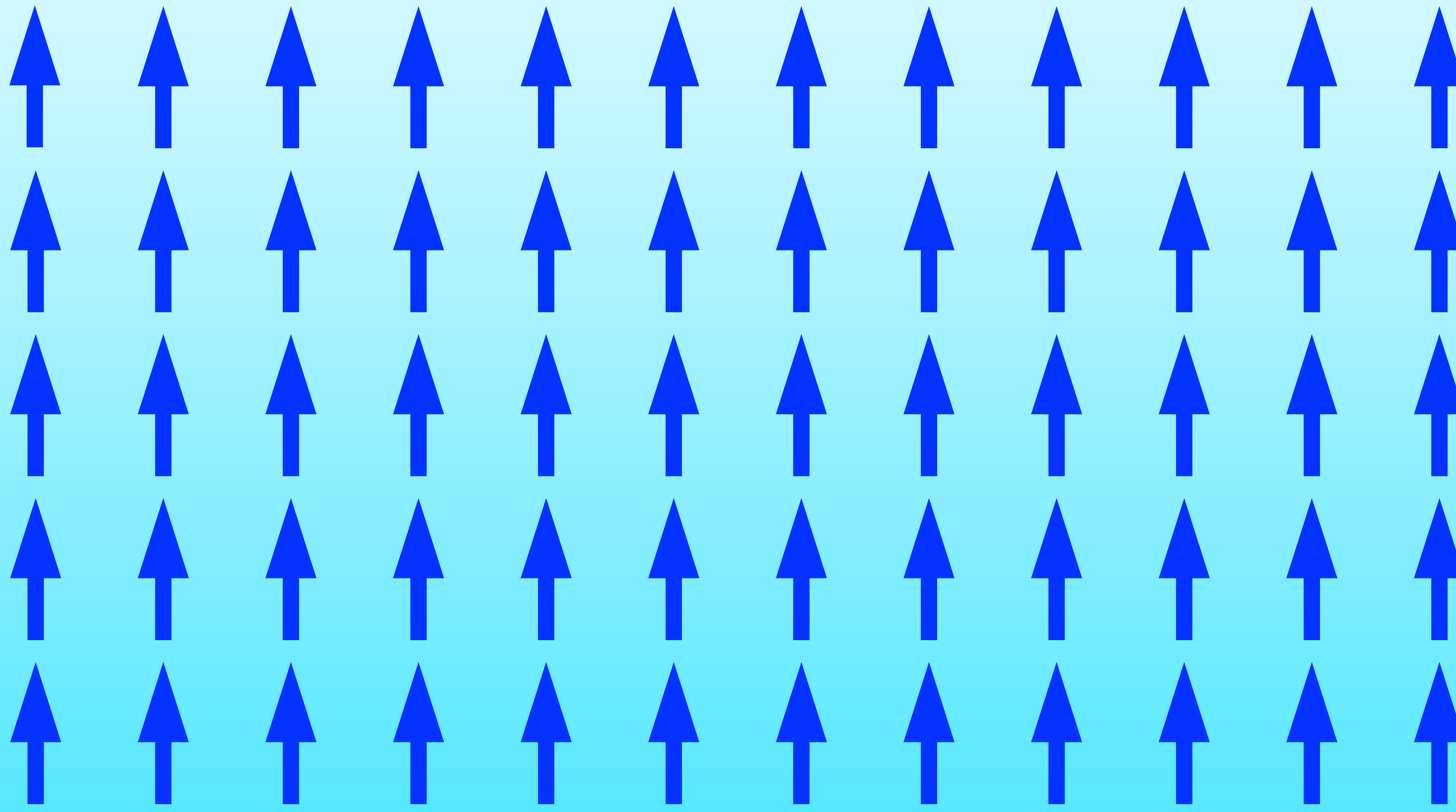
Ordered　　低温相　　*Broken Symmetry*



特定の方向を自発的に物質が選び出す！

温度低下による自発的対称性の破れ

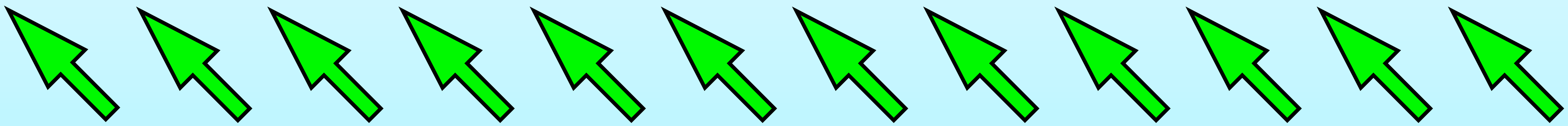
Ordered　　低温相　　*Broken Symmetry*



どの方向への対称性の破れ、秩序もやはり同等！！

温度低下による自発的対称性の破れ

Ordered　　低温相　　Broken Symmetry

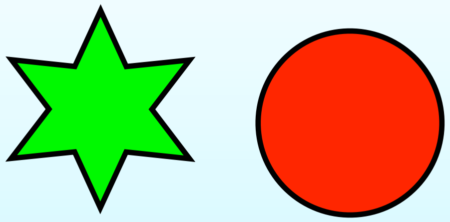


自発的対称性の破れは
自由度無限大になって初めて起こり得る
（マクロな系固有の現象）

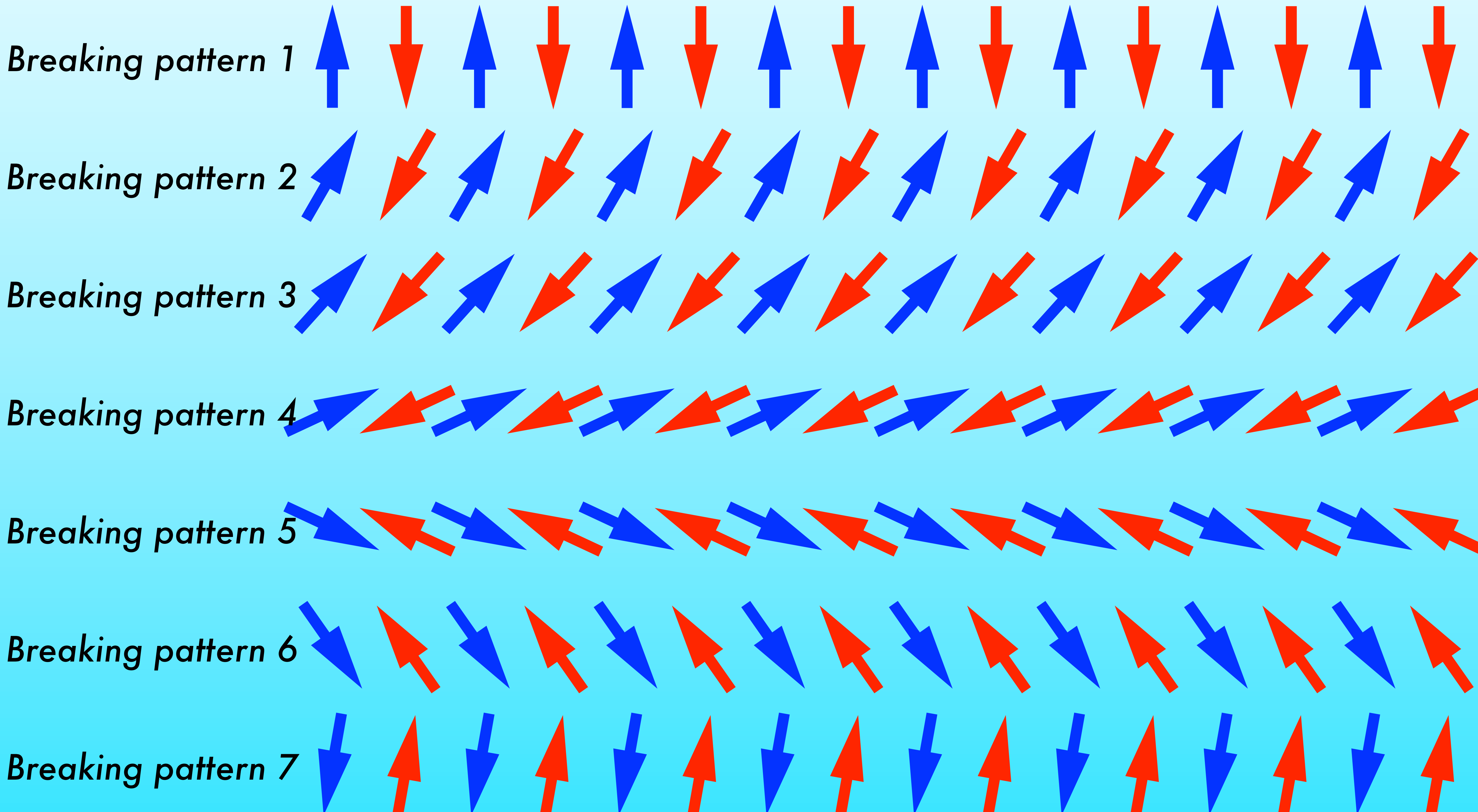
More is Different
by P.W.Anderson

どの方向への対称性の破れ、秩序もやはり同等！！

連続対称性の自発的破れとNambu-Goldstone Boson



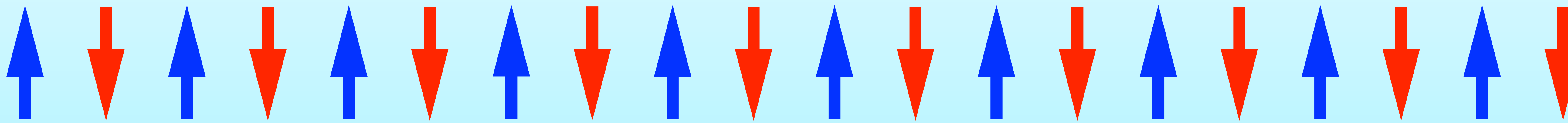
同等な方向が連続無限個ある！



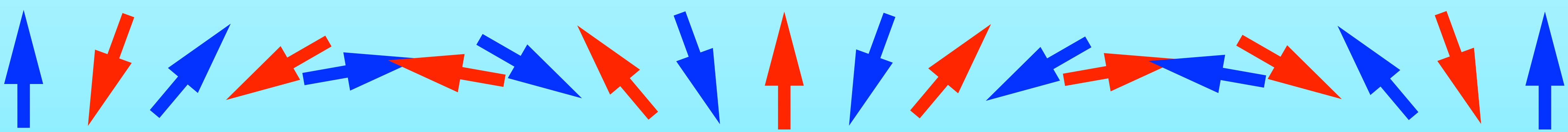
連続対称性の自発的破れとNambu-Goldstone Boson

対称性の破れのパターンをゆっくり乱す

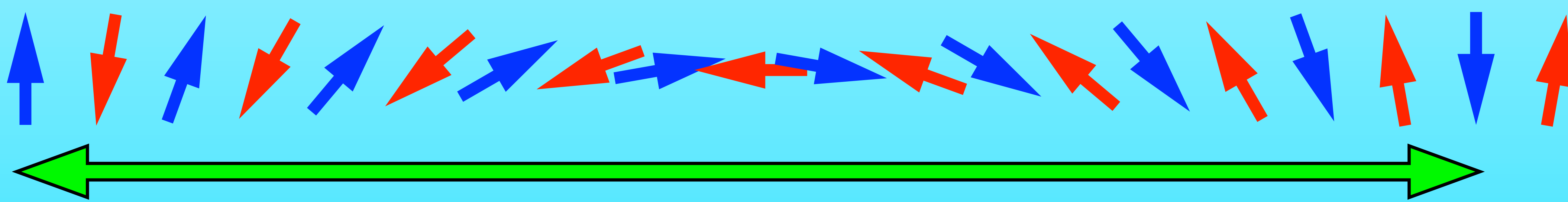
理想的な対称性の破れ



ゆっくりしたパターンの乱れ



もっとゆっくりしたパターンの乱れ



パターンの乱れの波長

連続対称性の自発的破れとNambu-Goldstone Boson

対称性の破れのパターンをゆっくり乱す

Symmetries are destined to be broken (Nambu)
It implies low energy excitations

長波長のゆっくりした乱れは殆どエネルギーのロスが無い

$\lambda \nearrow \quad \hbar\omega \searrow$ 音波的振る舞い **magnon** (磁性波)

$$\omega = ck$$

$k = 2\pi/\lambda$

phonon(格子振動の波)

Nambu-Goldstone Boson

Gapless excitation with
Spontaneous Symmetry Breaking

Spontaneous Broken Symmetry

Nobelprize.org

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By Year | Nobel Prize in Physics | Nobel Prize in Chemistry | Nobel Prize in Medicine | Nobel Prize in L

The Nobel Prize in Physics 2008

"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"

Photo: University of Chicago

Yoichiro Nambu

© The Nobel Foundation Photo: U. Montan

Makoto Kobayashi

© The Nobel Foundation Photo: U. Montan

Toshihide Maskawa

対称性の破れを使った相分類の大成功

Nambu-Goldstone
&
Landau-Ginzburg-Wilson　～1980

The Munich physics professor advised Planck against going into physics, saying, "in this field, almost everything is already discovered, and all that remains is to fill a few holes."

M. Planck　～1900

Wikipedia
The Free Encyclopedia

Quantum phases *without* symmetry breaking

Quantum/spin liquids

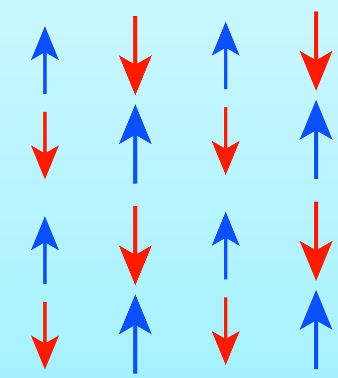
対称性の破れを越えて新しい概念へ

Why do we care topological phases ?

Characterization of phases

too much success

- Ginzburg-Landau theory
  - Local order parameter: $\langle \boldsymbol{S}(r) \rangle$
  - Symmetry breaking $\langle \boldsymbol{S}(r) \rangle \neq 0$



Magnetism ... perconductivity, charge/orbital ordering ...

Is ... isfactory ?

Quantum ... ids

Absence of symmetry b ... king

need something more:

Topological !

Quantum/Spin Liquids ?

- Quantum Liquids in Low Dimensional Quantum Systems
  - Low Dimensionality, Quantum Fluctuations
  - No (fundamental) Symmetry Breaking
  - No Local Order Parameter

New Type of Order
Topological Order!
X.-G.Wen '89

- Quantum Liquids in Condensed Matter
  - Integer & Fractional Quantum Hall States
  - Dimer Models of Fermions and Spins
  - Half filled Kondo Lattice
  - Kitaev model & Levin-Wen model
  - Anisotropic superfluids/superconductors (ABM, BW, p-wave)
  - Graphene, Weyl semi-metal
  - Topological insulators : quantum spin Hall states
  - Photonic crystals & Some of cold atoms ..

Quantum Liquids ?

- Quantum Liquids in Low Dimensional Quantum Systems
  - Low Dimensionality, Quantum Fluctuations
  - N Gapped ntal) Sy Gapless king
  - N der Para

New Type of Order

Topological Order!

X.-G.Wen '89

- Quantum Liquids in Condensed Matter
  - Integer & Fractional Quantum Hall States — Gapped
  - Dimer Models of Fermions and Spin — Gapped
  - Half filled Kondo Lattice — Gapped
  - Kitaev model & Levin-Wen model — Gapped/Gapless
  - Anisotropic superfluids/superconductors (ABM, BW, p-wave)
  - Graphene, Weyl semi-metal — Gapless
  - Topological insulators : quantum spin Hall states — Gapped
  - Photonic crystals & Some of cold atoms ..

"Insulators are stable" implies "Topological"

- Insulators : Gapped
  - Band insulators
  - Superconductors
  - Integer & Fractional Quantum Hall States
  - Integer spin chains (Haldane)
  - Dimer Models (Shastry-Sutherland)
  - Valence bond solid (VBS) states
  - Half filled Kondo Lattice
  - Spin Hall insulators
  - Kitaev model & string net

Absence of low energy excitations
Energy gap above the ground state

Lots of variety

Absence of fundamental symmetry breaking (mostly)

Quantum/spin liquids (gapped)

"*Insulators are stable*" implies "*Topological*"

insulator

Gapped: Nothing in the gap : cf. Nambu-Goldstone boson

No low lying excitations

No Response for small perturbation

~~gapless modes:~~
~~acoustic phonons~~
~~zero sounds~~
~~spin waves~~

??

? ? ?

Absence of low energy excitations
Energy gap above the ground state

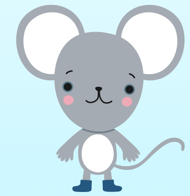
Lots of variety

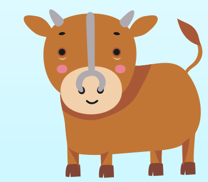
Absence of fundamental symmetry breaking (mostly)

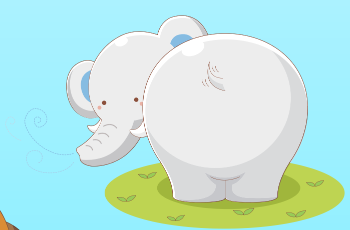
Quantum/spin liquids (gapped)

“*Insulators are stable*” *implies* “*Topological*”

- ☆ *Quantum liquids (gapped)*
  - ☆ *Band insulators*
  - ☆ *Superconductors*
  - ☆ *Integer & Fractional Quantum Hall States*
  - ☆ *Integer spin chains (Haldane)*
  - ☆ *Dimer Models (Shastry-Sutherland)*
  - ☆ *Valence bond solid (VBS) states*
  - ☆ *Half filled Kondo Lattice*
  - ☆ *Spin Hall insulators*
  - ☆ *Kitaev model & string net*







Topological Order

X.G.Wen ’89

Zoo

Something for classification

- ☑ *Topological order*
- ☑ *Edge states*
- ☑ *Berry connections*

講義予定

- 物理学の大事にするもの
  - 普遍性
  - 物性物理学？
  - ２１世紀の物理学
- 物理学における対称性とその破れ
  - 連続対称性
  - 対称性の破れと物性物理学
- 量子的な物質としてのグラフェン
  - 物質中の相対論的粒子
  - 相対論的粒子とエッジ状態
- トポロジカル相とは
  - 量子ホール効果と量子スピンホール効果
  - 端をみて中身を考える：バルク・エッジ対応

KUNGL.
VETENSKAPS-
AKADEMIEN
THE ROYAL SWEDISH ACADEMY OF SCIENCES

OCTOBER 5, 2010

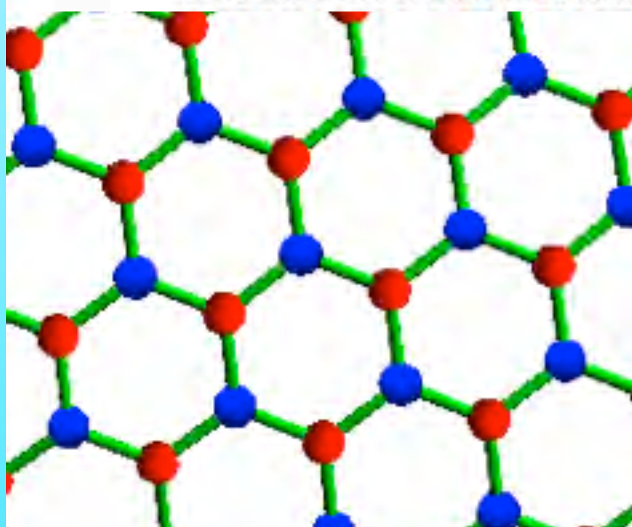
r 2010

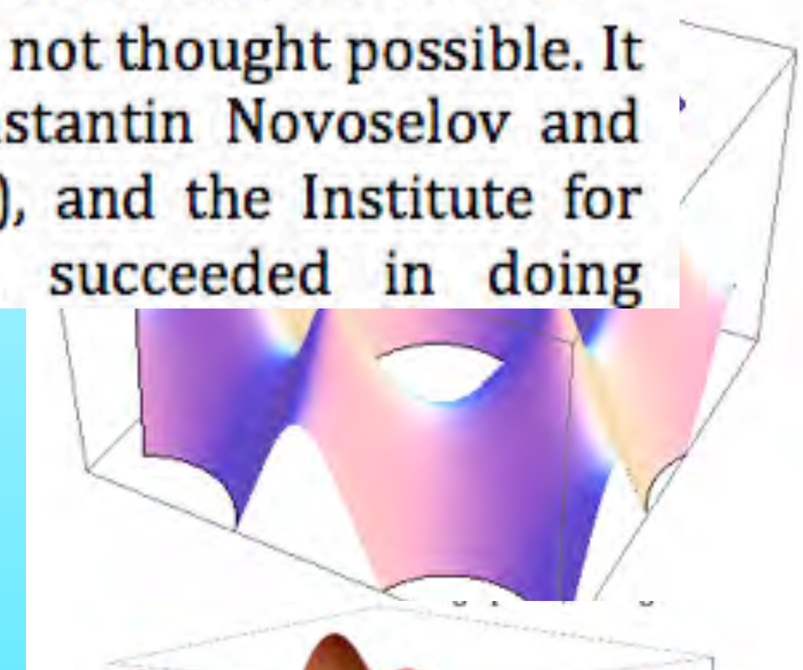
n!

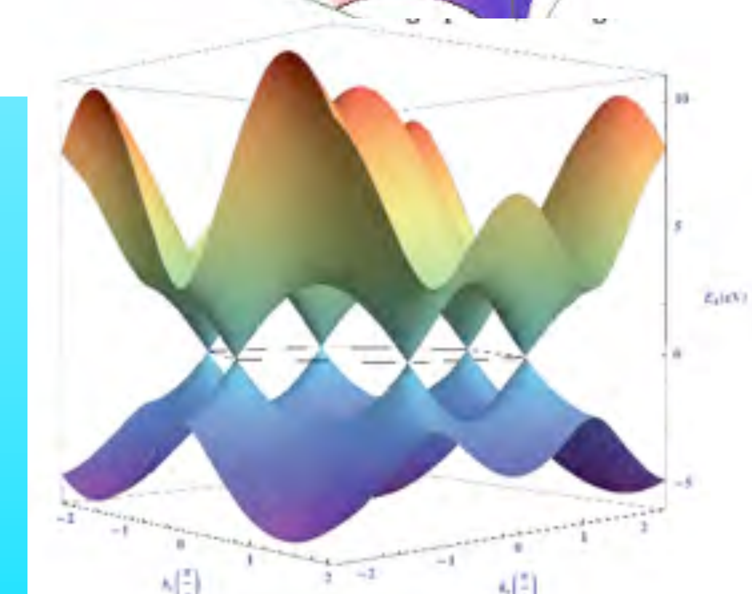
4. The discovery of graphene

Graphene had already been studied theoretically in 1947 by P.R. Wallace[18] as a text book example for calculations in solid state physics. He predicted the electronic structure and noted the linear dispersion relation. The wave equation for excitations was written down by J.W. McClure[19] already in 1956, and the similarity to the Dirac equation was discussed by G.W. Semenoff in 1984,[20] see also DiVincenzo and Mele.[21]

Before 2004, the isolation of stable sheets of graphene was not thought possible. It was therefore a complete surprise when Andre Geim, Konstantin Novoselov and their collaborators from the University of Manchester (UK), and the Institute for Microelectronics Technology in Chernogolovka (Russia), succeeded in doing

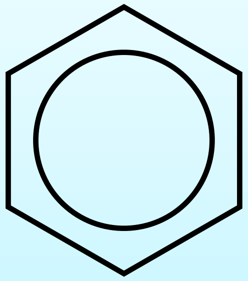


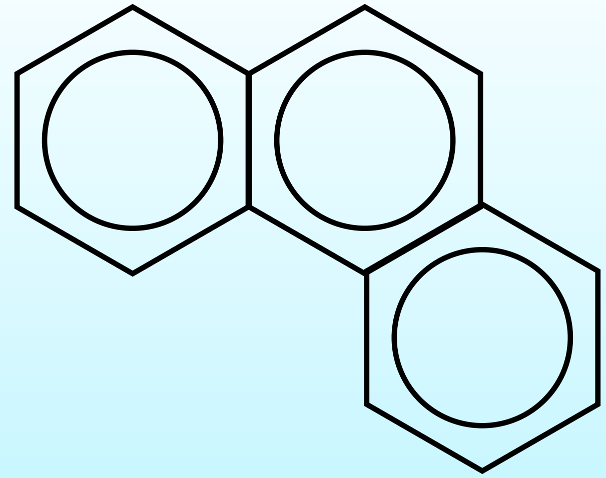


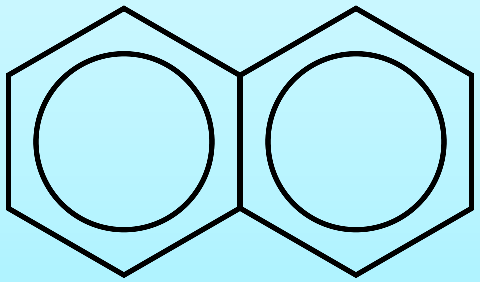


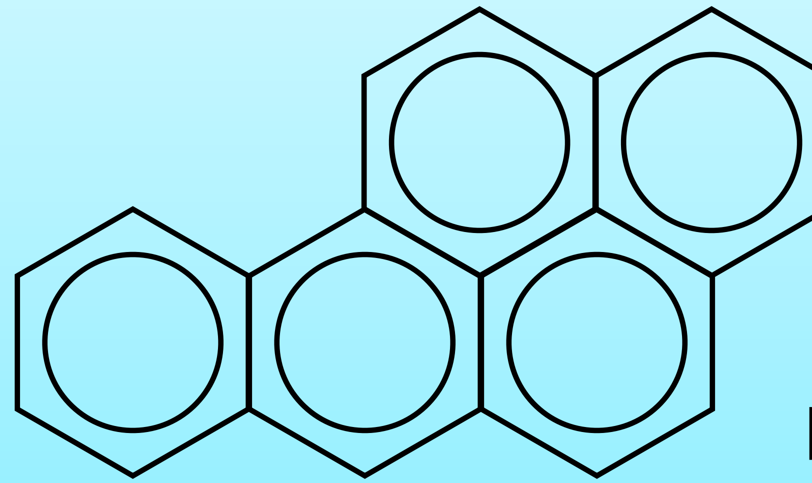
Graphene??

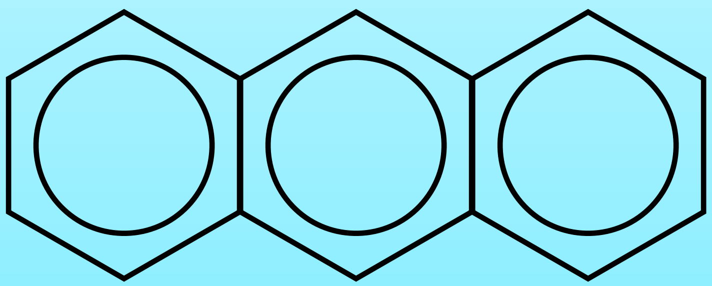
- ☆ π-electron systems

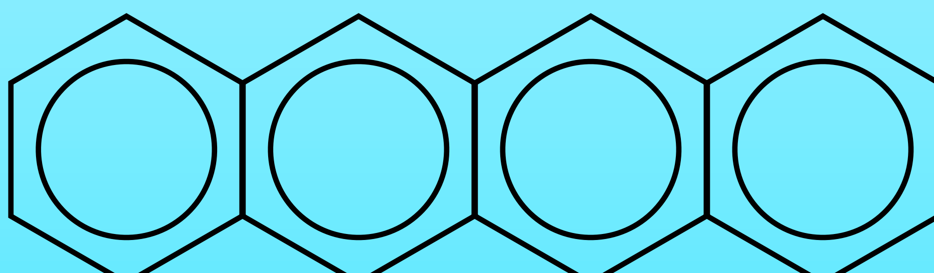

benzene

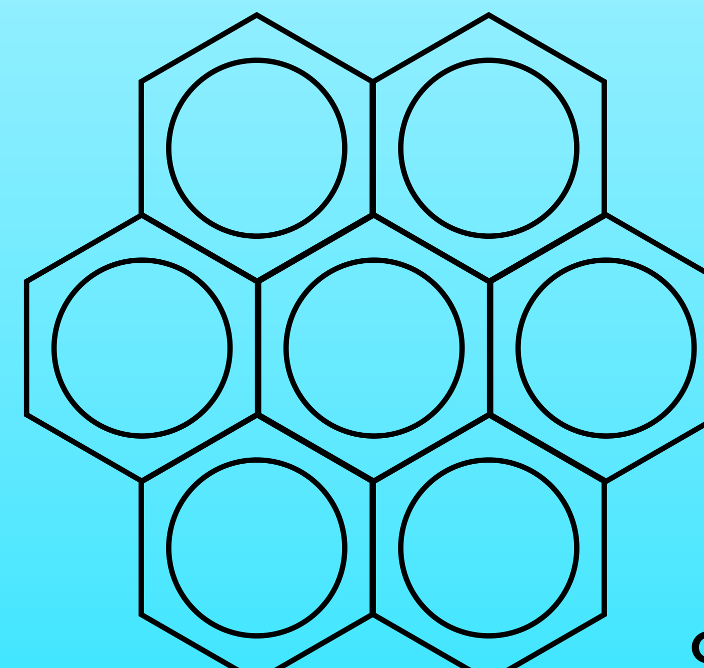

phenanthrene

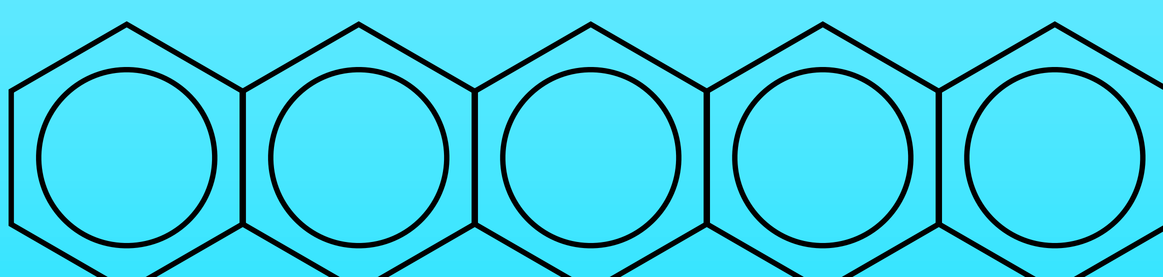

naphthalene


benzopyrene


anthracene

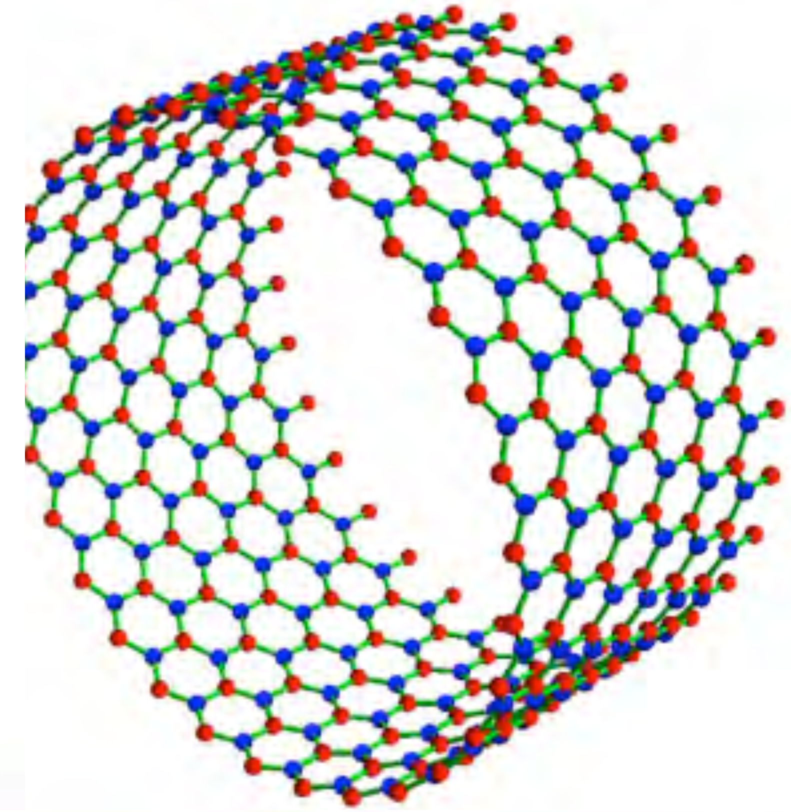

tetracene


coronene

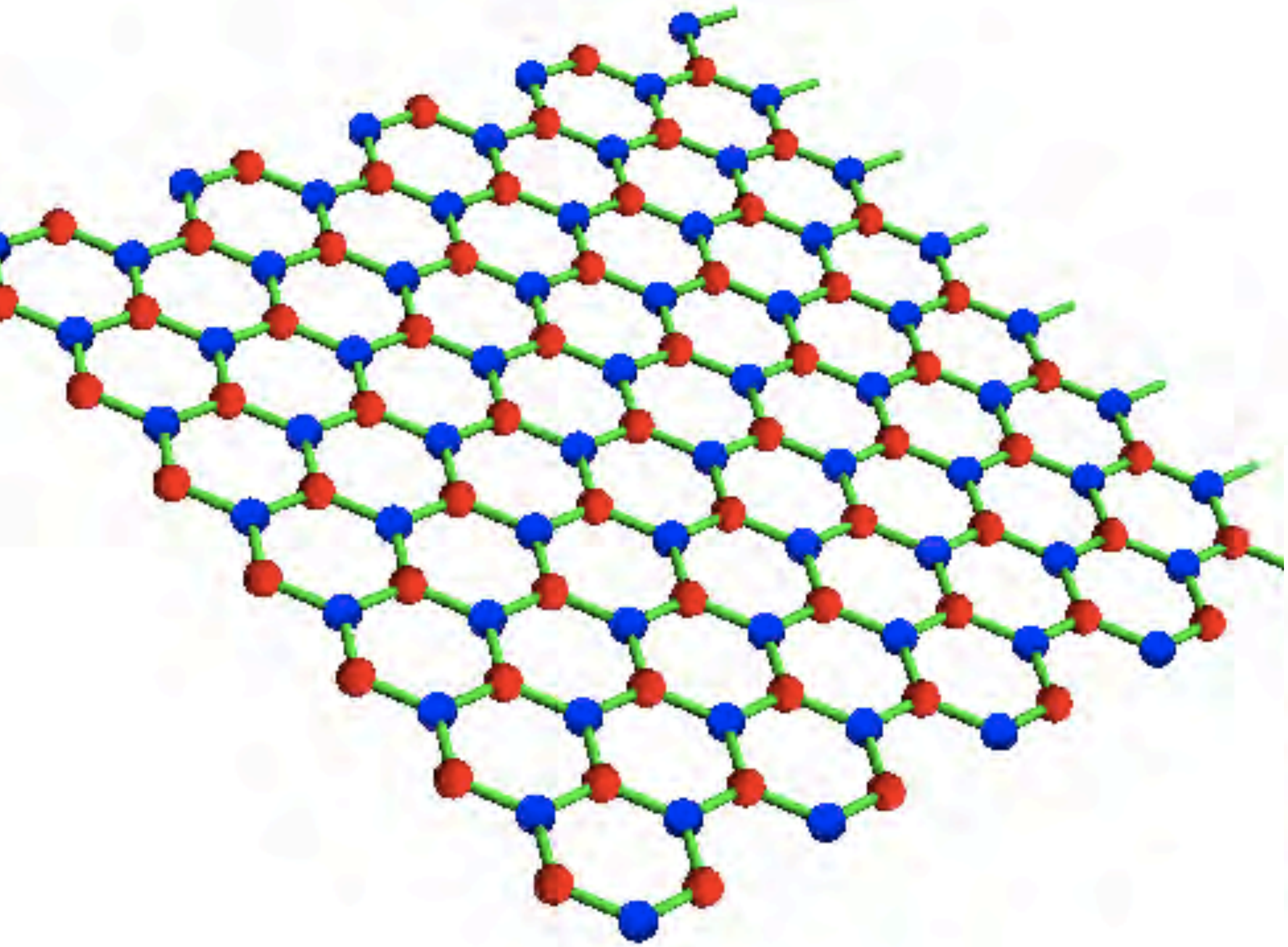

pentacene

Graphene??

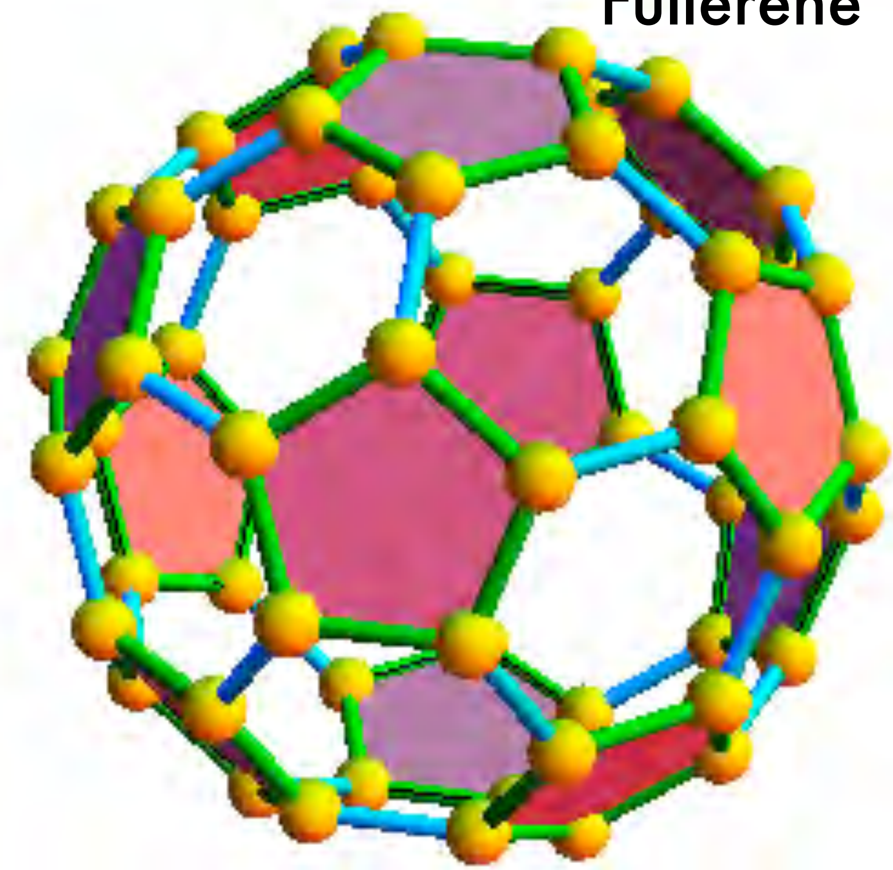
Carbon Nano-Tube



Graphene

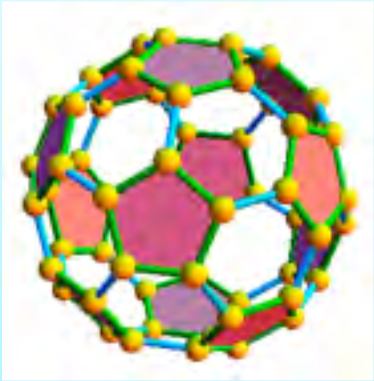


Fullerene



Carbons in Dimension 0,1,2,3,...

fullerene D=0

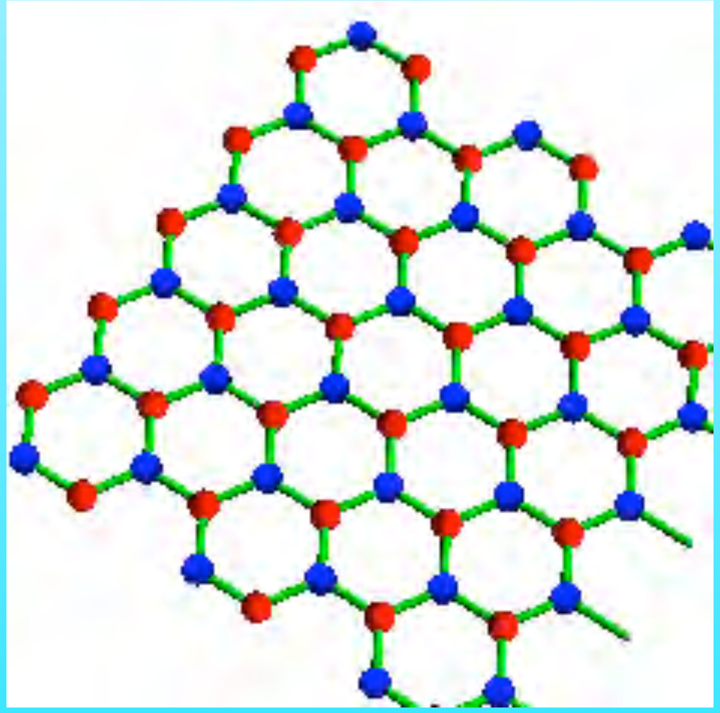


The Nobel Prize in Chemistry 1996
Robert F. Curl Jr., Sir Harold Kroto, Richard E. Smalley

polyacetylene D=1

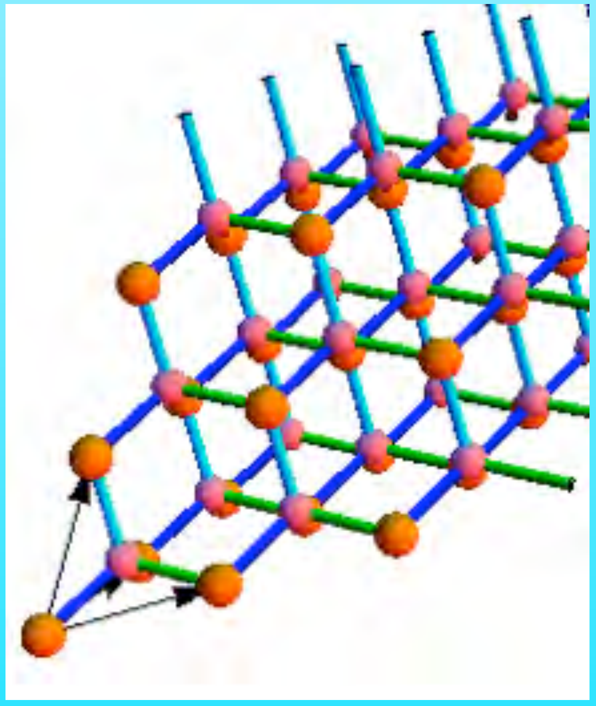
The Nobel Prize in Chemistry 2000
Alan Heeger, Alan G. MacDiarmid, Hideki Shirakawa

graphene D=2



The Nobel Prize in Physics 2010
Andre Geim, Konstantin Novoselov

diamond D=3



4 D graphene
for lattice
gauge theory

M.Creutz

JHEP04(2008)017

Graphene??

THEOREM (Landau, Mermin)

1,2D crystal is unstable against for long range fluctuations

Crystal

Translational symmetry is broken

Fluctuation:phonon

Nambu Goldstone mode

It's sound like :

longer wavelength, then energy cost is small

Strong fluctuation in 2 dimensions

Melting of the crystal

Broken Translational symmetry (Nambu-Goldstone)

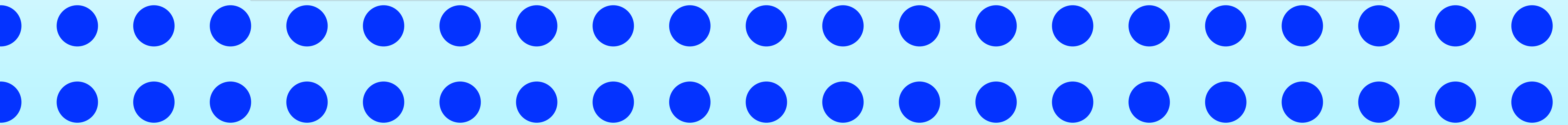
1D g

1D cr

nslation

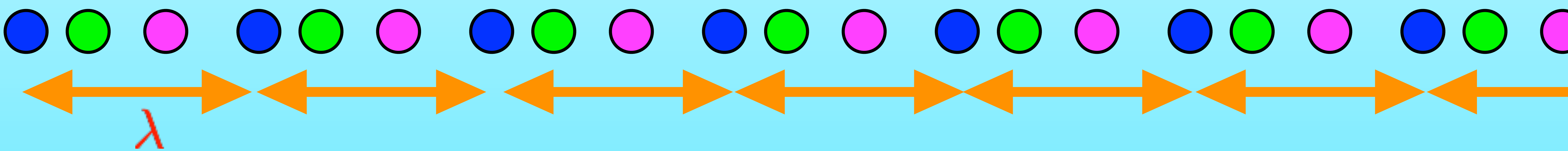
Broken symmetry :

It implies low energy excitations

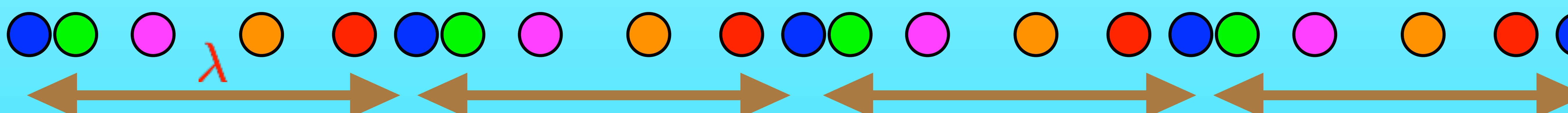


Translational symmetry is broken !

Modification & energy cost



short period modulation : high energy cost



long period modulation : low energy cost

phonon(Sound)

period $\lambda \to \infty$

almost no change

$$\text{Energy cost} \sim \omega \to 0$$

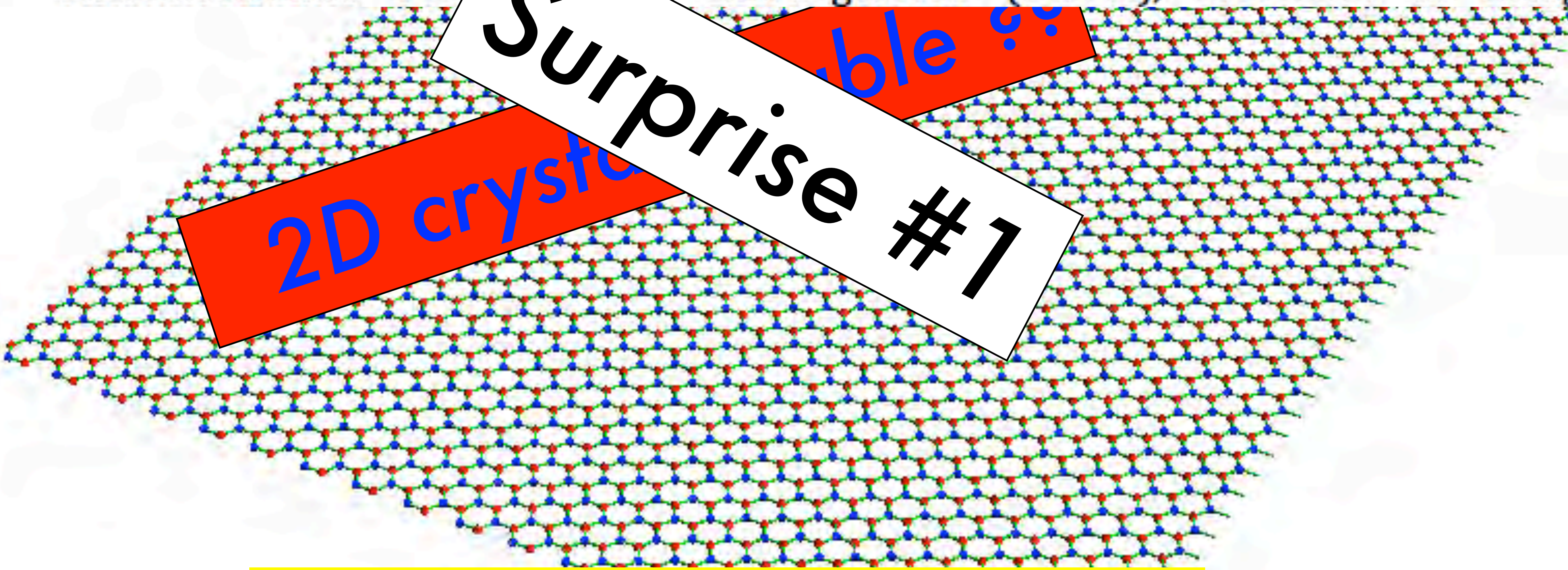
$$\omega \propto 1/\lambda$$

The Nobel Prize in Physics 2010
Andre Geim, Konstantin Novoselov

4. The discovery of graphene

Graphene had already been studied theoretically in 1947 by P.R. Wallace[18] as a text book example for calculations in solid state physics. He predicted the electronic structure and noted the linear dispersion relation. The wave equation for excitations was written down by J.W. McClure[19] already in 1956, and the similarity to the Dirac equation was discussed by G.W. Semenoff in 1984,[20] see also DiVincenzo and Mele.[21]

Before 2004, the isolation of stable sheets of graphene was not thought possible. It was therefore a complete surprise when Andre Geim, Konstantin Novoselov and their collaborators fro the University of Manchester (UK), and the Institute for Microelectronics Tech in Chernogolovka (Russia), succeeded in doing



Impossible to realize

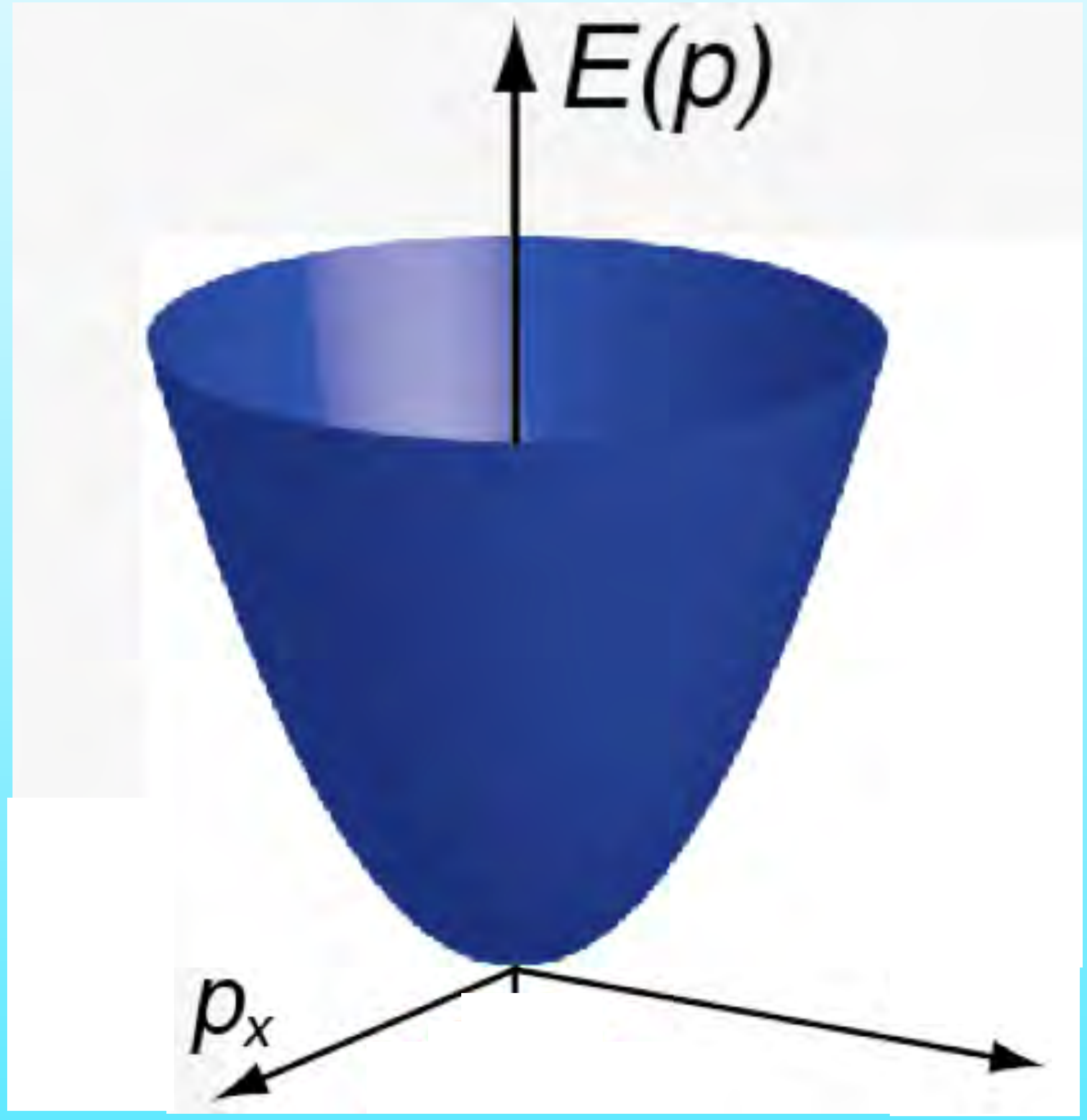
What is so special for Graphene ??

QED (quantum electrodynamics) in a PENCIL

Condensed matter realization of relativistic particles

Energy - momentum relation

E p



Classical particles

$$E = \frac{p^2}{2m}$$

Relativistic particles Einstein

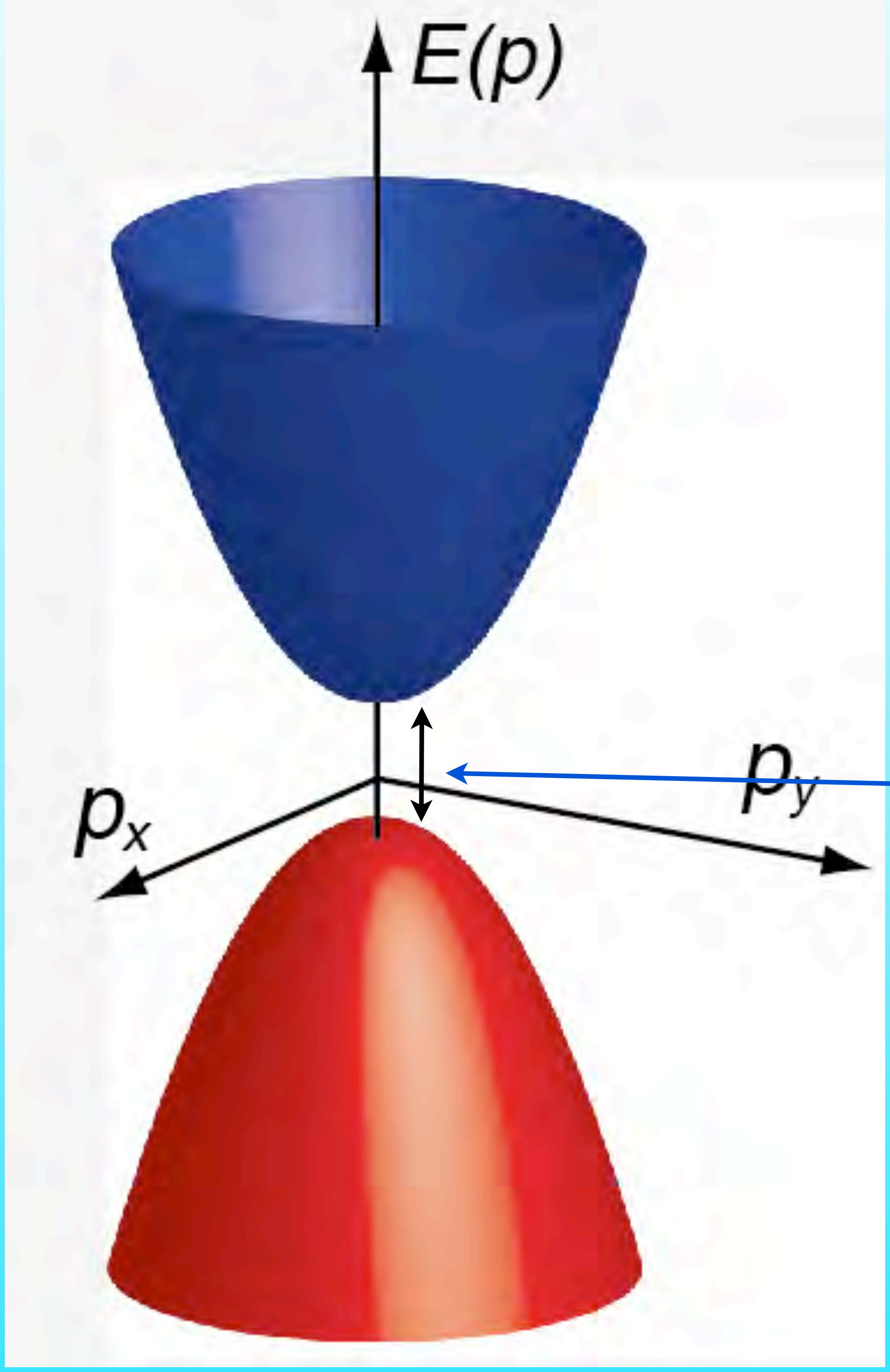
$$E = mc^2$$ at rest

c :light velocity

$$E^2 = (mc^2)^2 + (cp)^2$$ with momentum

Energy - momentum relation

Usual semiconductors (Si, GaAs, ...)



band gap

If m is large,

$$E^2 = (mc^2)^2 + (cp)^2$$ with momentum

$$E = \pm(mc^2)\sqrt{1+(p/mc)^2} \approx \pm E_c$$

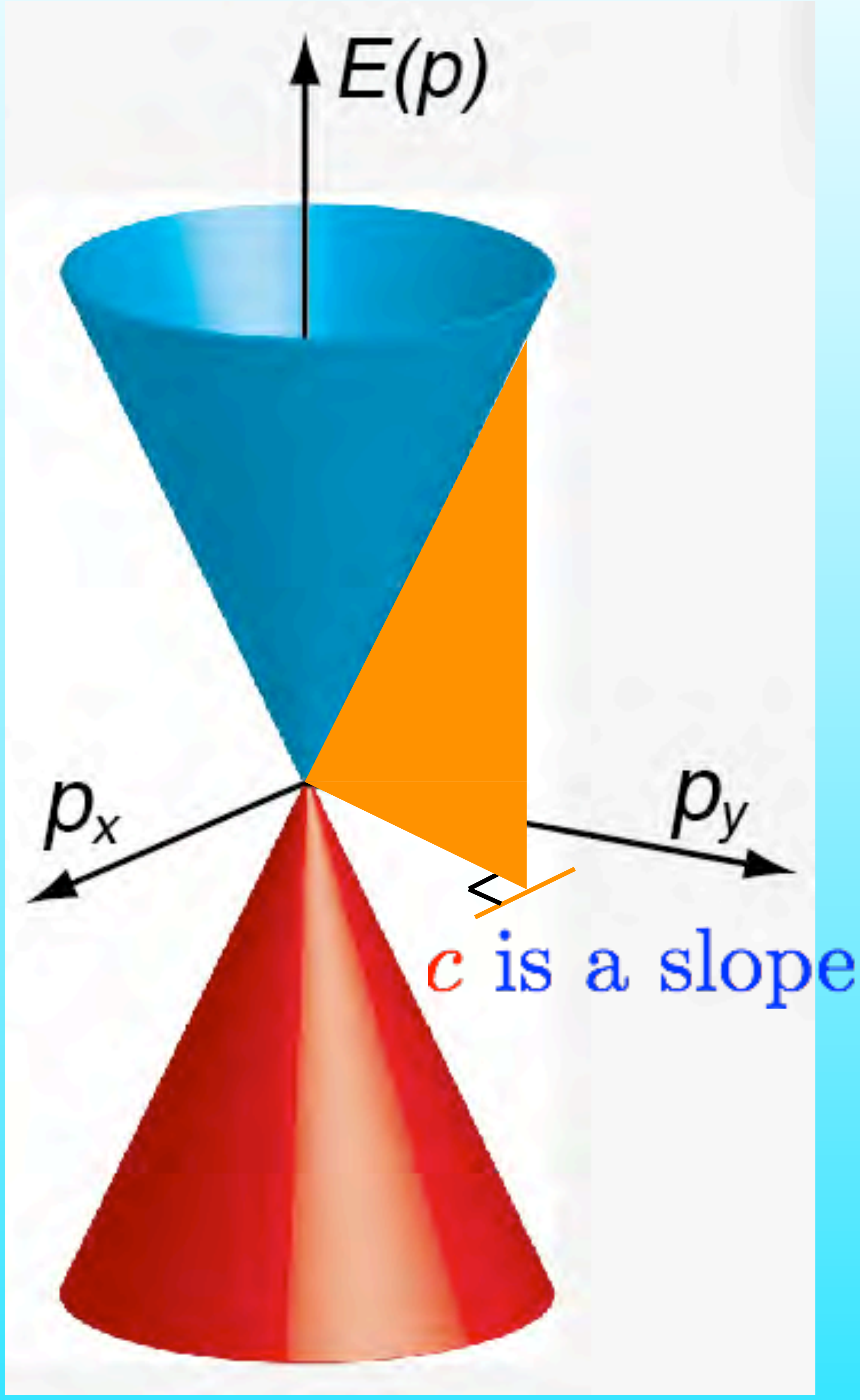
$$E_c = \frac{p^2}{2m} + mc^2$$

Negative energy states

Anti-particles

Zero gap semiconductor

Graphene



Energy momentum relation

$$E = \pm c|p|, \quad |p| = \sqrt{p_x^2 + p_y^2}$$

$$E^2 = (mc^2)^2 + (cp)^2$$

When $m = 0$ $\quad E = \pm c|p|$

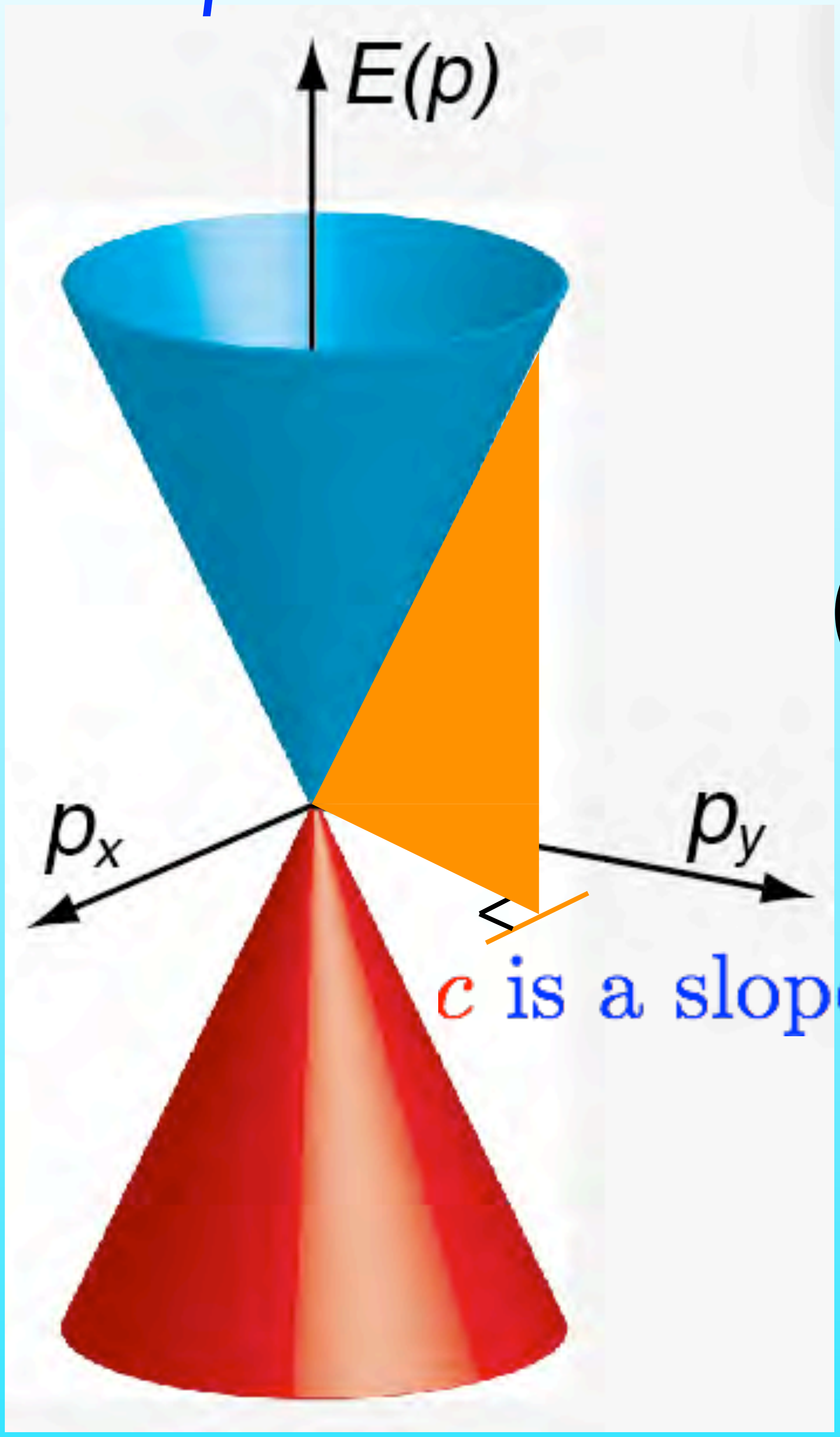
c is not a real light velocity

(just a slope: $\sim c_{\mathrm{light}}/300$ for graphene)

QED (quantum electrodynamics) in a PENCIL

Quantum description of Relativistic particles

Graphene



Dirac equation

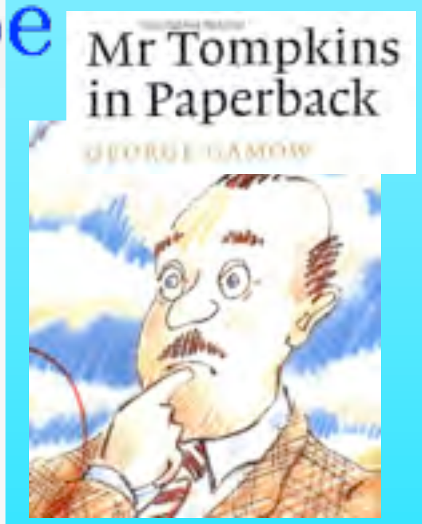
c.f. Newton eq.

c is not a real light velocity

(just a slope: $\sim c_{\mathrm{light}}/300$ for graphene)

"Mr. Tompkins in Wonderland" by G. Gamow

Story of the world where the light speed is about the speed of a bike



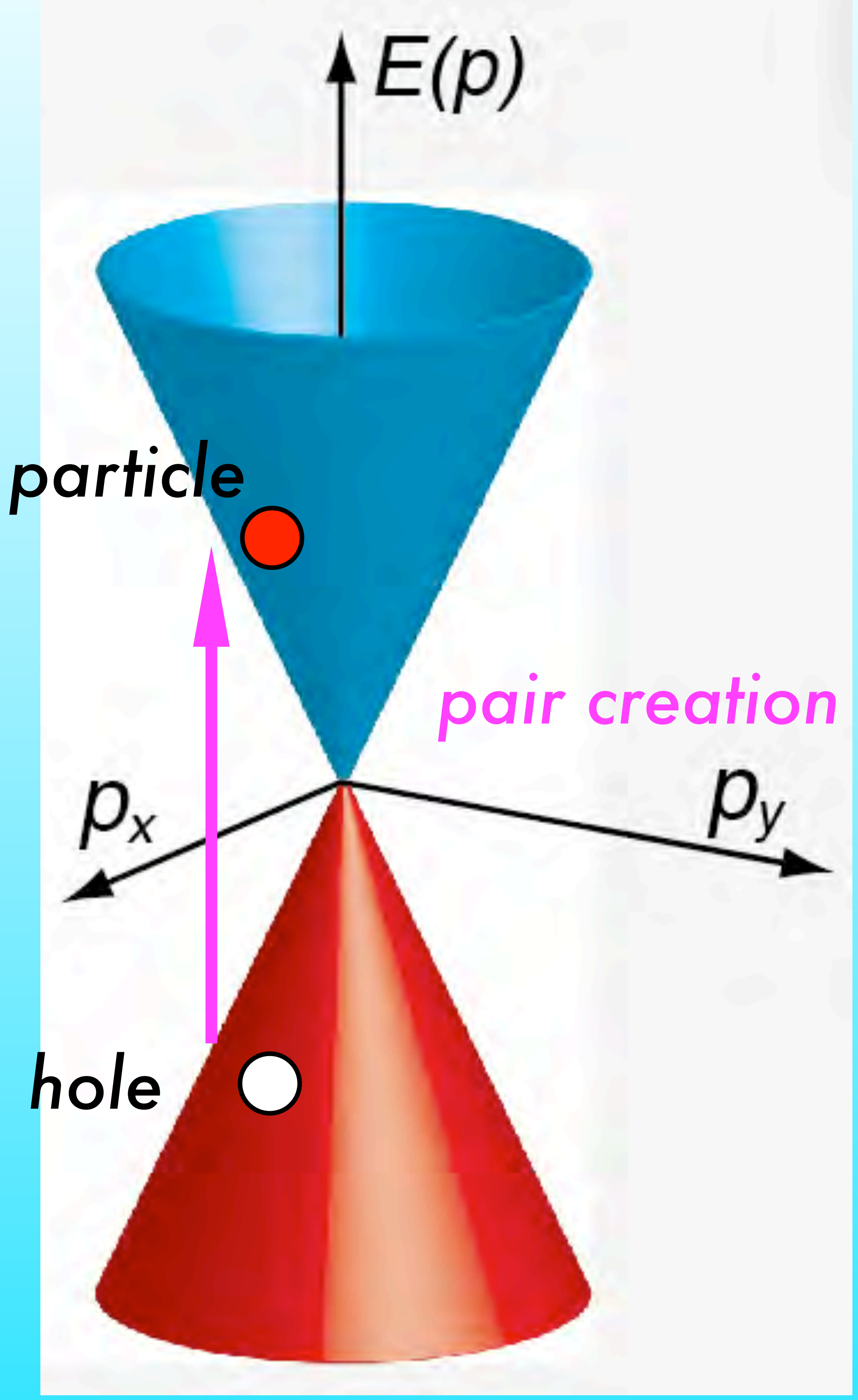
Lorentz contraction

on the bike

from www.amazon.com

Fermions & Anti-particles

Dirac fermions



Fermion :

It can not occupy a single seat
with somebody else

Pauli exclusion principle

粒子は2個は入れない！

Dirac sea

Red part is singly occupied :
never observed directly
only holes(anti-particles) can be seen

hypothesis, fantasy ?

Graphene

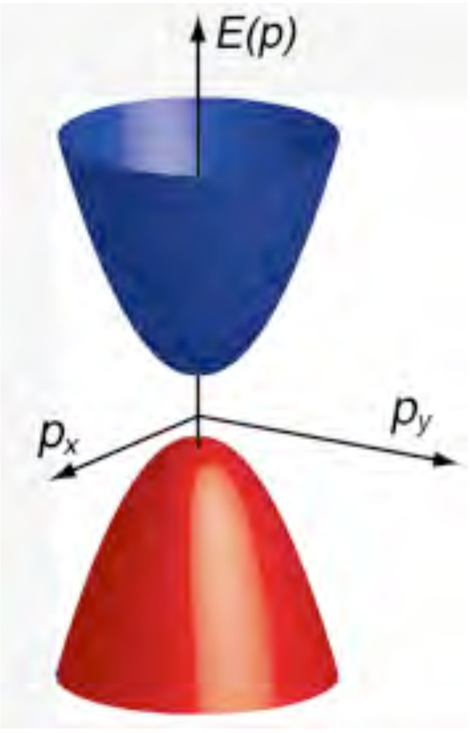
Just a filled valence band
quite standard for semiconductors

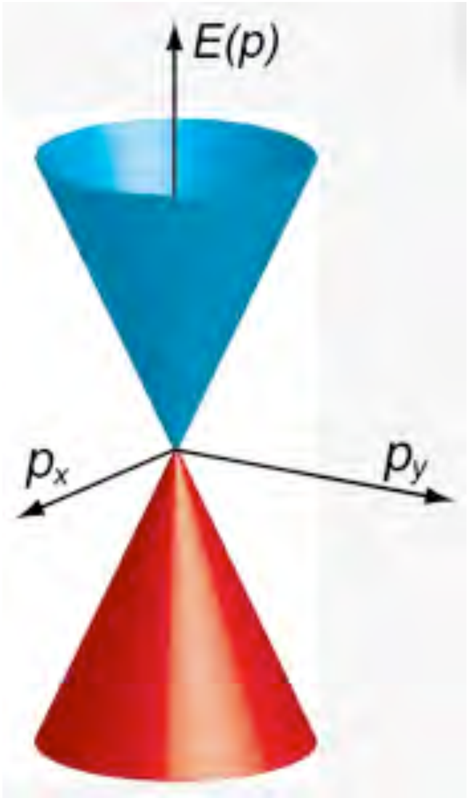
pair annihilation　　LED

P.A.M. Dirac, 1928

half electrons

$$\sqrt{Si} = Graphene$$





P.A.M. Dirac, 1928

half electrons: Dirac equation

$$\sqrt{\mathit{Si}} = \mathrm{Graphene}$$

$$\sqrt{p_x^2 + p_y^2} = \begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix}$$

$$\because \begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix}^2 = (p_x^2 + p_y^2)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2 成分が必要：スピンの発見（「スピンはめぐる」朝永）

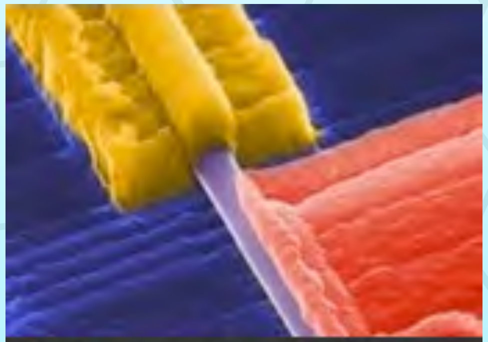
$$c\begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix}\psi = E\psi \qquad E = \pm cp,\ p = \sqrt{p_x^2 + p_y^2}$$

One more half electrons

Majorana fermions

Breakthrough of the Year, 2012

Science AAAS



Majorana Fermions

Science 25 May 2012:
Vol. 336 no. 6084 pp. 1003-1007

Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

V. Mourik[1,*], K. Zuo[1,*], S. M. Frolov[1], S. R. Plissard[2], E. P. A. M. Bakkers[1,2], L. P. Kouwenhoven[1,†]

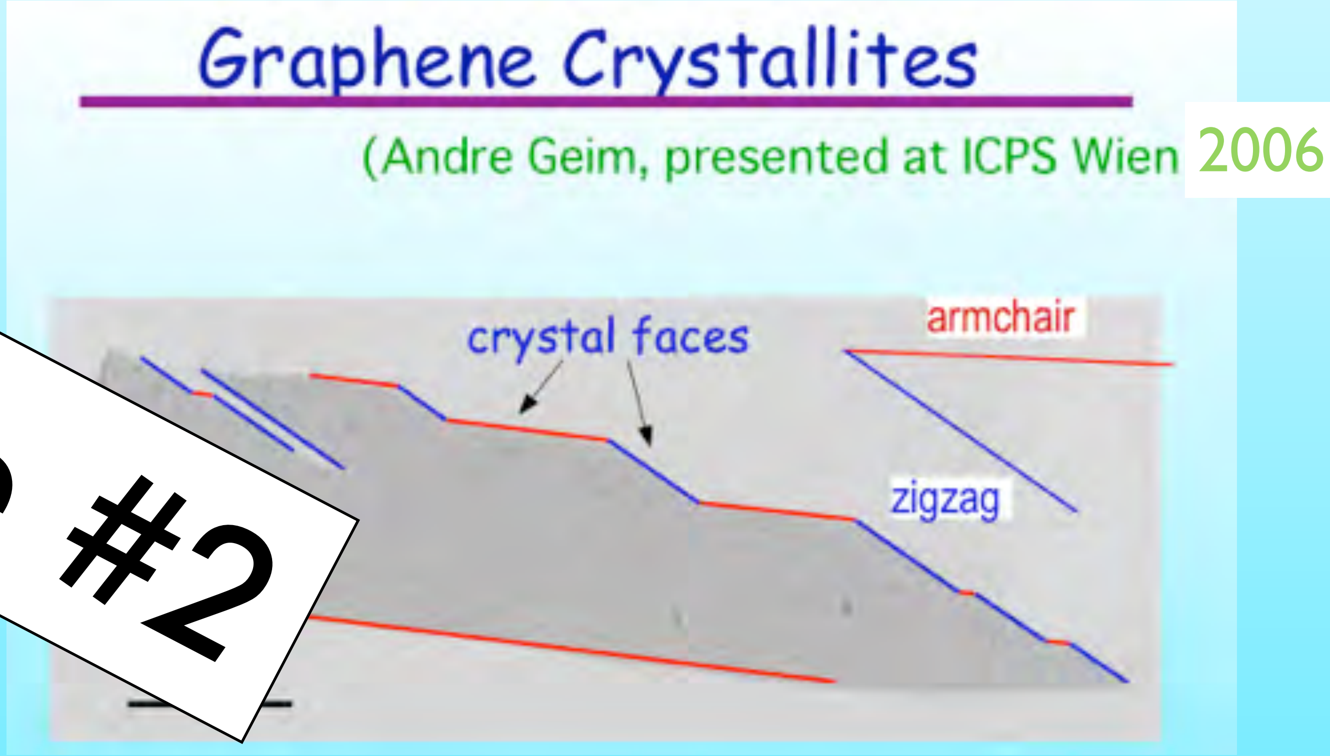
Majorana fermions $= \mathrm{Re}/\mathrm{Im}$ *(electrons)*

量子計算機？　超並列（ルール違反！）

非*Neumann* 型

Graphene

- ☆ Condensed Matter realization of Dirac Fermions
- ☆ QED (quantum electrodynamics) in a PENCIL
  Schwinger-Feynman-Tomonaga



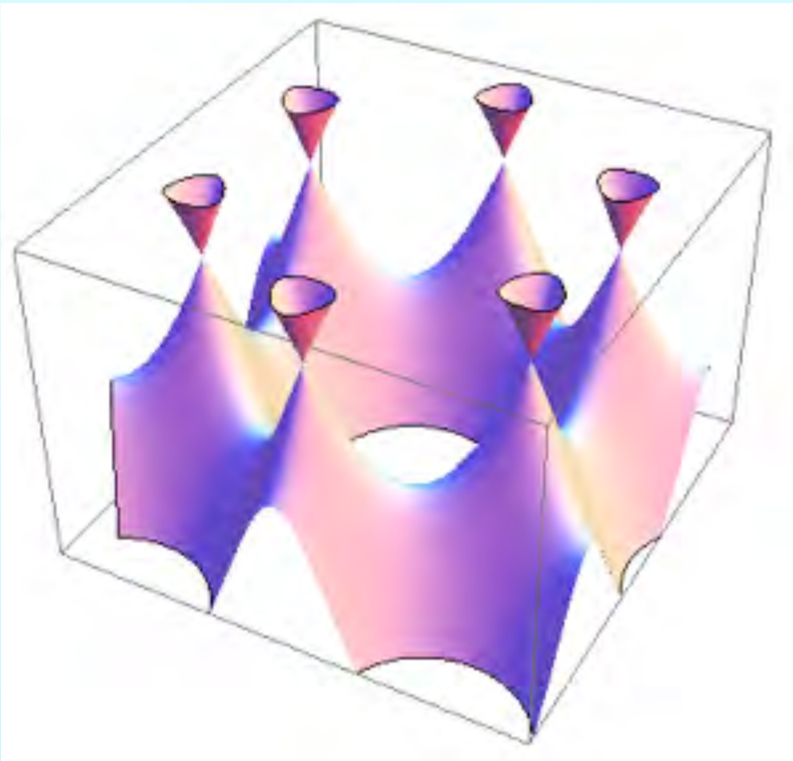
Surprise #2

Lots of fancy ideas of high energy particle physicists are realized and checked in our condensed matter Lab.

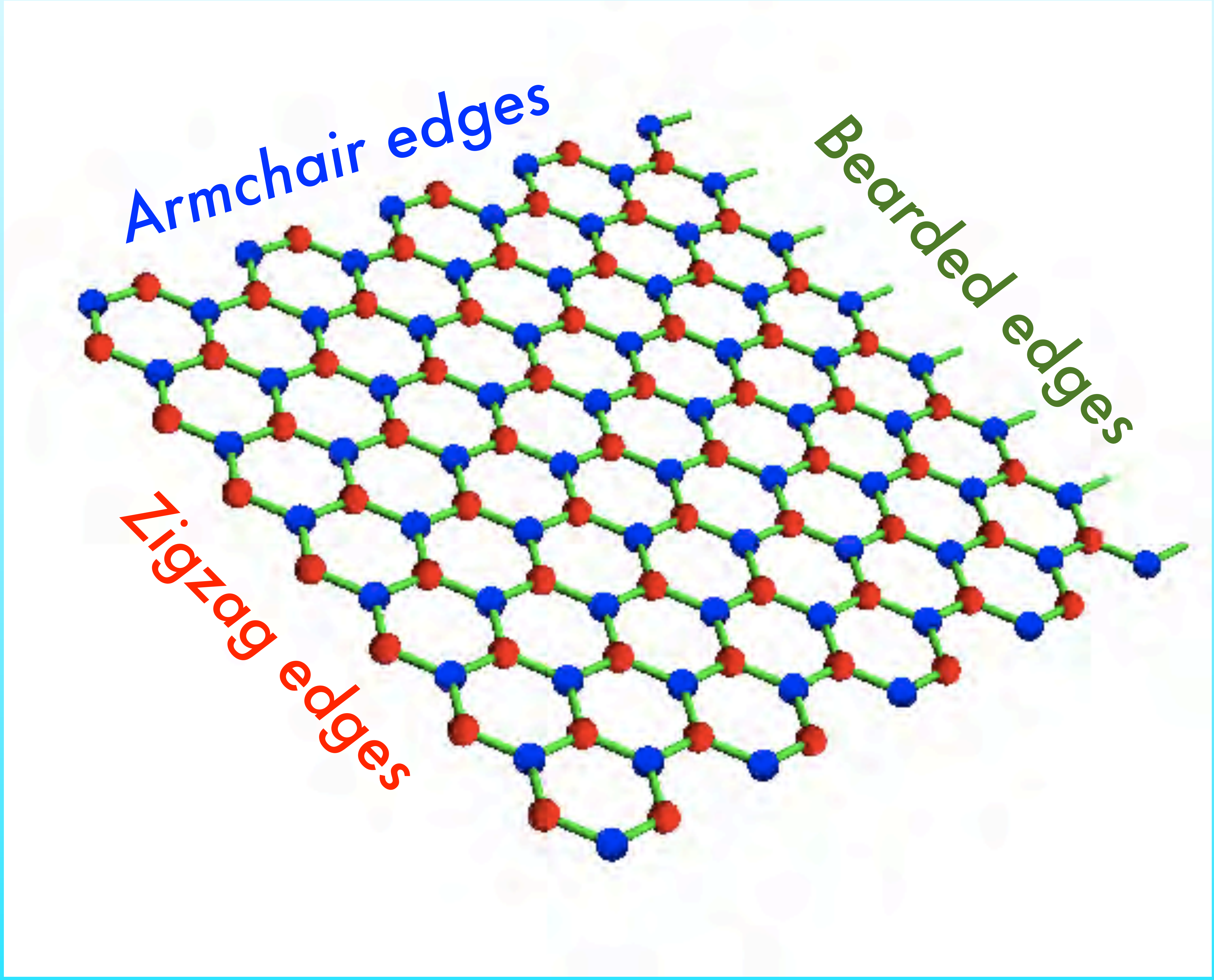
Klein paradox
Quantum Hall effects
Gauge structures

いろいろな境界とエッジ状態

Graphene

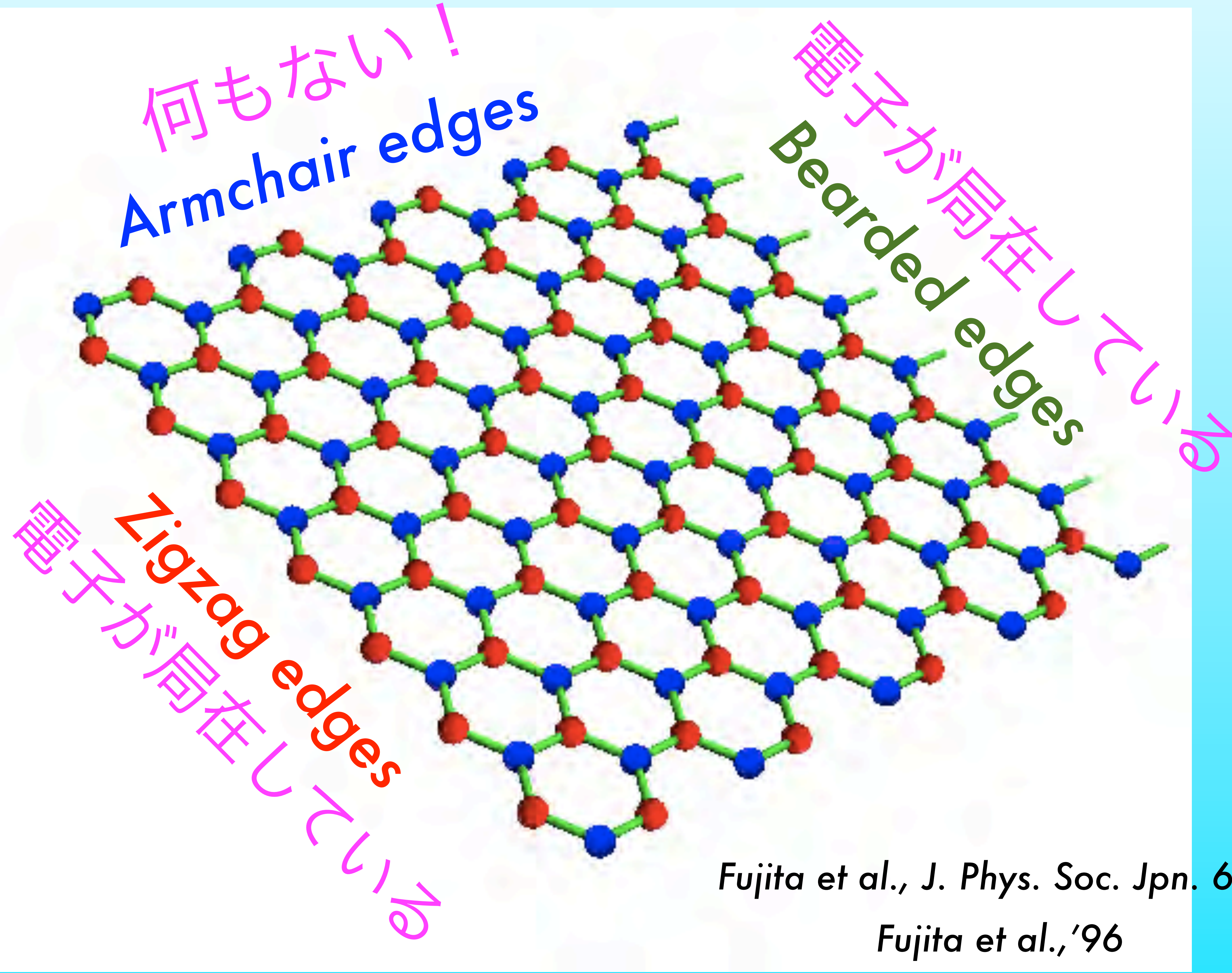


Several types in edges



いろいろな境界とエッジ状態

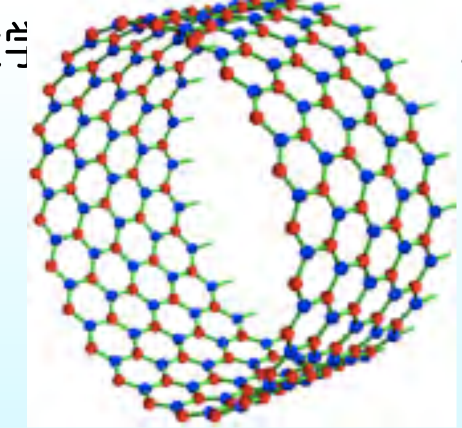
Several types in edges



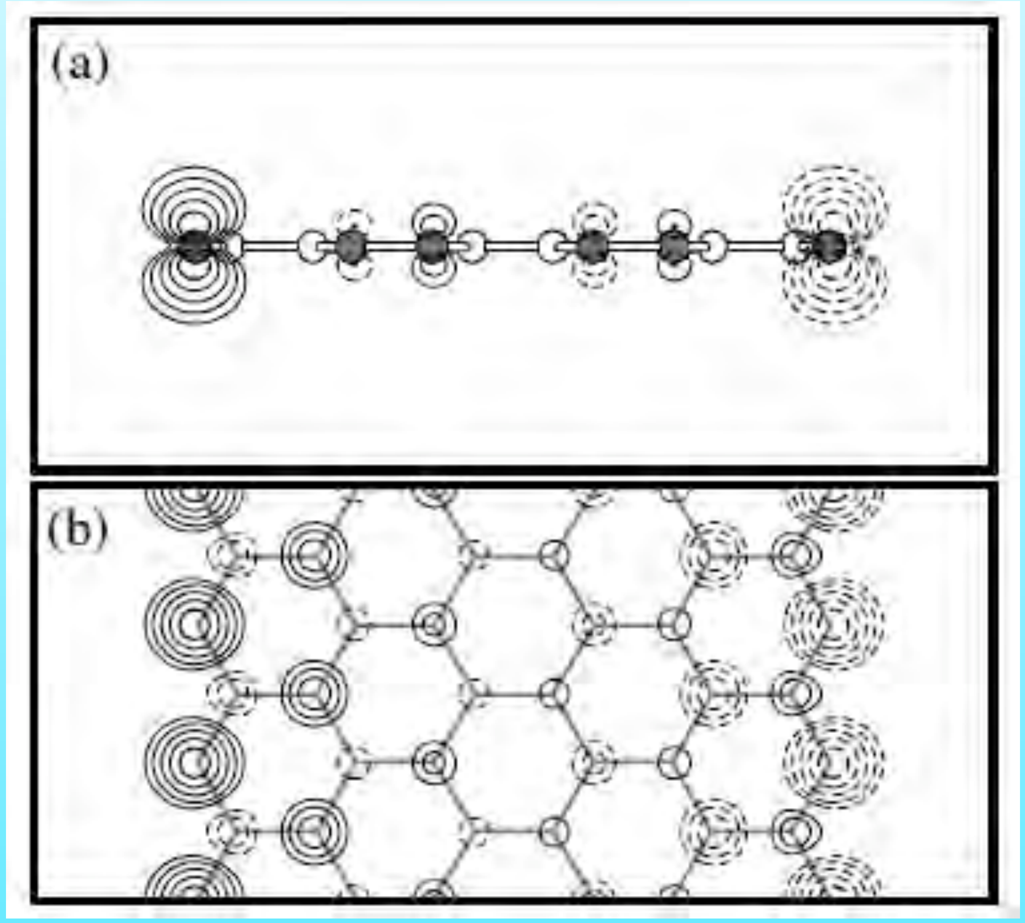
Fujita et al., J. Phys. Soc. Jpn. 65, 1920 (1996)

Fujita et al.,’96

Edge states: It's real !

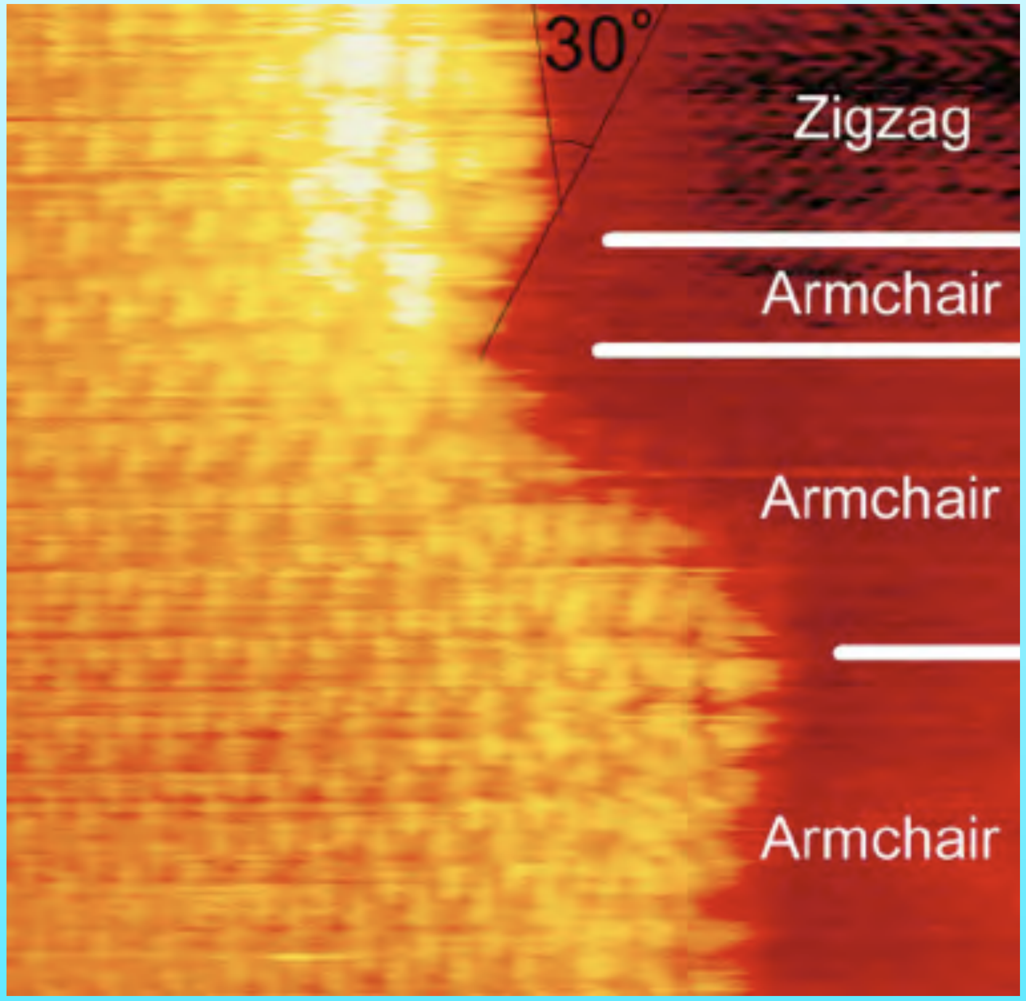


First principle calculation



Okada and Oshiyama,
Phys. Rev. Lett. 87, 146803 (2001)

STM image



Kobayashi et al,
Phys. Rev. B71, 193406 (2005)

Why the Edge States are there??

Accidental ?

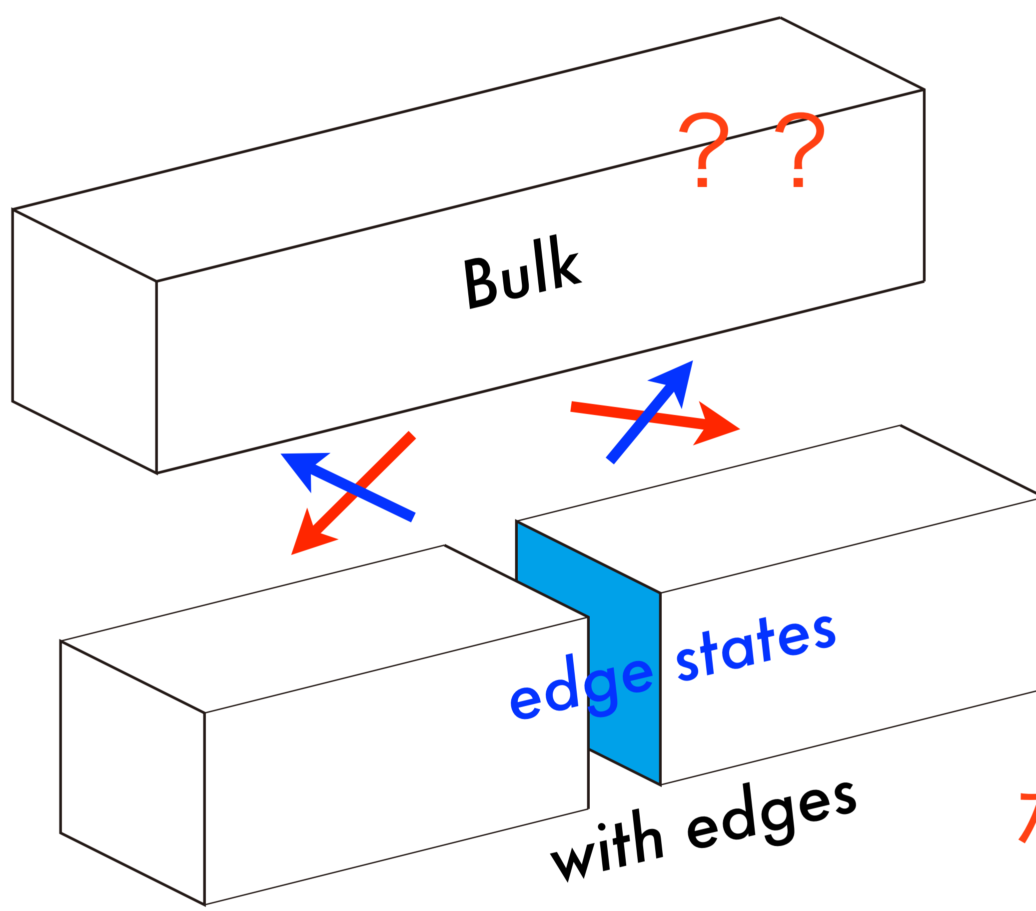
NO !

Inevitable reasons

Universal Structures behind:

Bulk determines the edges

Bulk-Edge Correspondence



cut into two !

端をみて

なかみを考える！

Z_2 Berry phases determine the zero modes

Lattice analogue of Witten's SUSY QM

S.Ryu & Y.Hatsugai, '02
YH'06

Berry phase for each k_y

$$\gamma(k_y) = \int_{k_y:\mathrm{fixed}} dk_x\, A(k_x, k_y) \qquad A = \langle \psi(k) | \nabla_k \psi(k) \rangle$$

Zak

Require Local Chiral Symmetry
(ex. bipartite)

$$\{\Gamma, H\} = \Gamma H + H\Gamma = 0$$

$\Rightarrow$

Z_2 quantization 1D

$$\gamma(k_y) = \begin{cases} \pi \\ 0 \end{cases}$$

c.f. Z_2 in 3D TR inv. case

$\boxed{\gamma(k_y) = \pi}$ $\Rightarrow$ **Zero energy localized states EXIST**

: There exists odd number of zero modes

Bulk-edge correspondence: "Bulk determines the edges"

講義予定

- 物理学の大事にするもの
  - 自然科学のなかの物理学
  - 多様性と普遍性
- 物理学における対称性とその破れ
  - 連続対称性
  - 対称性の破れと物性物理学
- 量子的な物質としてのグラフェン
  - 物質中の相対論的粒子
  - 相対論的粒子とエッジ状態
- トポロジカル相とは
  - 量子ホール効果と量子スピンホール効果
  - 端をみて中身を考える：バルク・エッジ対応

Quantum Hall Effect *'80, K.v.Klitzing et al.*

Quantization of the Hall conductance σ_{xy} with anomalous accuracy: $I = \sigma_{xy} V$

topologically cylinder

The Nobel Prize in Physics 1985
Klaus von Klitzing

Share this: 6

The Nobel Prize in Physics 1985

Klaus von Klitzing
Prize share: 1/1

The Nobel Prize in Physics 1985 was awarded to Klaus von Klitzing *"for the discovery of the quantized Hall effect"*.

The Nobel Prize in Physics 1998
Robert B. Laughlin, Horst L. Störmer, Daniel C. Tsui

Share this: 5

The Nobel Prize in Physics 1998

Robert B. Laughlin
Prize share: 1/3

Horst L. Störmer
Prize share: 1/3

Daniel C. Tsui
Prize share: 1/3

The Nobel Prize in Physics 1998 was awarded jointly to Robert B. Laughlin, Horst L. Störmer and Daniel C. Tsui *"for their discovery of a new form of quantum fluid with fractionally charged excitations"*.

3

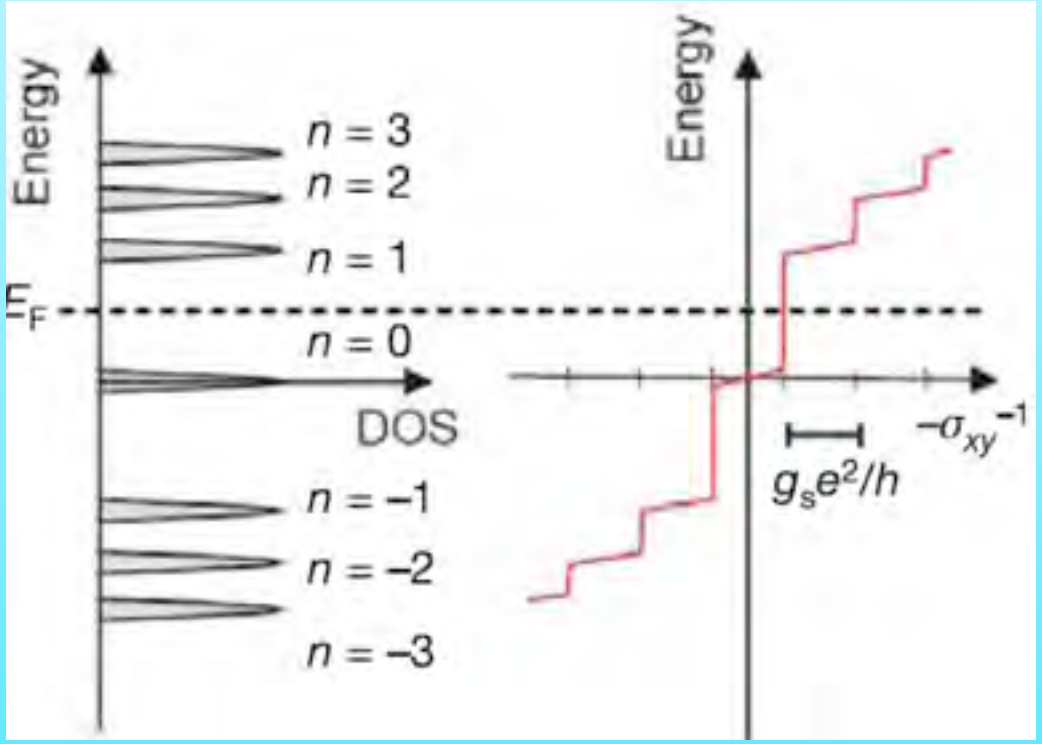
MAGNETIC FIELD [T]

R. Willett et al.

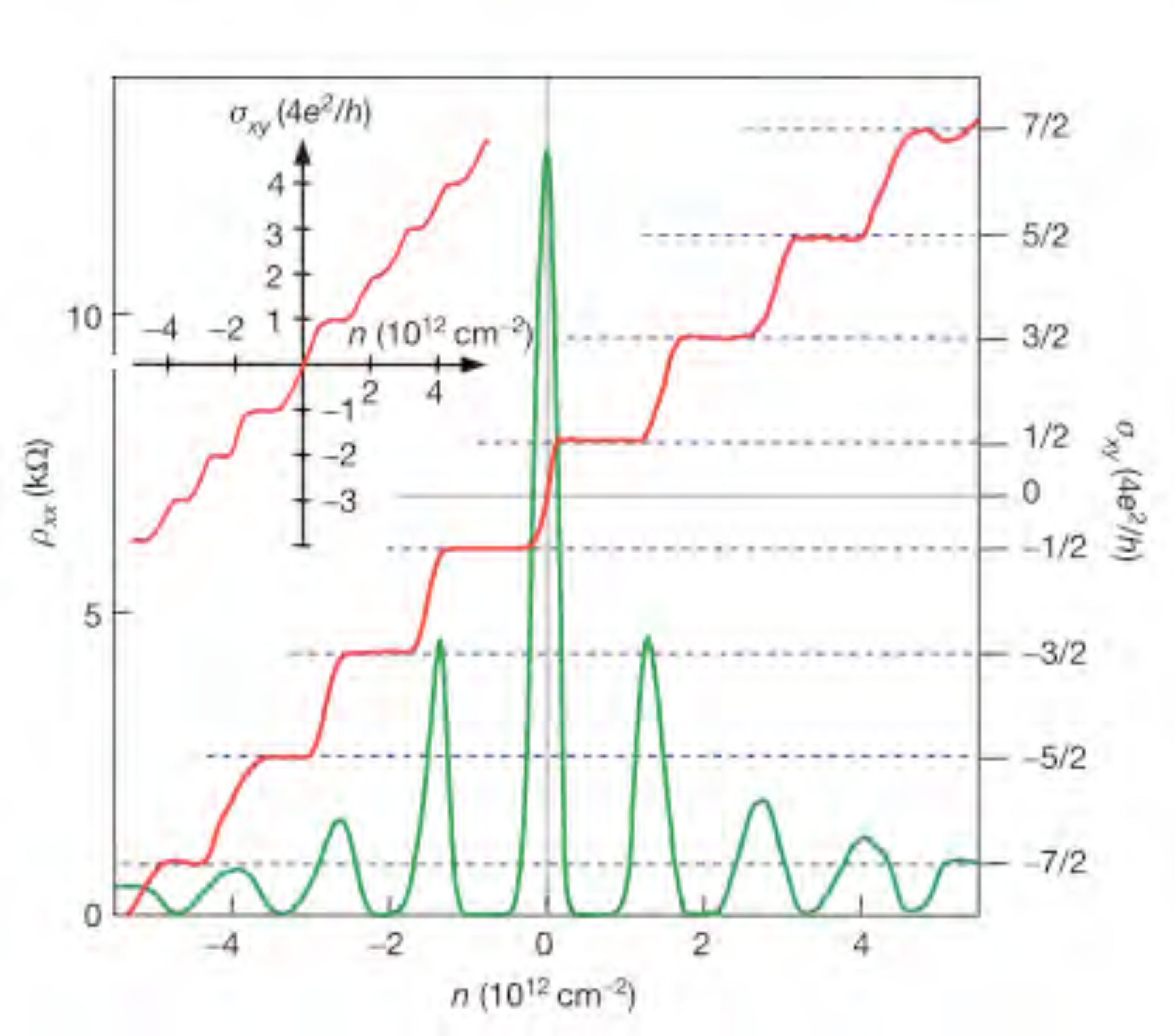
Graphene

☆ *Anomalous QHE of gapless Dirac Fermions*

$$\sigma_{xy} = \frac{e^2}{h}(2n+1),\ \ n = 0, \pm 1, \pm 2, \cdots$$
$$= 2\frac{e^2}{h}(n+\frac{1}{2})$$



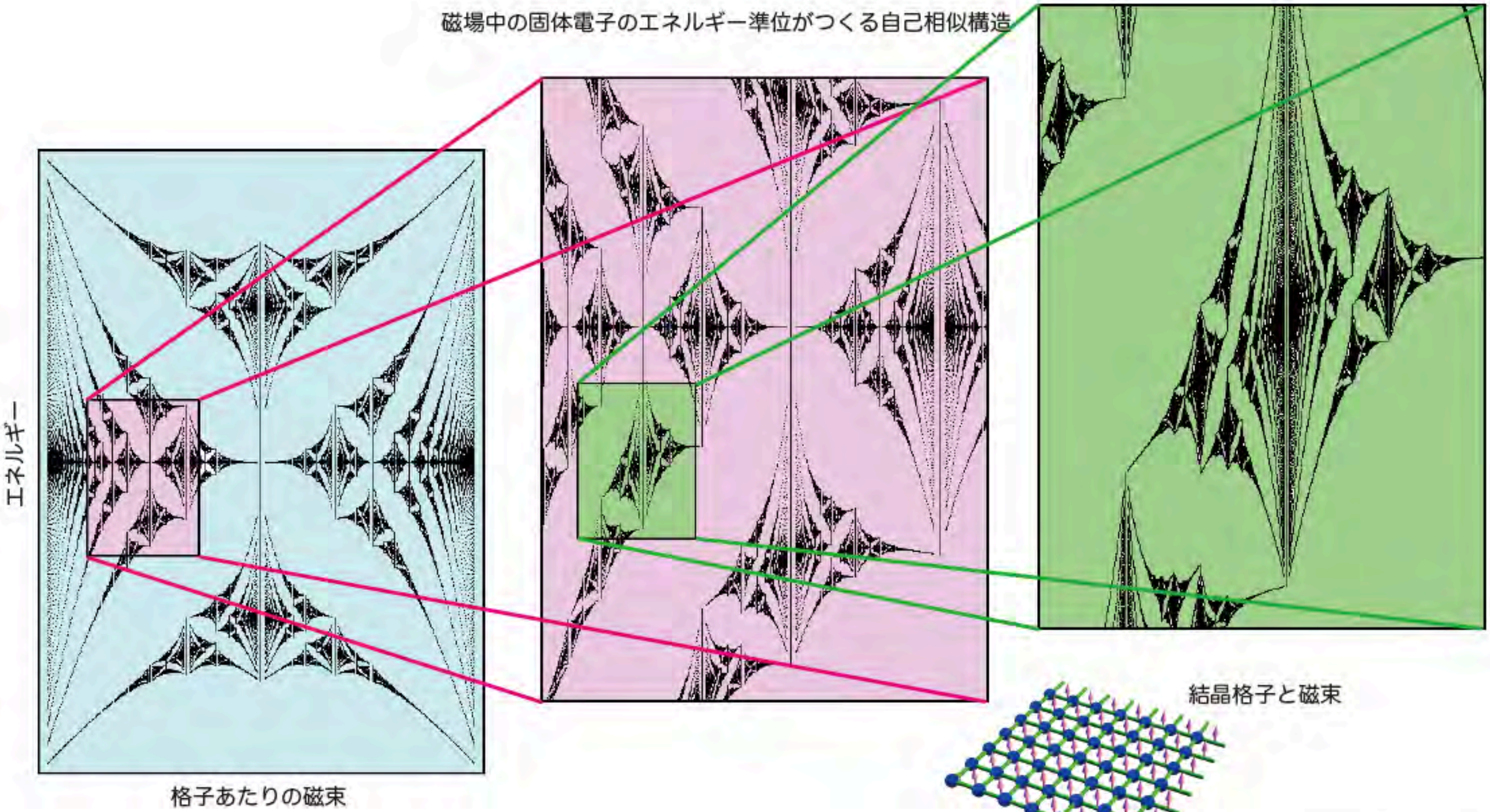
Zhang et al. Nature 2005



Novoselov et al. Nature 2005

物質中のフラクタル
Hofstadter の蝶

磁場中の固体電子のエネルギー準位がつくる自己相似構造



by Y. Hatsugai

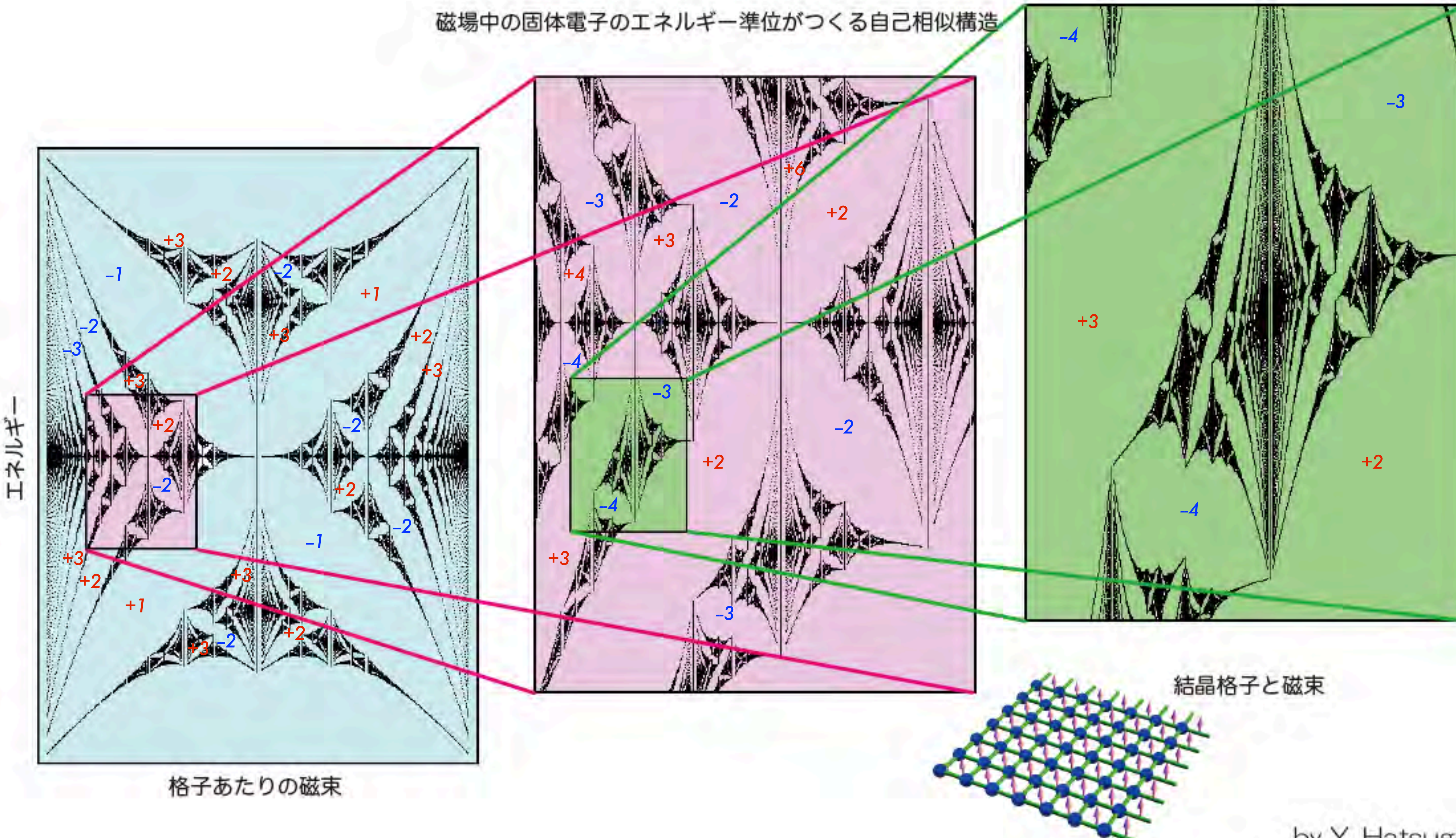
$$\sigma_{xy} = \frac{e^2}{h} \times (\mathrm{integer})$$

Klitzing et al. '80

Thouless-Kohmoto-Nightingale-den Niij '82

物質中のフラクタル
Hofstadter の蝶

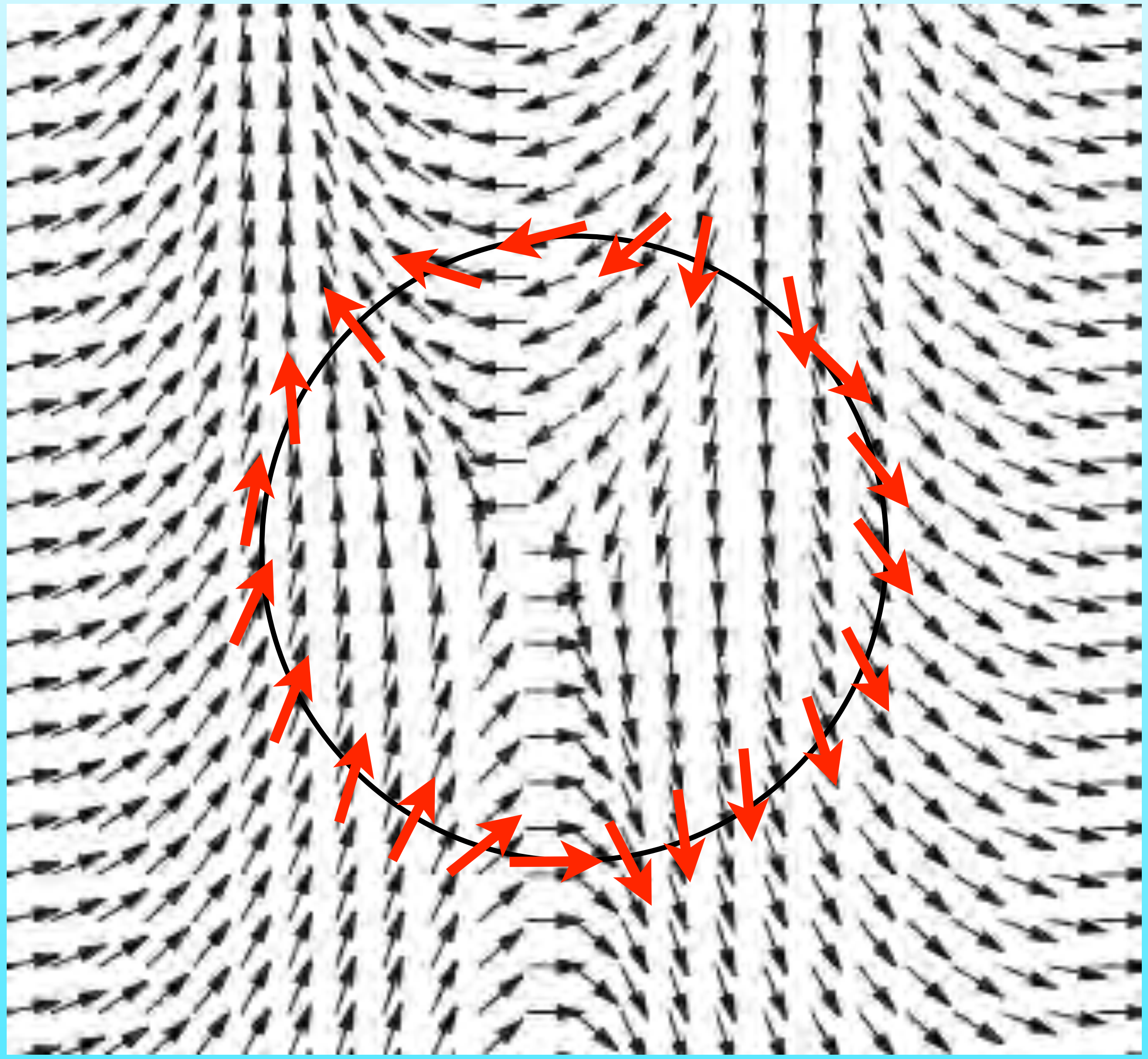
磁場中の固体電子のエネルギー準位がつくる自己相似構造



by Y. Hatsugai

Why is it ?

It is topological ! Can't be fractional



winding number -1

Singular point : stable

Berry connection, gauge structure

Eigen vector? $$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = E\begin{pmatrix} a \\ b \end{pmatrix}$$ $E = \pm 1$

OK

$E = 1$ $$\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -a+b \\ a-b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

If

Gauge Freedom

M.V.Berry

Can

!!!!

Is it all ? No! $$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ i \end{pmatrix}, \begin{pmatrix} 0 \\ e^{i\theta} \end{pmatrix}$$

phase is arbitrary

Hall conductance by “Berry connection”

☆ “Classical” Observables　　: charge/spin density

$$\langle \mathcal{O} \rangle_G = \langle G|\mathcal{O}|G\rangle = \langle G'|\mathcal{O}|G'\rangle = \langle \mathcal{O} \rangle_{G'} \qquad |G'\rangle = |G\rangle e^{i\phi}$$

☆ “Quantum” Observables !　　: interference

$$\langle G_1|G_2\rangle = \langle G_1'|G_2'\rangle e^{i(\phi_1-\phi_2)}$$

$$\langle G|G+dG\rangle = 1 + \langle G|dG\rangle$$

$A = \langle G|dG\rangle$:Berry Connection

$i\gamma = \int A$:Berry Phase

$C = \frac{1}{2\pi i}\int dA$: Chern number

Hall conductance by bulk

$$\sigma_{xy} = \frac{e^2}{h} \times C$$

TKNN’82
(Thouless-Kohmoto-Nightingale-den Niij ’82)

Geometrical phases !

Bulk σ_{xy} of the Filled Fermi sea & Dirac Sea

☆ Integration of the NonAbelian Berry Connection of the "Fermi Sea" & "Dirac Sea"

Numerical advantage for graphene

$$H_j(k)|\psi_j(k)\rangle = \epsilon_j(k)|\psi_j(k)\rangle$$

✓ Berry connections

$$\Psi = (\ |\psi_1\rangle, \cdots, |\psi_M\rangle)$$

Collect M states below the Fermi level

$$A_{FS} \equiv \Psi^\dagger d\Psi = \begin{pmatrix} \langle\psi_1^\dagger|d\psi_1\rangle & \cdots & \langle\psi_1^\dagger|d\psi_M\rangle \\ \vdots & \ddots & \vdots \\ \langle\psi_M^\dagger|d\psi_1\rangle & \cdots & \langle\psi_M^\dagger|d\psi_M\rangle \end{pmatrix}$$

Matrix vector potential of the Fermi (Dirac) Sea

Non Abelian extension for the Chern numbers

$$\sigma_{xy} = \frac{e^2}{h}\frac{1}{2\pi i}\int_{T^2} \mathrm{Tr}_M dA_{FS}$$

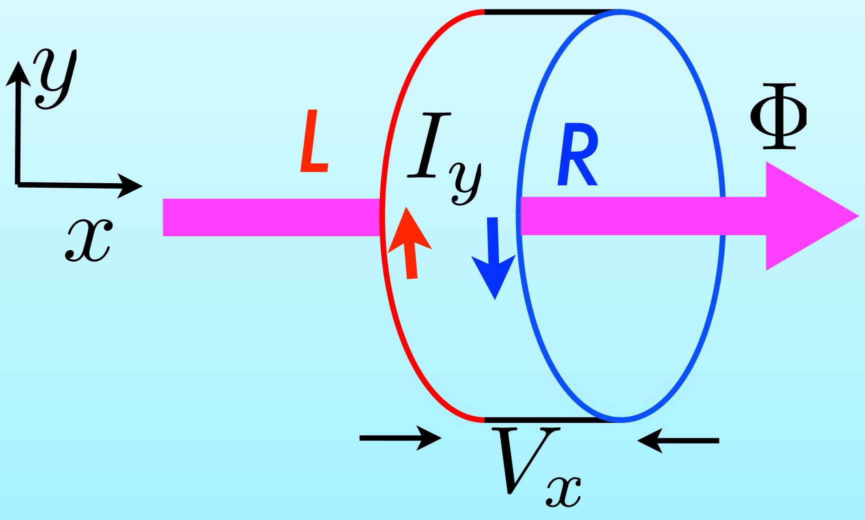
Thouless-Kohmoto-Nightingale-den Niij '82 Niu-Thouless-Wu'84 Hatsugai '93, '04

Stability of the quantized Hall Conductance

☆ Gauge invariance and quantization of σ_{xy}

Laughlin '81

adiabatic process to increase Φ



$$A \to A' = A + \nabla\Phi, \quad \delta\Phi = \int_{\bigcirc} (A' - A)$$

$$\psi \to \psi' = e^{i2\pi\delta\Phi/\Phi_0}\,\psi$$ one particle state

$$\delta\Phi = \Phi_0 = \frac{e}{h} \longrightarrow \psi' = \psi$$

flux quantum

Gauge transformation

Byers-Yang formula

$$I_y = \frac{\Delta E}{\Delta\Phi} = \frac{neV_x}{h/e} = \underbrace{\boxed{n(\frac{e^2}{h})}}_{\sigma_{yx}} V_x = \sigma_{yx} V_x$$

$$\Delta\Phi = \Phi_0 = \frac{h}{e}, \quad \Delta E = n \cdot eV_x$$

All states are invariant up to phase after the process:

Some n states are carried from L to the R

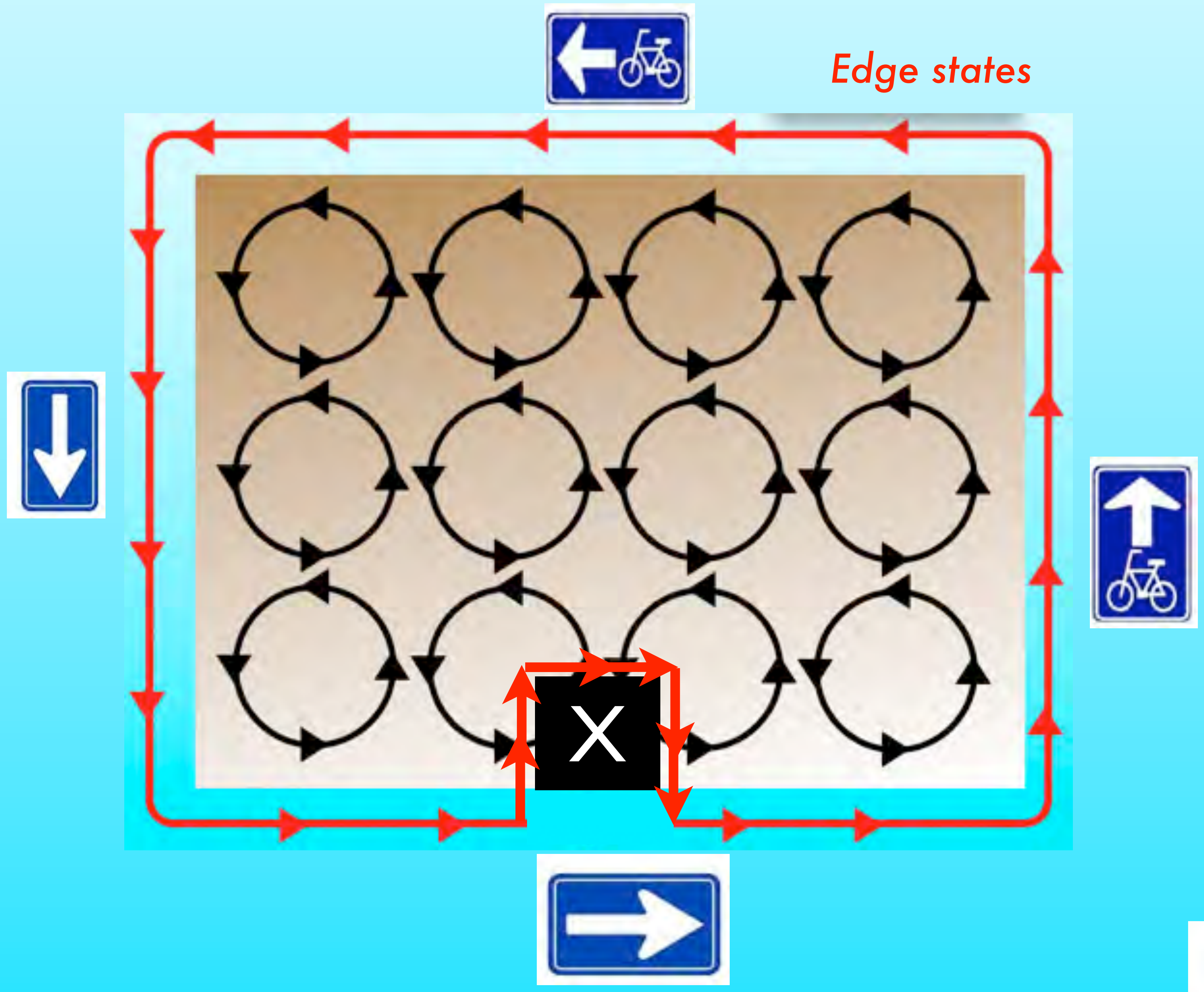
n : generic integer (but undetermined)

Another view by edge states

Edge states are topologically stable Edge

Cyclotron motion by Lorentz force $\boldsymbol{F} = -e\boldsymbol{v} \times \boldsymbol{B}$

Currents are canceled in the bulk but induces a boundary current



Edge states are chiral

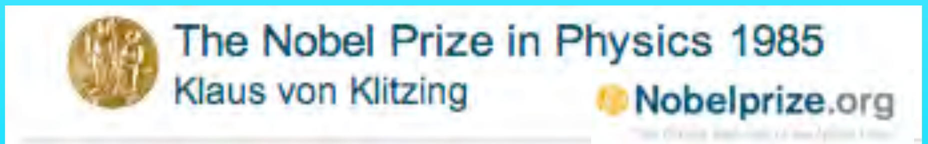
One way going !!

Cannot stop !

No back scattering

Stable for impurities !!

Topological stability
of
Chiral edge states



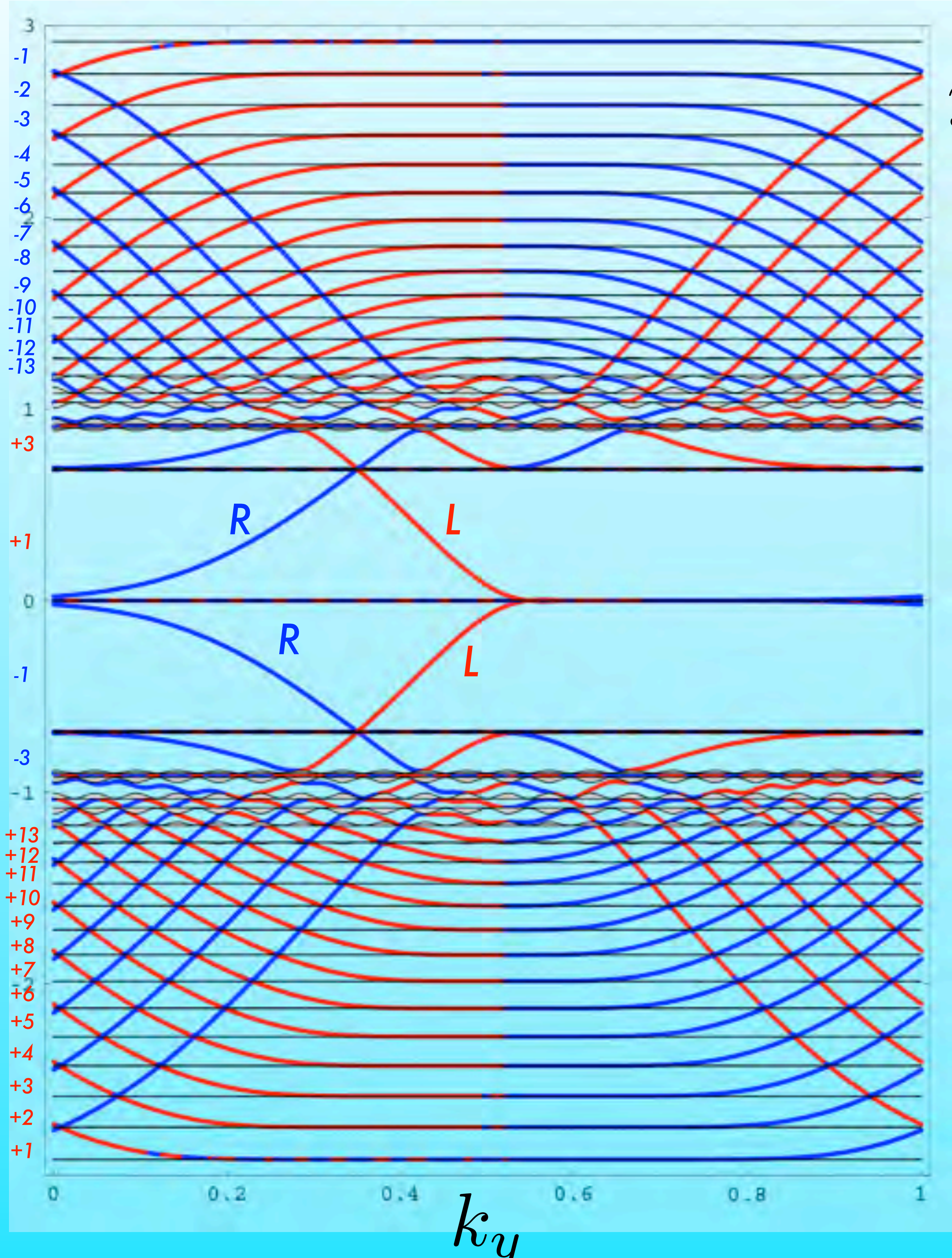
Edge States of Graphene

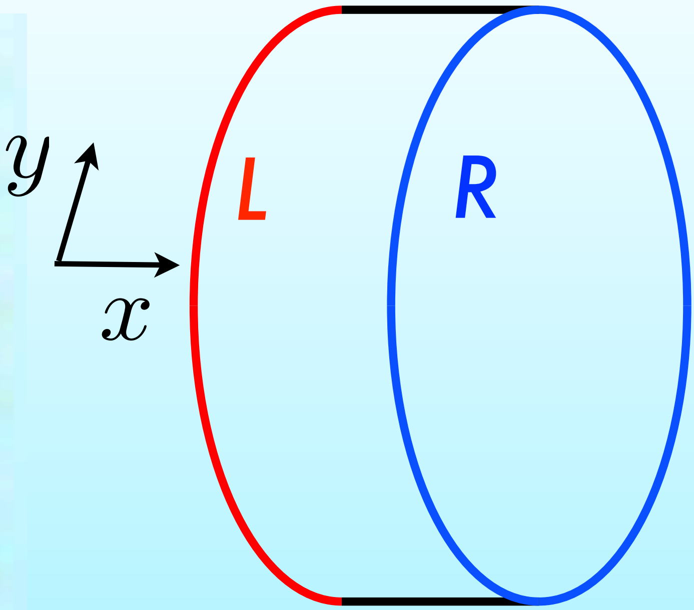
Standard Quantization (hole)

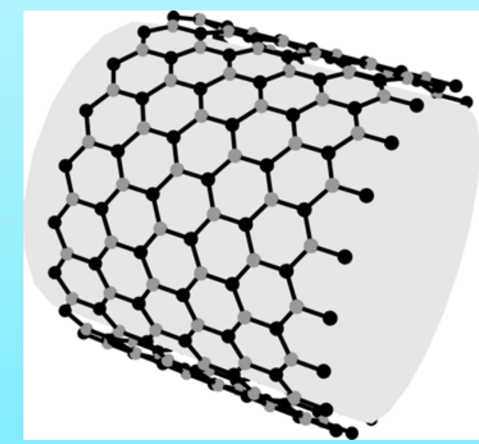
$\phi = 1/21$

Dirac Type Quantization

Standard Quantization



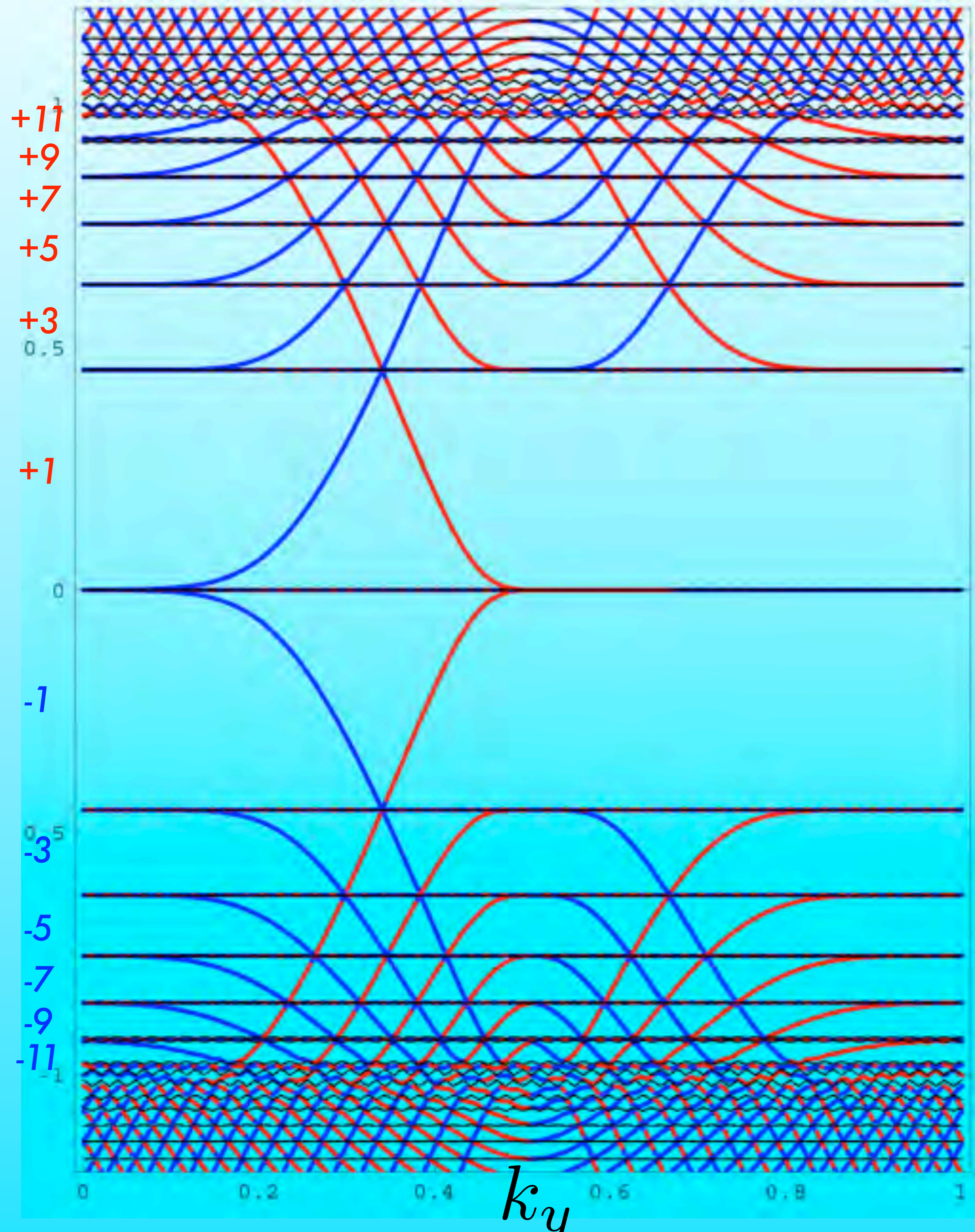


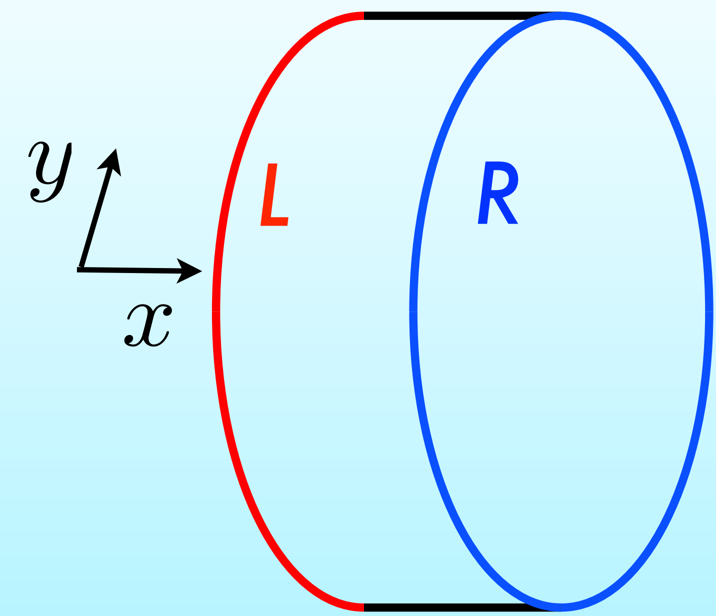


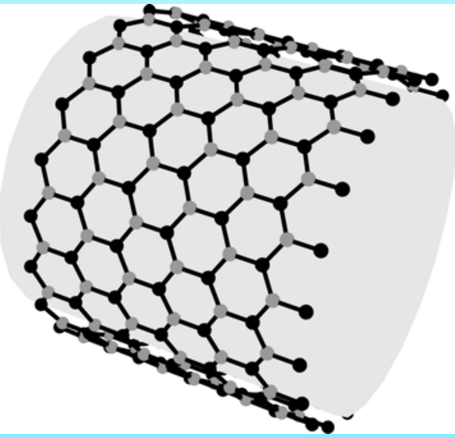
Edge States of Graphene

$\phi = 1/51$

Edge States being consistent with Dirac Type Quantization



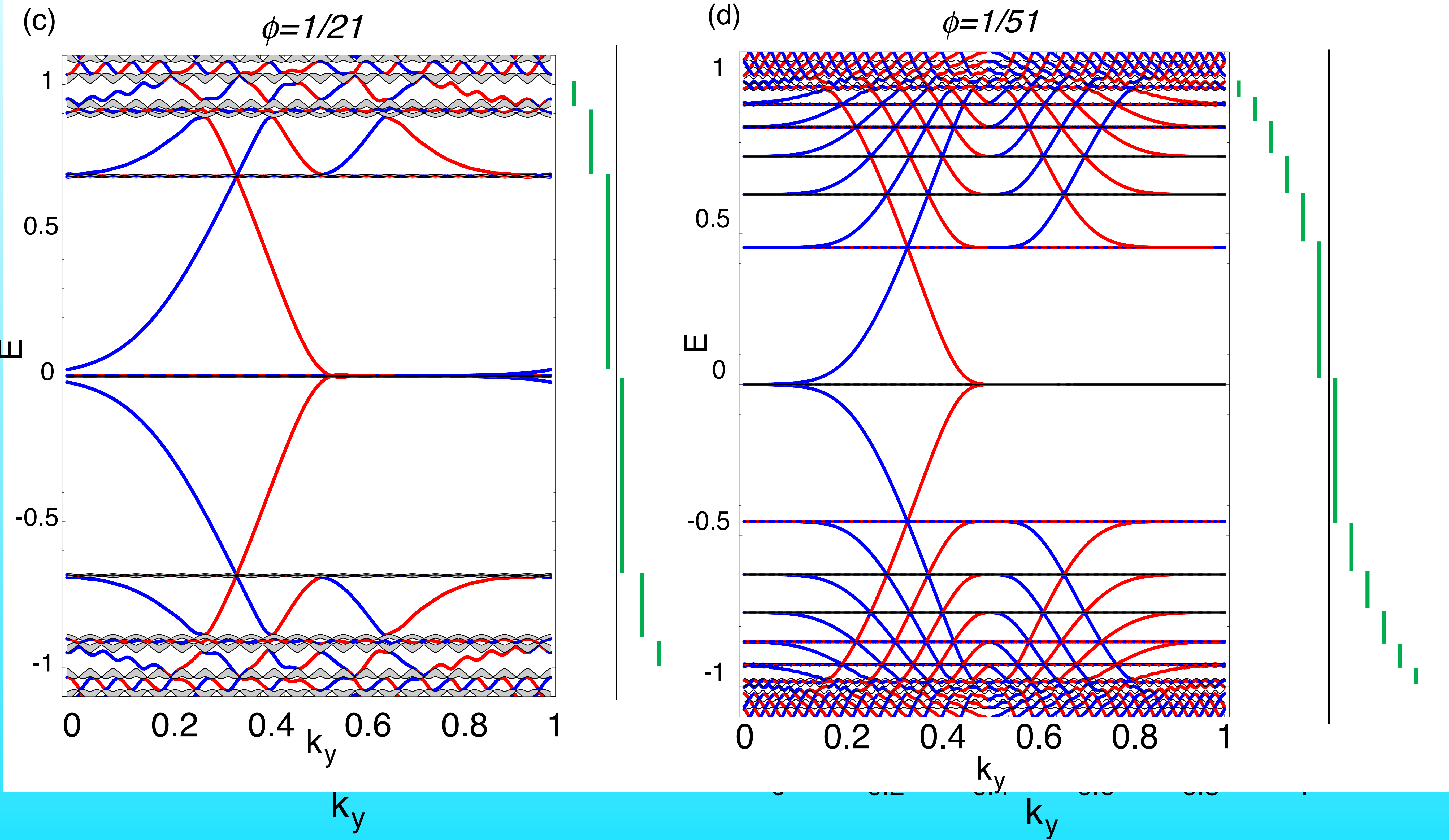




Bulk – Edge Correspondence

$$\sigma_{xy}{}^{\text{bulk}} = \sigma_{xy}{}^{\text{edge}}$$

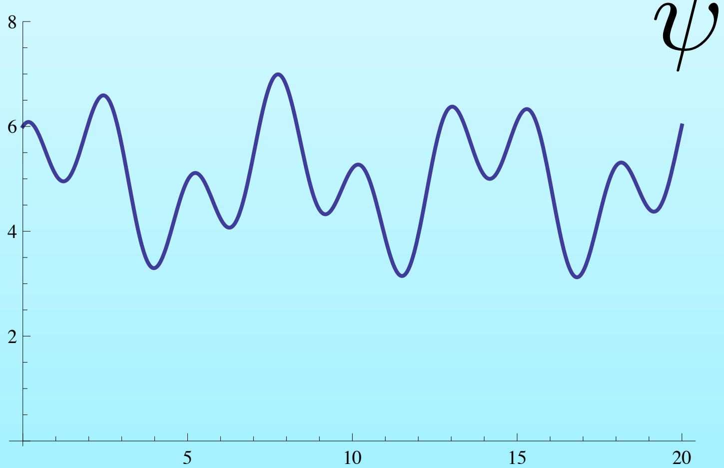
Graphene



Right / left to the symmetry

Extended states

unnormalizable

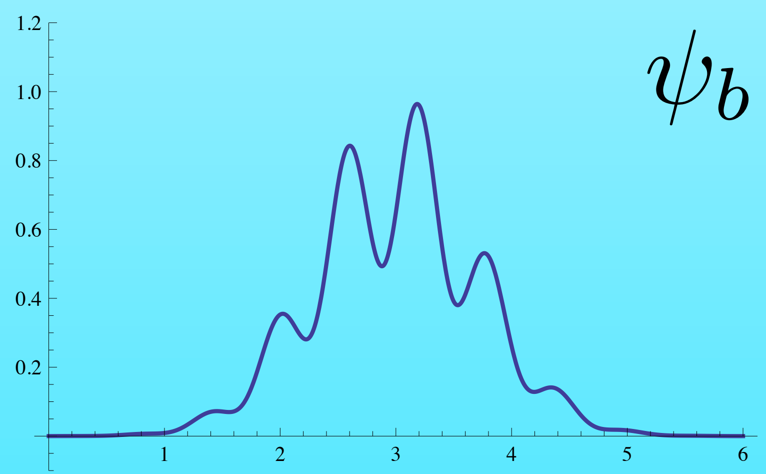


$$\psi_e(r) \sim \frac{1}{\sqrt{V}} e^{ikr} \longrightarrow 0 \quad (V \to \infty)$$

V: Volume

Bound states / Edge states

normalizable



$$\psi_b(r) \sim \frac{1}{\sqrt{a_0^3}} e^{-r/a_0}$$

a_0 :size of the bound state

☑ Clear difference only in the infinite system

Why do we care edge states?

Why the Edge States are there??

Accidental ?

NO !

Inevitable reasons

Physical Structures behind:

“Bulk determines the edges”

“Edge determines the bulk”

Bulk-Edge Correspondence

$$\sigma_{xy}^{\mathrm{bulk}} = \sigma_{xy}^{\mathrm{edge}}$$

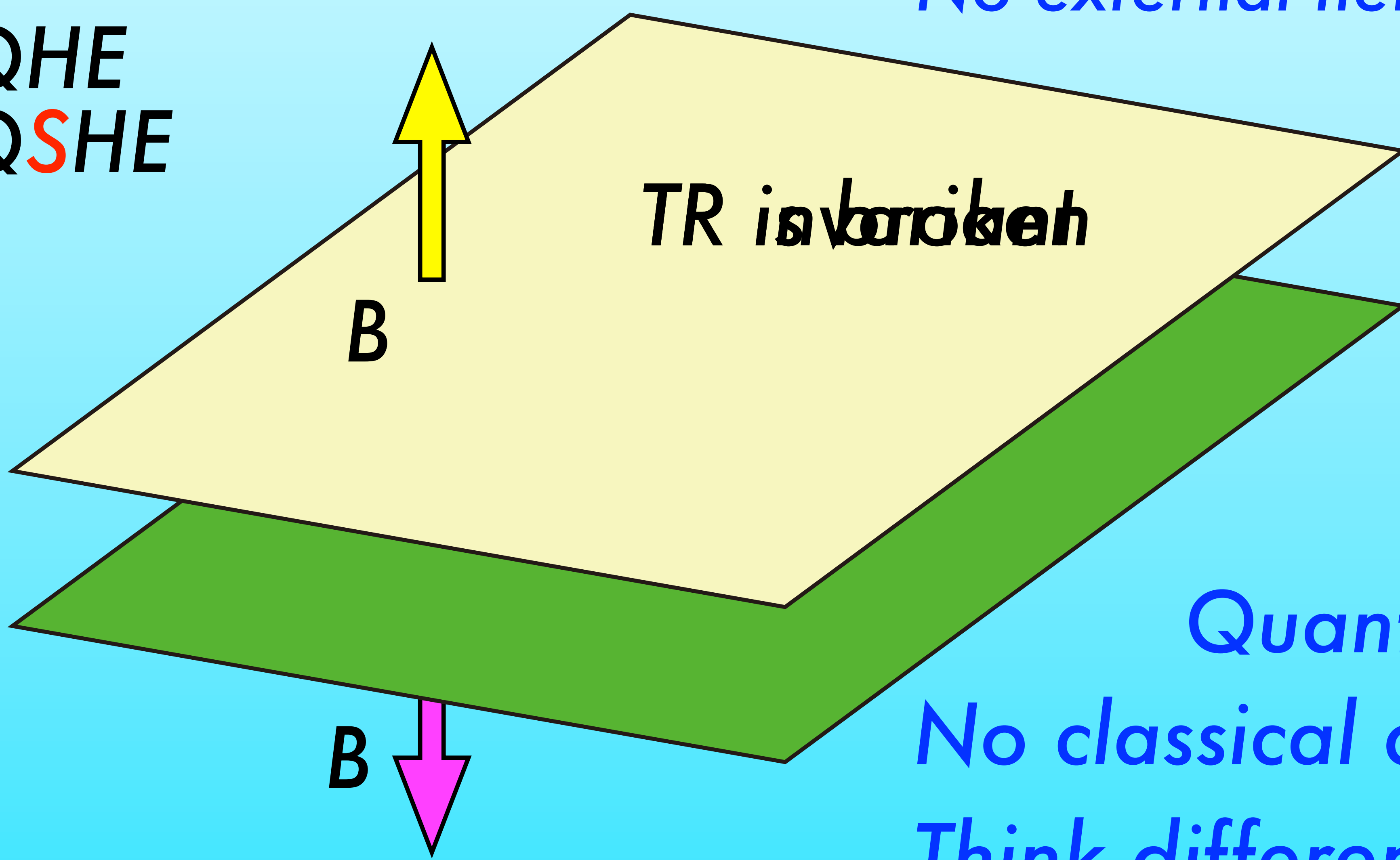
"Quantum" Spin Hall Effect

= Quantum Hall Effect *without magnetic field*

= Quantum Hall Effect *with time-reversal invariance*

No external net magnetic field

QHE
QSHE



spin up

spin down

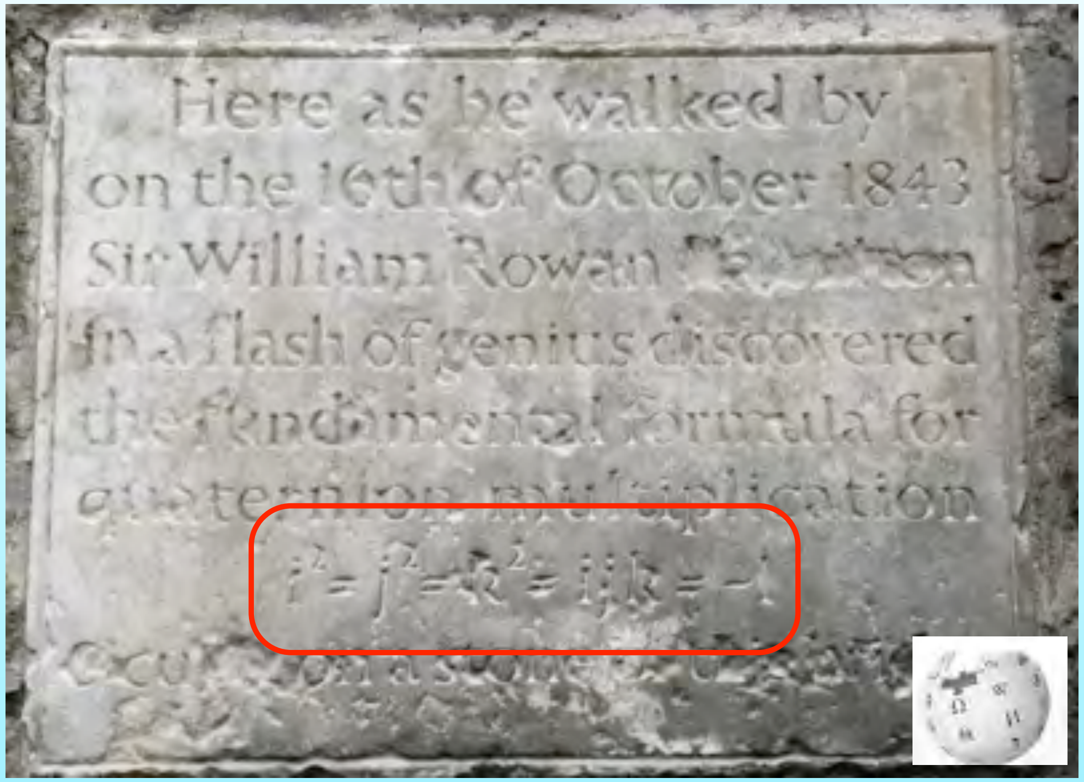
Quantum effects !

No classical correspondent !

Think different

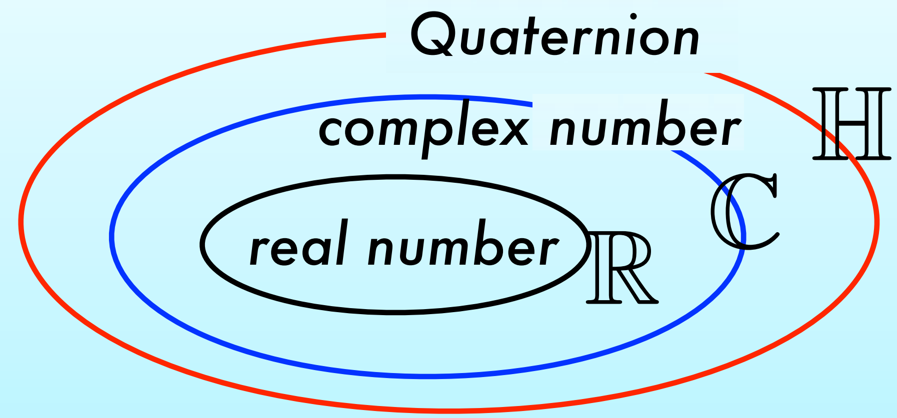
Following the QHE

Time Reversal & Quaternions



Hamilton 四元数発見の碑

F.J.Dyson '61–



No magic, neither crazy
just Pauli matrices

$$H = \begin{bmatrix} t & \Delta \\ -\Delta^* & t^* \end{bmatrix} = (\mathrm{Re}t) I_2 + (\mathrm{Im}t)\, i\sigma_z + (\mathrm{Re}\Delta)\, i\sigma_y + (\mathrm{Im}\Delta)\, i\sigma_x$$

$$\cong (\mathrm{Re}t)\, 1 + (\mathrm{Im}t) i_{\mathbb{H}} + (\mathrm{Re}\Delta)\, j_{\mathbb{H}} + (\mathrm{Im}\Delta)\, k_{\mathbb{H}}$$

: **Quaternion (四元数)**

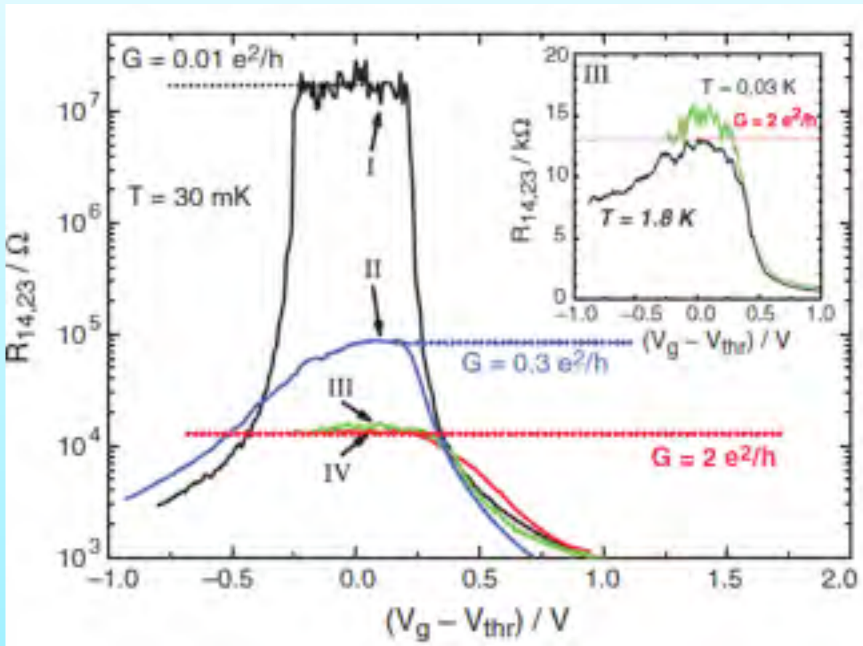
$$i_{\mathbb{H}} \cong i\sigma_z, \;\; j_{\mathbb{H}} \cong i\sigma_y, \;\; k_{\mathbb{H}} \cong i\sigma_z$$

$$i_{\mathbb{H}}^2 = k_{\mathbb{H}}^2 = k_{\mathbb{H}}^2 = i_{\mathbb{H}} j_{\mathbb{H}} k_{\mathbb{H}} = -1$$

Quaternion 2×2 Matrix, Yang Monopole & quantization: YH, NJP12, 065004 (2010)

Topological Insulator

Topological insulator : Quantum Spin Hall state

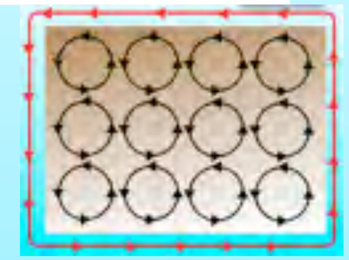


Quantum Spin Hall Insulator State in HgTe Quantum Wells

Markus König,[1] Steffen Wiedmann,[1] Christoph Brüne,[1] Andreas Roth,[1] Hartmut Buhmann,[1] Laurens W. Molenkamp,[1*] Xiao-Liang Qi,[2] Shou-Cheng Zhang[2]

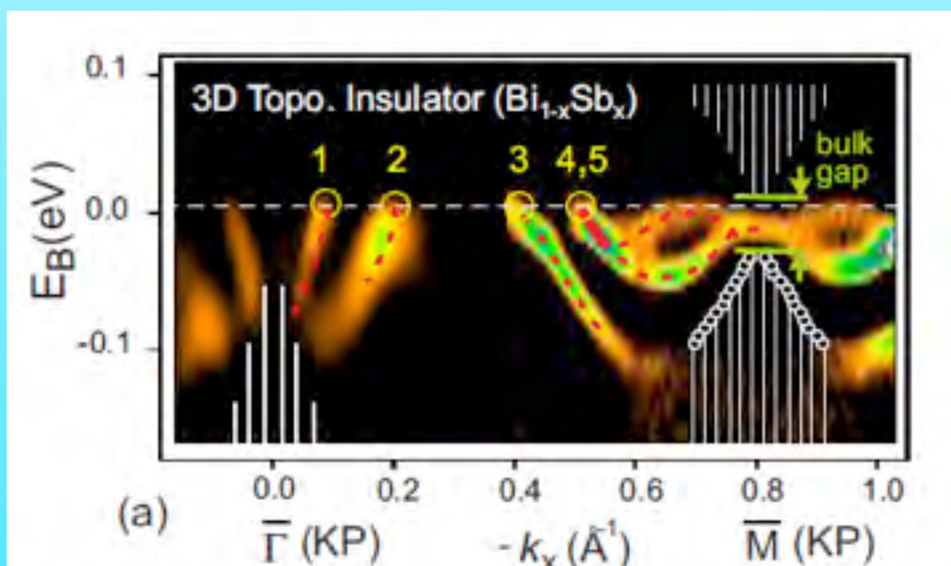
Science **318**, 766 (2007)

2D



1d boundary

TNG 3D



Hsieh, D., D. Qian, L. Wray, Y. Xia, Y. S. Hor, R. J. Cava, and M. Z. Hasan, 2008, Nature (London) **452**, 970.

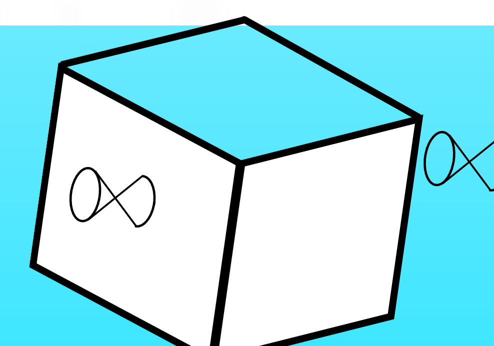
TOPOLOGICAL INSULATORS 378 NATURE PHYSICS | VOL 5 | JUNE 2009 |

The next generation

Spin-orbit coupling in some materials leads to the formation of surface states that are topologically protected from scattering. Theory and experiments have found an important new family of such materials.

Joel Moore

2d boundary



Also higher dim.: domain wall fermions: Lattice gauge theory

Why the Edge States are there??

Accidental ?

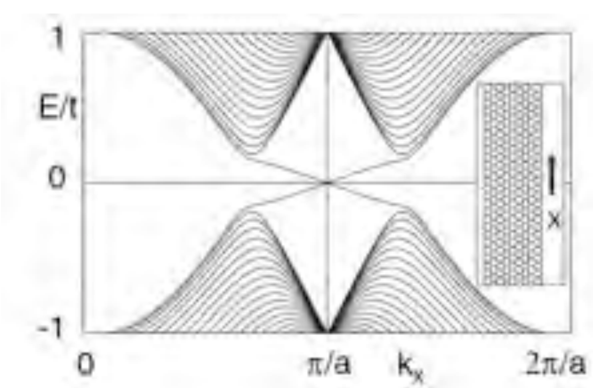
NO !

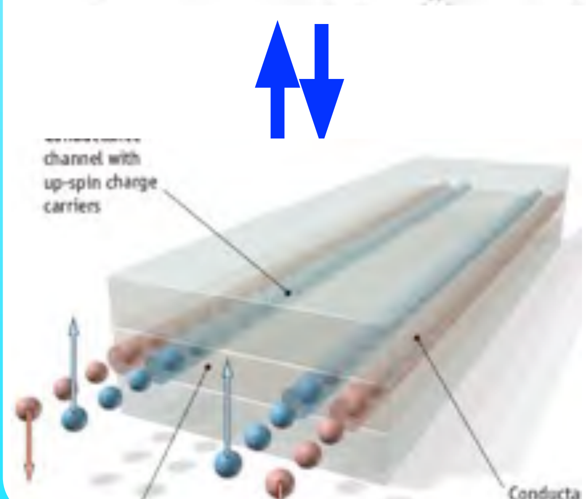
Inevitable reasons

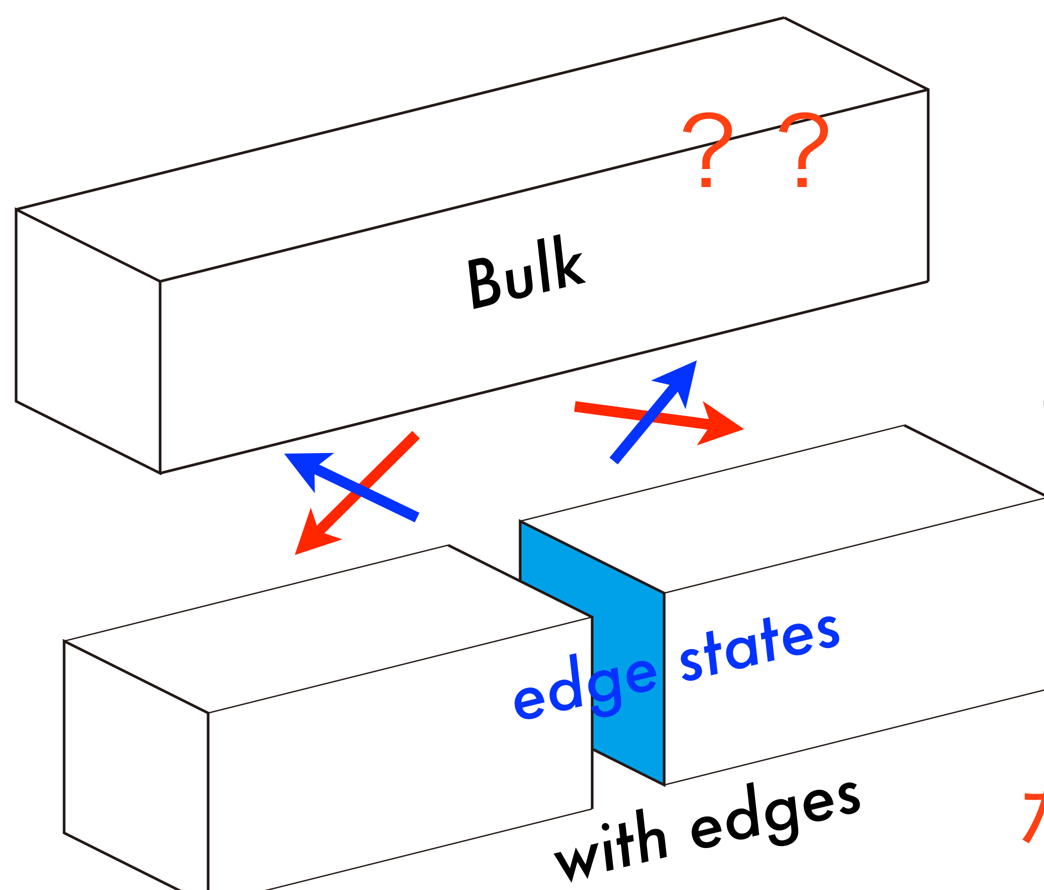
Universal Structures behind:

Bulk determines the edges

2D　　Bulk-Edge Correspondence







cut into two !

端をみて

なかみを考える！

Why the Edge States are there??

Accidental ?

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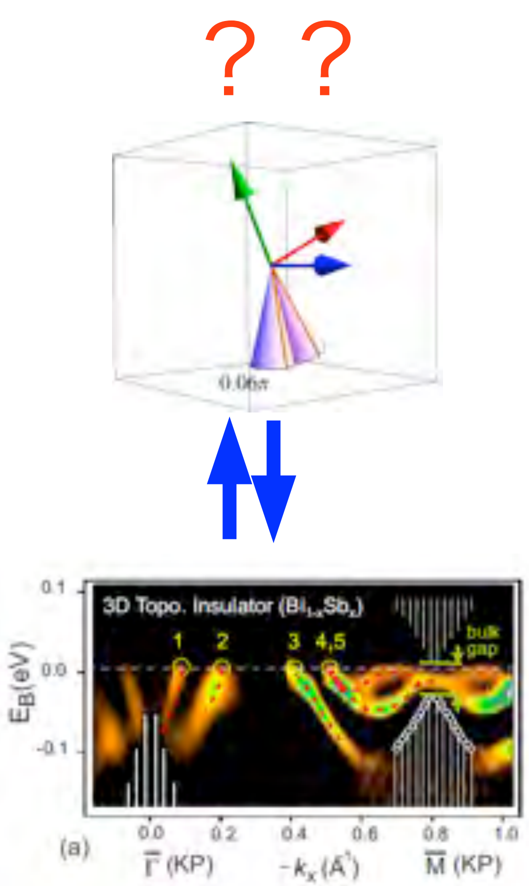
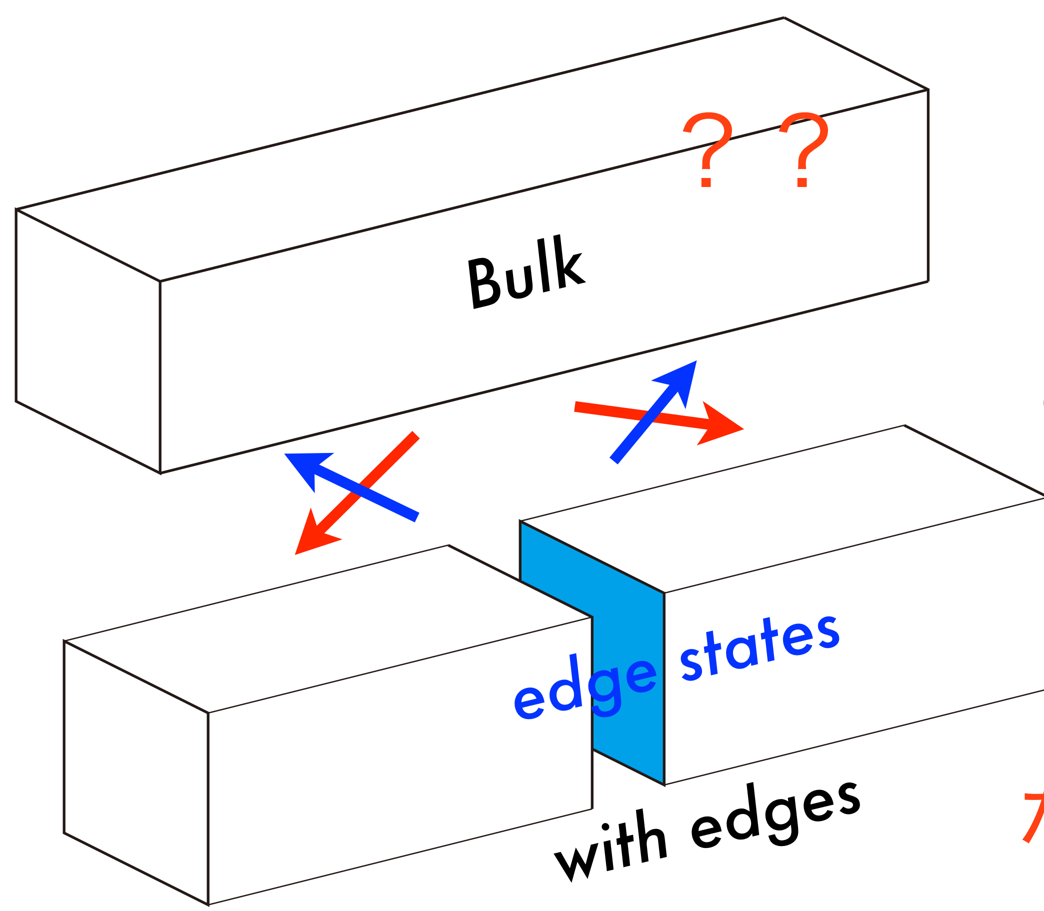
Bulk determines the edges

TNG　3D　Bulk-Edge Correspondence





cut into two !

端をみて

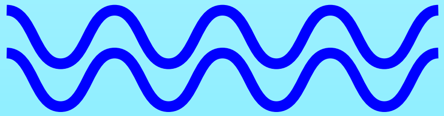
なかみを考える！

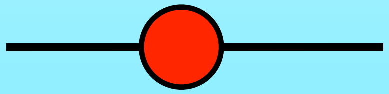
エッジ状態：そこにあるには何かの理由が！

バルク・エッジ対応

YH '93 : IQHE (Hofstadter)
Kitaev '01 Majorana chains
Ryu-YH '02 Graphene, d-wave superconductivity
Qi-Wu-Zhang '06, General theorem

Universality





Bulk state (scattering state) Bulk Gap Non trivial Vacuum	Control with each other	Edge state (Bound state) Particles in the gap

can not be independent

Lots of examples for bulk-edge correspondence

- ✓ Friedel sum rule　　*Levinson's theorem, Friedel's sum rule*
- ✓ Polarization　　*King-Smith & Vanderbilt*
- ✓ Solitons in polyacetylene　　*Su-Schriefer-Heeger '79*
- ✓ Q(S)HE　　*Halperin '82　YH '93　Kane-Mele'05　Bernevig-Hughes-Zhang '06*
- ✓ Haldane spin chain (Kennedy triplet)　　*Kennedy '90*
- ✓ ZBCP & Majorana　　*Hu, '94　Kitaev, '01*
- ✓ Fujita state in graphene　　*Fujita et al.'96　Ryu-YH'02*
- ✓ Cold atoms in optical lattice & edge states　　*Scarola-Das Sarma., '07*
- ✓ Photonic crystals & edge states　　*Haldane & Raghu, '08,　Wang et al., '08,*
- ✓ Spin Ladder　　*Arikawa-Tanaya-Maruyama, YH '09*

AND more...

Lots of examples for bulk-edge correspondence

- ✓ Frie
- ✓ Pola
- ✓ Soli
- ✓ Q(S ... 06
- ✓ Hal
- ✓ ZBC
- ✓ Fuji
- ✓ Col ... '07
- ✓ Photonic crystals & edge states — *Haldane & Raghu, '08, Wang et al., '08,*
- ✓ Spin Ladder — *Arikawa-Tanaya-Maruyama, YH '09*

AND more...

Lots of variety

Absence of symmetry breaking

Bulk: Insulators !

Quantum/spin liquids (gapped)

打てども響かず

対称性の破れを使った相分類の大成功

Nambu-Goldstone
&
Landau-Ginzburg-Wilson　～1980

The Munich physics professor advised Planck against going into physics, saying, "in this field, almost everything is already discovered, and all that remains is to fill a few holes."

WIKIPEDIA
The Free Encyclopedia

M. Planck　～1900

Quantum phases *without* symmetry breaking
Quantum/spin liquids

対称性の破れを越えて新しい概念へ

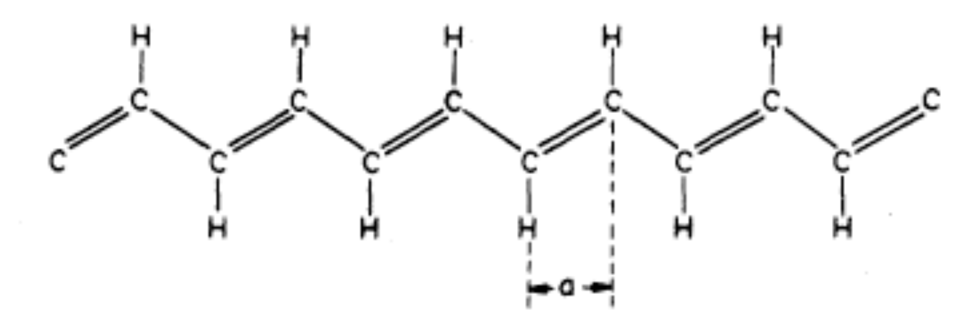
☆ *Solitons in polyacetylene*

The Nobel Prize in Chemistry 2000
Alan Heeger, Alan G. MacDiarmid, Hideki Shirakawa

VOLUME 42, NUMBER 25 PHYSICAL REVIEW LETTERS 18 JUNE 1979

Solitons in Polyacetylene

W. P. Su, J. R. Schrieffer, and A. J. Heeger
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 15 March 1979)



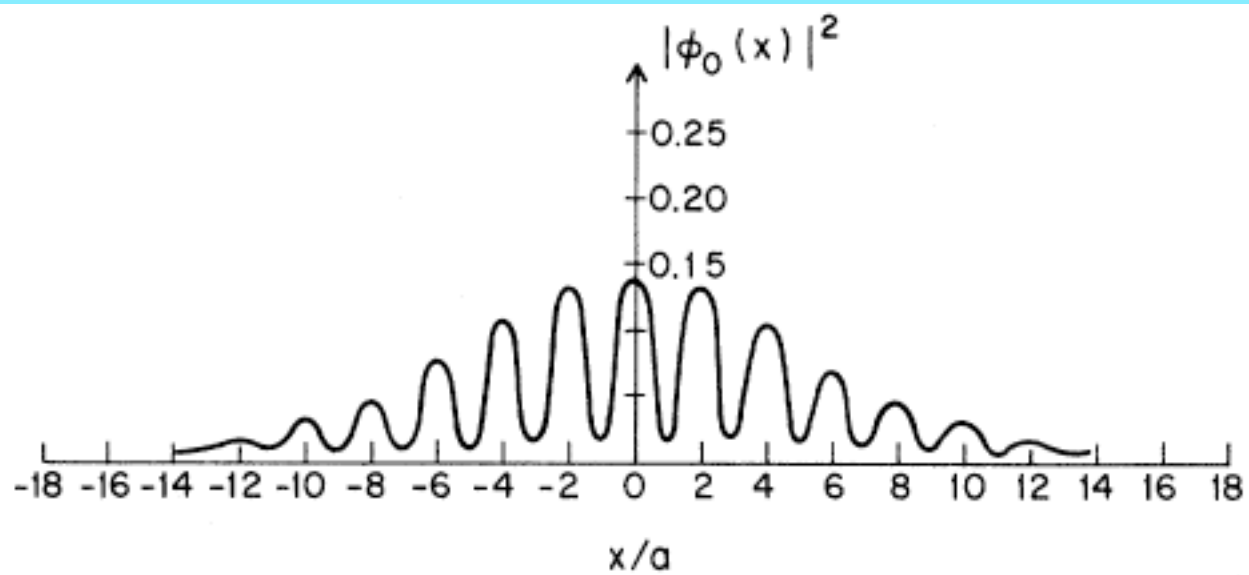
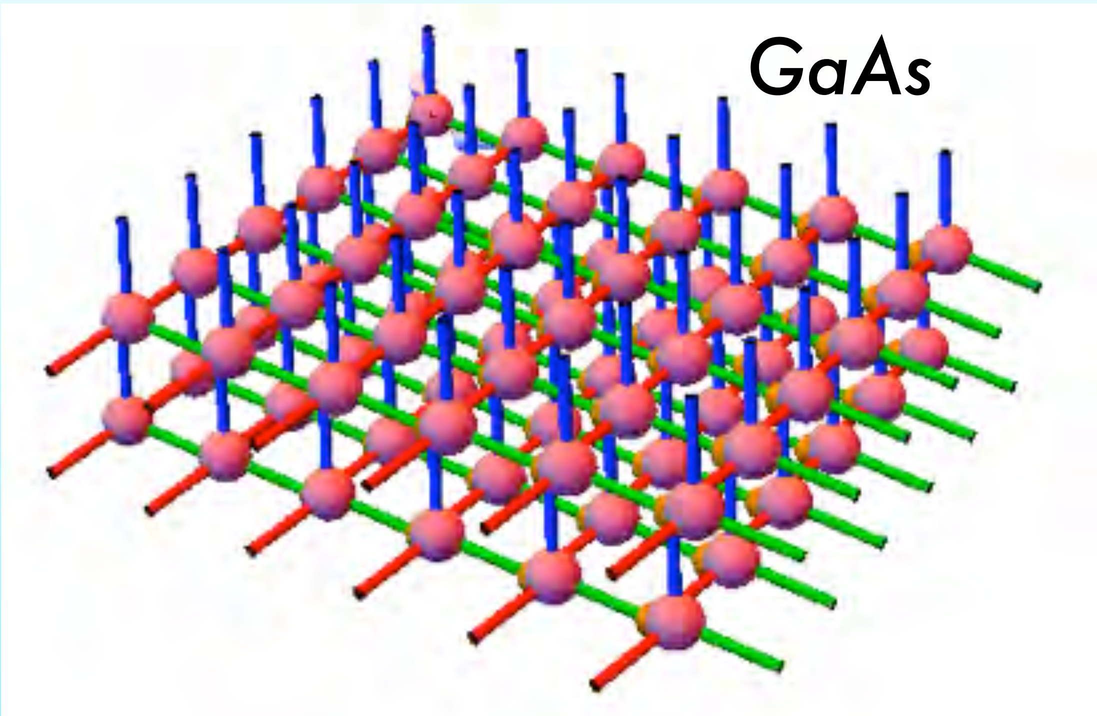
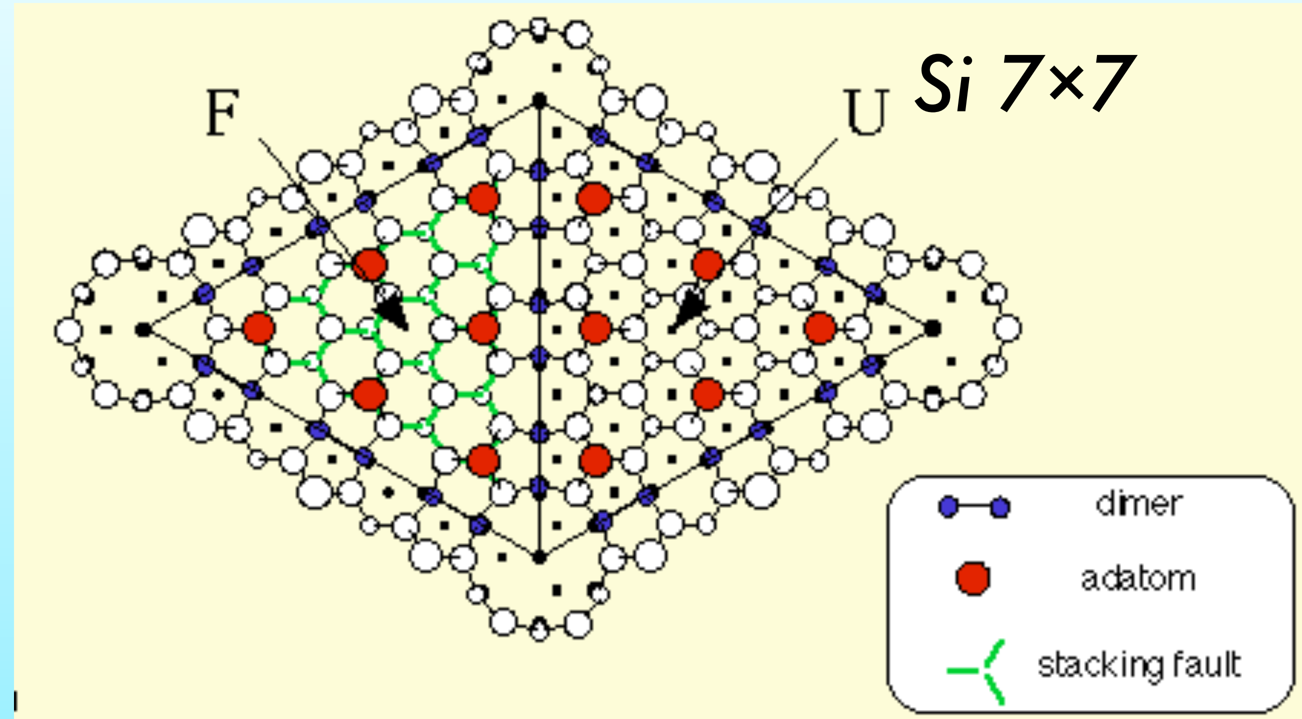


FIG. 3. Probability distribution of the localized electronic state at the center of the gap.

格子欠陥と近傍のソリトン

☆ *Surface states of Semiconductors & polarization*





PHYSICAL REVIEW B　　VOLUME 47, NUMBER 3　　15 JANUARY 1993-I

Theory of polarization of crystalline solids

R. D. King-Smith and David Vanderbilt

Macroscopic polarization in crystalline dielectrics: the geometric phase approach

Raffaele Resta

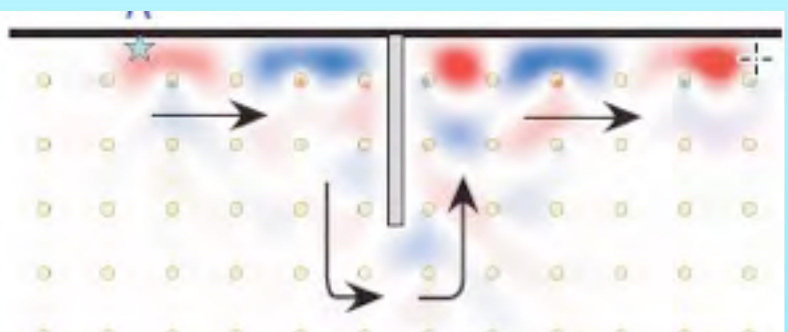
誘電体と分極

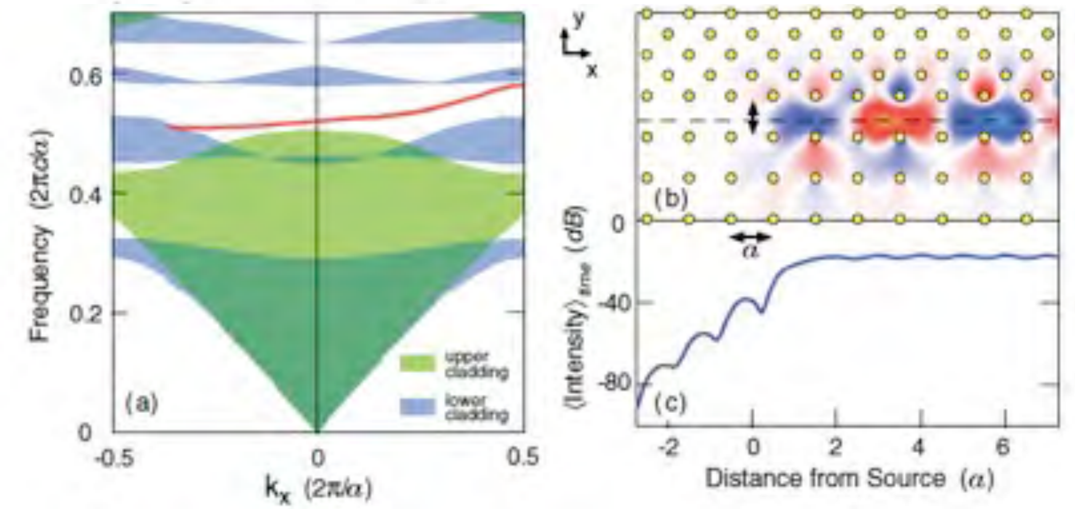
+ −+ −+ −+ −+ −+ −+ −+ −+ −+ −

PRL 100, 013905 (2008) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2008

Reflection-Free One-Way Edge Modes in a Gyromagnetic Photonic Crystal

Zheng Wang, Y. D. Chong, John D. Joannopoulos, and Marin Soljačić





Vol 461 | 8 October 2009 | doi:10.1038/nature08293

Observation of unidirectional backscattering-immune topological electromagnetic states

Zheng Wang[1]*, Yidong Chong[1]†*, J. D. Joannopoulos[1] & Marin Soljačić[1]

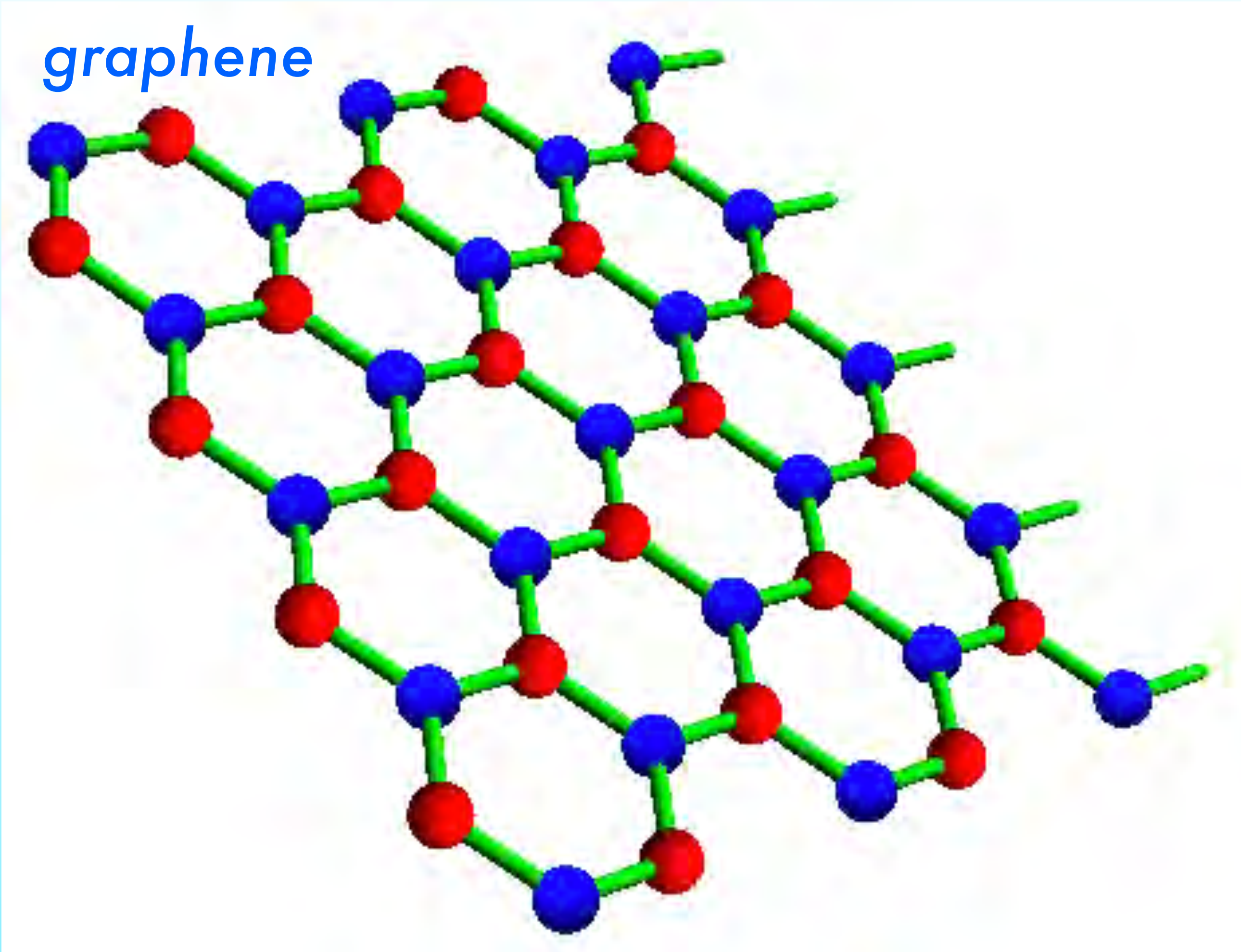
人工格子

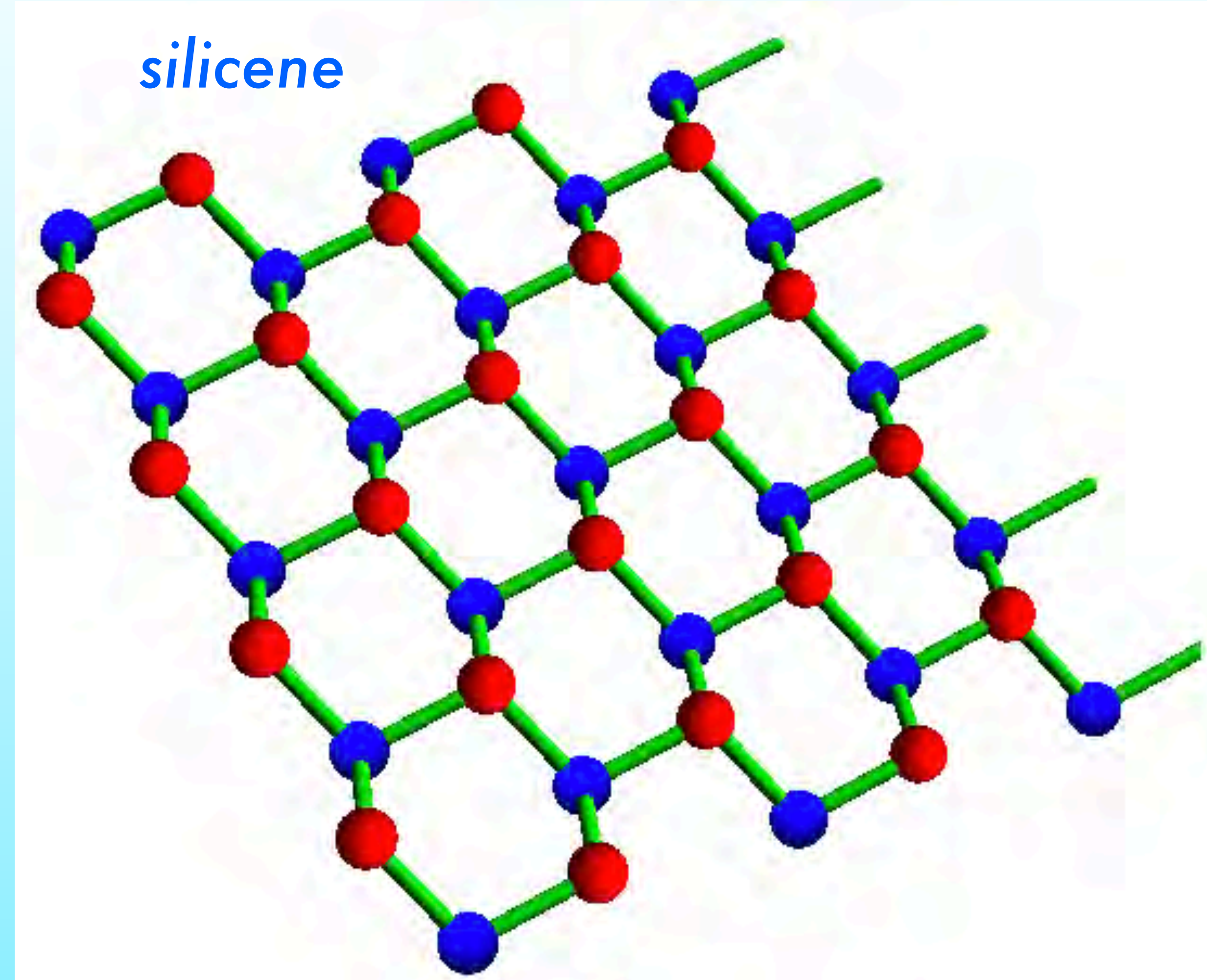
PRL 100, 013904 (2008) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2008

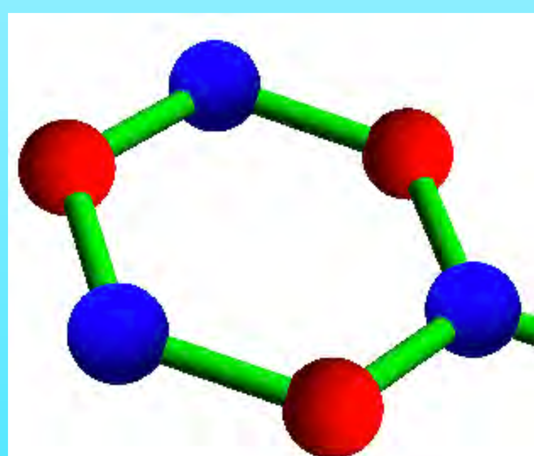
Possible Realization of Directional Optical Waveguides in Photonic Crystals with Broken Time-Reversal Symmetry

F. D. M. Haldane and S. Raghu*

グラフェン　　　　　　　　シリセン





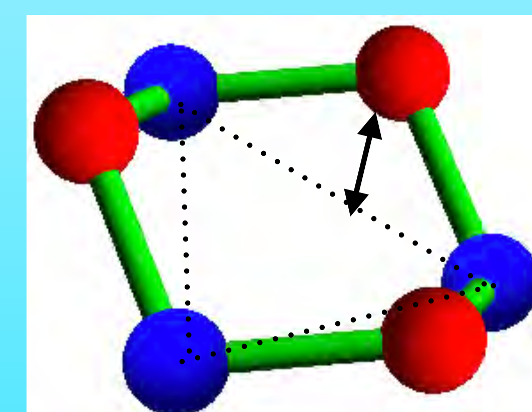


planar

sp^2

buckled

sp^3-like



Similarity & Variety

Lots of examples for bulk-edge correspondence

- ✓ Frie
- ✓ Pola
- ✓ Solit
- ✓ Q(S)
- ✓ Hald
- ✓ ZBC
- ✓ Fujit
- ✓ Cold

Need something more

Topological order !

Bulk-edge correspondence

Quantum entanglement

Condensed matter physics

in 21-st Century

- ✓ Photonic crystals & edge states — Haldane & Raghu, '08, Wang et al., '08,
- ✓ Spin Ladder — Arikawa-Tanaya-Maruyama, YH '09

AND more...