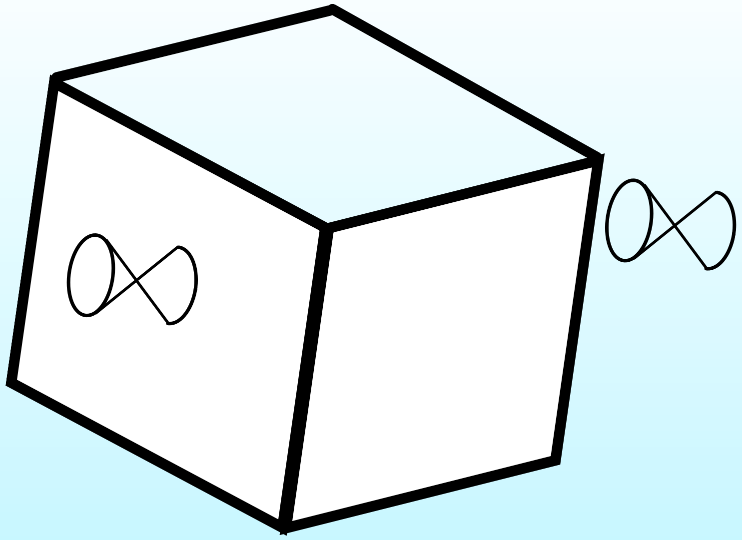
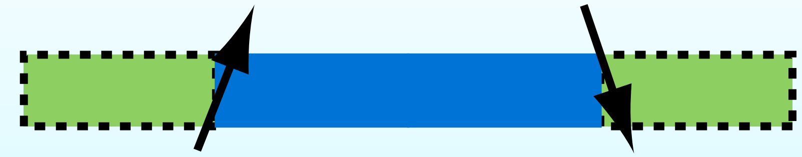
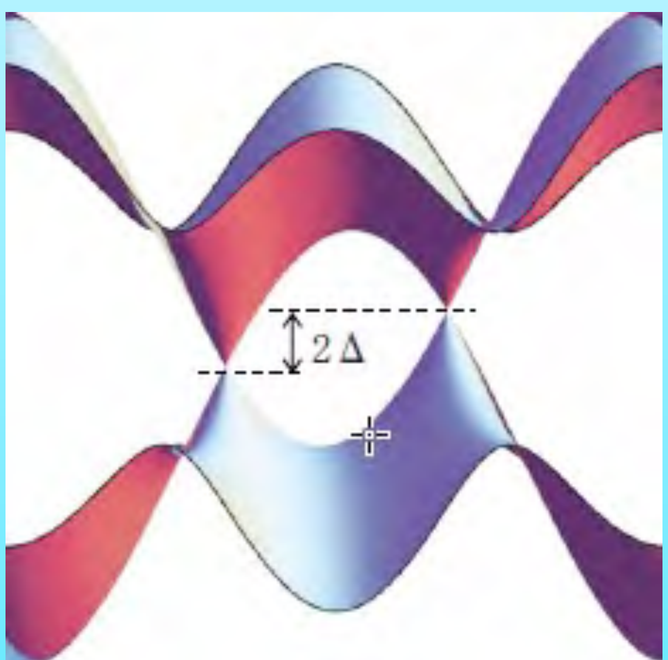
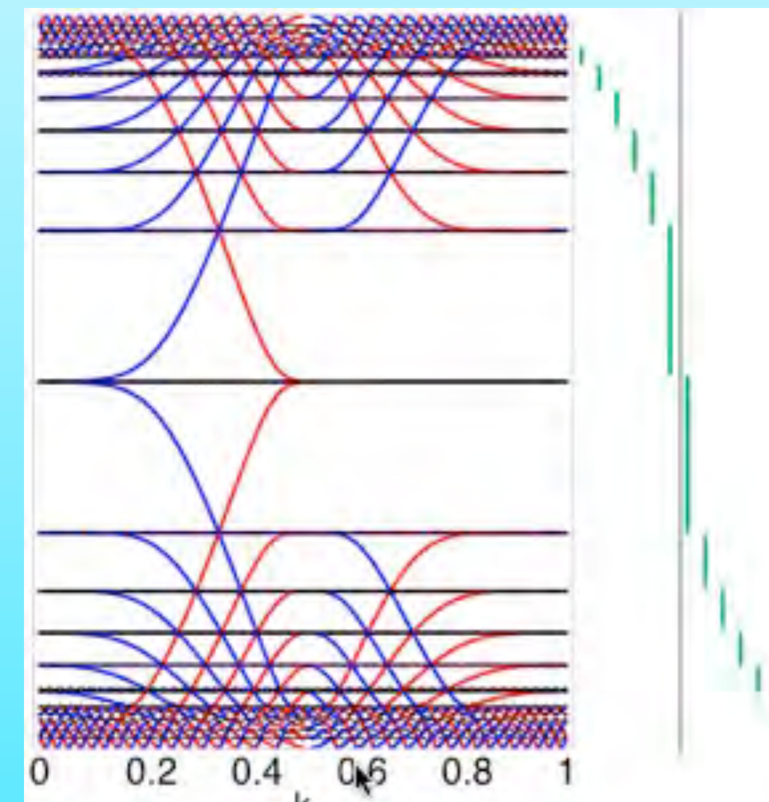




Bulk-edge correspondence in real world





Institute of Physics
University of Tsukuba
JAPAN
Yasuhiro Hatsugai

HIROSHIMA UNIVERSITY

University of Tsukuba 筑波大学

Outline

- Are insulators boring ?
  - Zoo of insulators : variety to universality
  - Classification of the insulators
- Zoo of boundary states
  - From the textbook to the research
  - Bulk-Edge correspondence
- A lucky example: Bulk-Edge correspondence
  - Quantum Hall states
  - Graphene
- Applications
  - Topological quantum phase transitions
  - How to observe 1/2 Hall conductance of Dirac fermions
  - Edge state device ?

Are insulators boring ??

- Metal is useful.

copper, silver, gold: good conductors

doped semiconductors

Gapless excitations above the ground state

Lots of applications

Definition (today)

- Insulators : Gapped

Energy gap above the ground state

- Band insulators
- Superconductors ?
- Integer & Fractional Quantum Hall States
- Integer spin chains (Haldane)
- Dimer Models (Shastry-Sutherland)
- Valence bond solid (VBS) states
- Half filled Kondo Lattice
- Spin Hall insulators (TI in a narrow sense)
- Kitaev model & string net

"Topological insulators"

Are insulators boring ??

- Insulators : Gapped
  - Band insulators
  - Superconductors
  - Integer & Fractional Quantum Hall States
  - Integer spin chains (Haldane)
  - Dimer Models (Shastry-Sutherland)
  - Valence bond solid (VBS) states
  - Half filled Kondo Lattice
  - Spin Hall insulators
  - Kitaev model & string net

Absence of low energy excitations
Energy gap above the ground state

Lots of variety

Absence of fundamental symmetry breaking (mostly)

Quantum/spin liquids (gapped)

Are insulators boring ??

Gapped: Nothing in the gap : cf. Nambu-Goldstone boson

No low lying excitations

No Response against small perturbation

gapless modes:
acoustic phonons
zero sounds
spin waves

??

? ? ?

Absence of low energy excitations
Energy gap above the ground state

Lots of variety

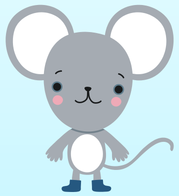
Absence of fundamental symmetry breaking (mostly)

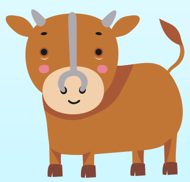
No responses against for small perturbation

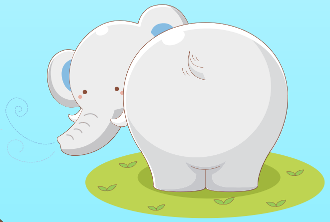
Are insulators boring ??

☆ Quantum liquids (gapped)

- ☆ Band insulators
- ☆ Superconductors
- ☆ Integer & Fractional Quantum Hall States
- ☆ Integer spin chains (Haldane)
- ☆ Dimer Models (Shastry-Sutherland)
- ☆ Valence bond solid (VBS) states
- ☆ Half filled Kondo Lattice
- ☆ Spin Hall insulators
- ☆ Kitaev model & string net





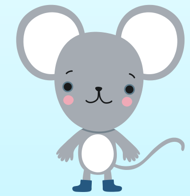
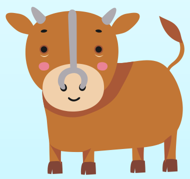


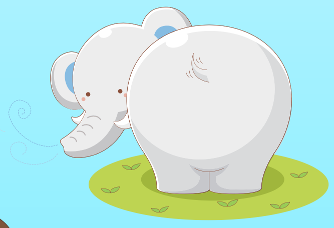
Zoo

Are insulators boring ??

- Quantum liquids *(gapped)*

- *Band insulators*
- *Superconductors*
- *Integer & Fractional Quantum Hall States*
- *Integer spin chains (Haldane)*
- *Dimer Models (Shastry-Sutherland)*
- *Valence bond solid (VBS) states*
- *Half filled Kondo Lattice*
- *Spin Hall insulators*
- *Kitaev model & string net*



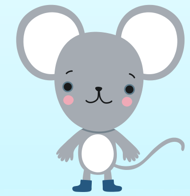
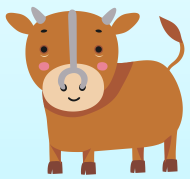
Zoo

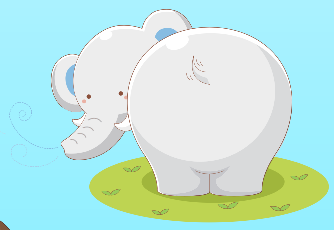
Something for classification

Are insulators boring ??

☆ Quantum liquids (gapped)

- ☆ Band insulators
- ☆ Superconductors
- ☆ Integer & Fractional Quantum Hall States
- ☆ Integer spin chains (Haldane)
- ☆ Dimer Models (Shastry-Sutherland)
- ☆ Valence bond solid (VBS) states
- ☆ Half filled Kondo Lattice
- ☆ Spin Hall insulators
- ☆ Kitaev model & string net



Topological Order

X.G.Wen '89

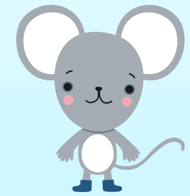
Zoo

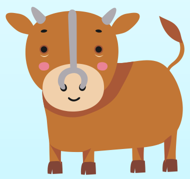
Something for classification

Are insulators boring ??

☆ Quantum liquids (gapped)

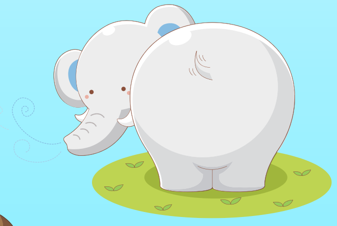
- ☆ Band insulators
- ☆ Superconductors
- ☆ Integer & Fractional Quantum Hall States
- ☆ Integer spin chains (Haldane)
- ☆ Dimer Models (Shastry-Sutherland)
- ☆ Valence bond solid (VBS) states
- ☆ Half filled Kondo Lattice
- ☆ Spin Hall insulators
- ☆ Kitaev model & string net





Topological Order

X.G.Wen '89



Zoo

Something for classification

- ☐ Topological order
- ☐ Berry connections
- ☑ Edge states

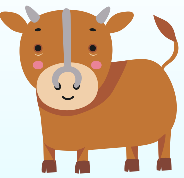
Zoo of Boundary (Edge) States in Cond. Mat.

From textbook examples to new discoveries

- Levinson's theorem to the Friedel's sum rule
- Surface states of Semiconductors & polarization
- Solitons in polyacetylene
- Edge states in quantum Hall effects
- Local moments in integer spin chains near the impurities
- Zero bias conductance peaks of the d-wave superconductors
- Zero energy localized states of graphene
- Quantum Spin Hall Edge states
- Edge states in 2D cold atoms in optical lattice
- One-way edge modes in gyromagnetic photonic crystals
- Spin Ladder with ring exchanges

☆ Levinson's theorem to the Friedel's sum rule

- Surface states of Semiconductors & polarization
- Solitons in polyacetylene
- Edge states in quantum Hall effects
- Local moments in integer spin chains near the impurities
- Zero bias conductance peaks of the d-wave superconductors
- Zero energy localized states of graphene
- Quantum Spin Hall Edge states
- Edge states in 2D cold atoms in optical lattice
- One-way edge modes in gyromagnetic photonic crystals
- Spin Ladder with ring exchanges



☆ Levinson's theorem to the Friedel's sum rule

PHYSICAL REVIEW VOLUME 121, NUMBER 4 FEBRUARY 15, 1961

Friedel Sum Rule for a System of Interacting Electrons

J. S. LANGER
Carnegie Institute of Technology, Pittsburgh, Pennsylvania

AND

V. AMBEGAOKAR*
Westinghouse Research Laboratories, Pittsburgh, Pennsylvania

(Received October 3, 1960)

The Friedel sum rule is derived for a system of interacting electrons in a periodic potential.

I. INTRODUCTION

IN a recent paper[1] one of the authors (J. S. L.) has derived an expression for the impurity-resistance of a metal using as a model an interacting Fermi fluid and randomly placed scattering centers. As expected, the current-carrying excitations of the fluid had wave numbers just at the Fermi surface and were scattered by screened impurities. The usual independent-electron

of metallic properties. For example, the Friedel sum rule[2,3] relates the total charge displaced in the field of a fixed impurity to the scattering by that impurity of a free electron at the Fermi momentum k_F. In particular, the rule states that the number of displaced electrons N_D is given by

$$N_D = \frac{2}{\pi} \sum_l (2l+1) \delta_l(k_F), \quad (1)$$

☆ One-way edge

☆ Spin Ladder wi

bound states ⇔ phase shift

☆ Surface states of Semiconductors & polarization

☆ Levinson's theorem to the Friedel's sum rule

☆ Solitons in polyacetylene

☆ Edge states in quantum Hall effects

☆ Local moments in integer spin chains near the impurities

☆ Zero bias conductance peaks of the d-wave superconductors

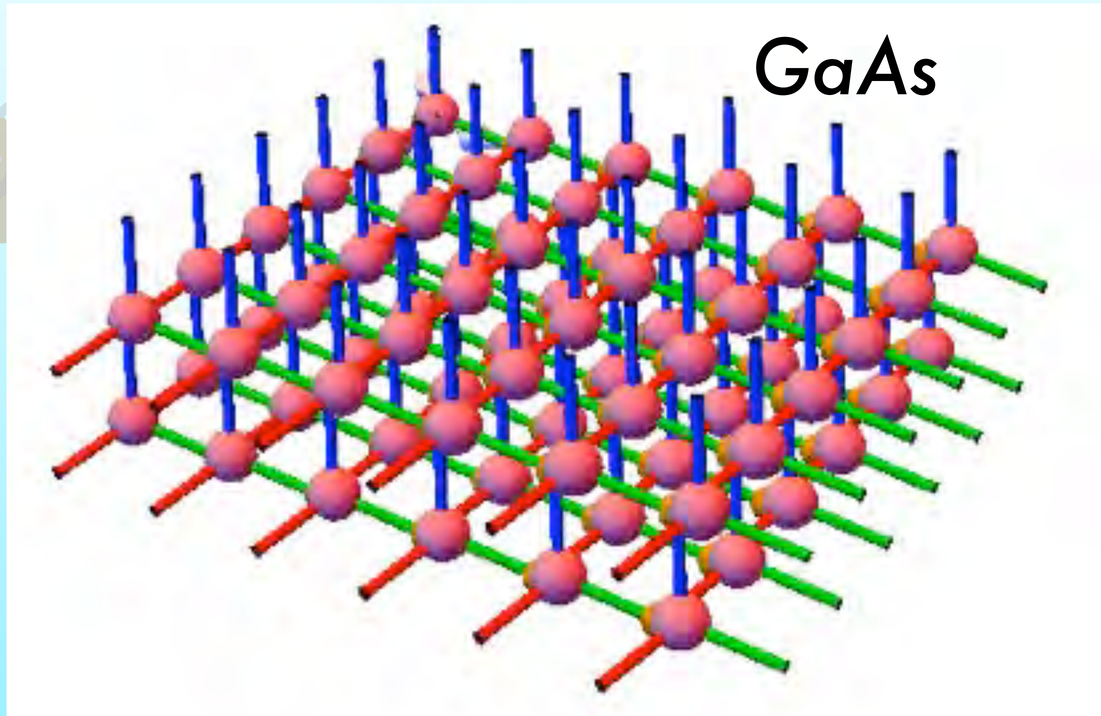
☆ Zero energy localized states of graphene

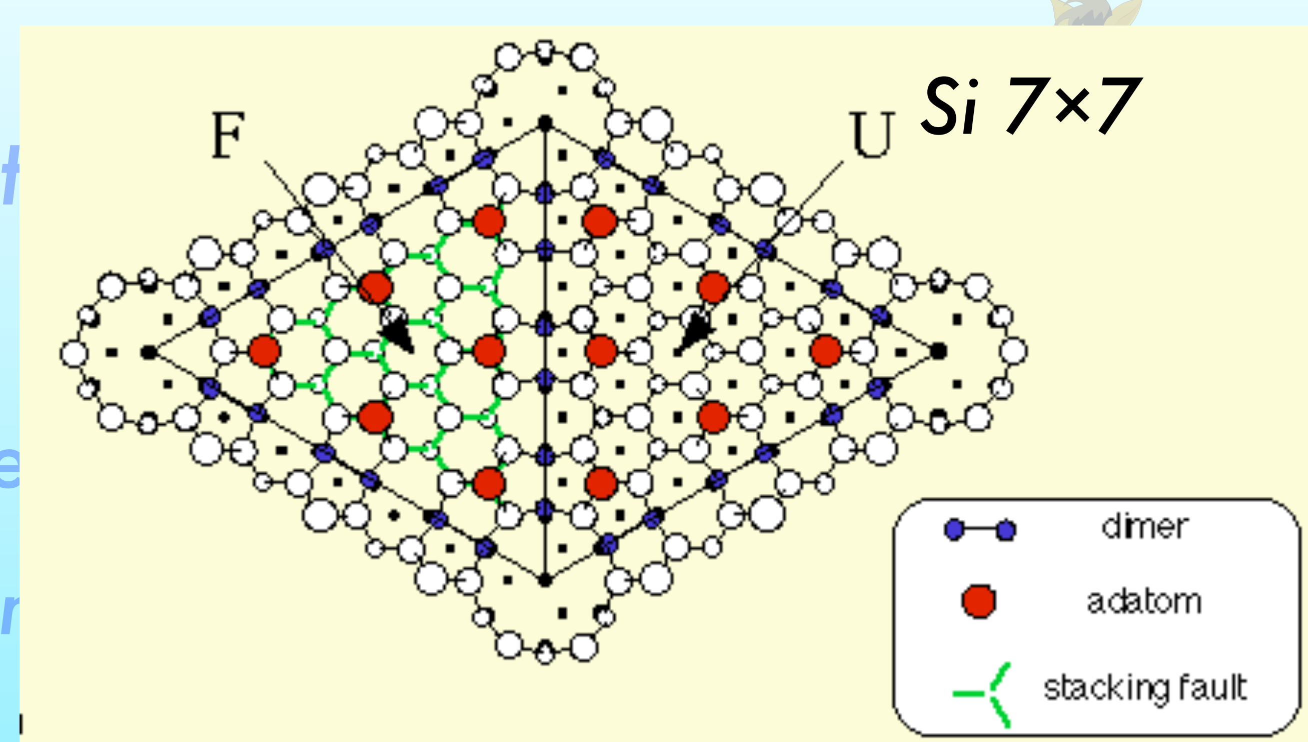
☆ Quantum Spin Hall Edge states

☆ Edge states in 2D cold atoms in optical lattice

☆ One-way edge modes in gyromagnetic photonic crystals

☆ *Surface states of Semiconductors & polarization*





PHYSICAL REVIEW B VOLUME 47, NUMBER 3 15 JANUARY 1993-I

Theory of polarization of crystalline solids

R. D. King-Smith and David Vanderbilt

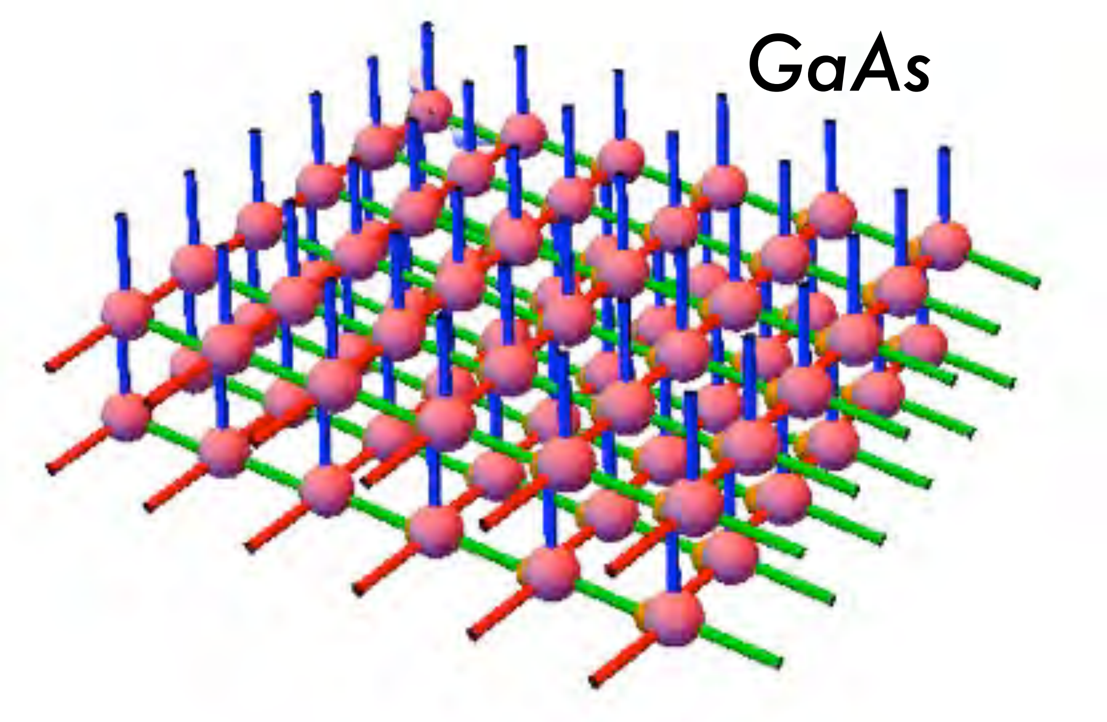
Macroscopic polarization in crystalline dielectrics: the geometric phase approach

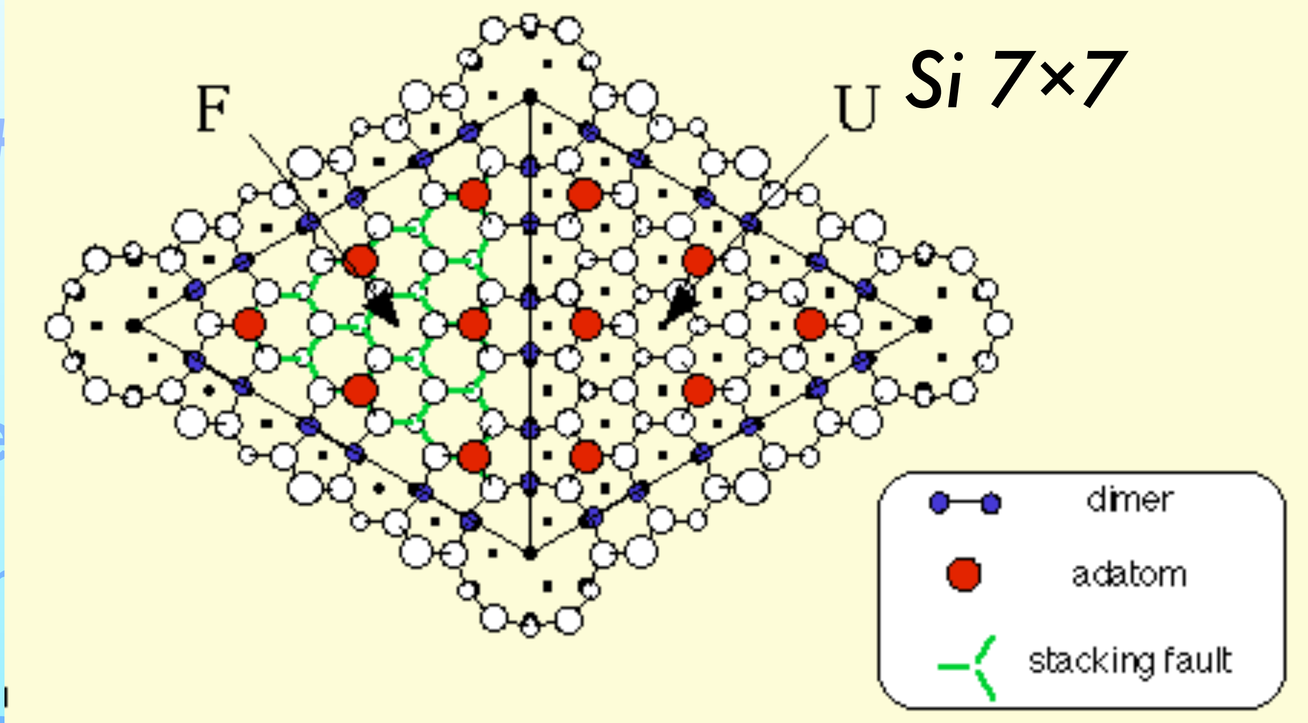
Raffaele Resta

Reviews of Modern Physics, Vol. 66, No. 3, July 1994

+ −+ −+ −+ −+ −+ −+ −+ −+ −+ −

☆ *Surface states of Semiconductors & polarization*





PHYSICAL REVIEW B VOLUME 47, NUMBER 3 15 JANUARY 1993-I

Theory of polarization of crystalline solids

R. D. King-Smith and David Vanderbilt

Macroscopic polarization in crystalline dielectrics: the geometric phase approach

Raffaele Resta

Reviews of Modern Physics, Vol. 66, No. 3, July 1994

Berry phase ⟷ *Polarization*

+ −+ −+ −+ −+ −+ −+ −+ −+ −+ −

photonic crystals

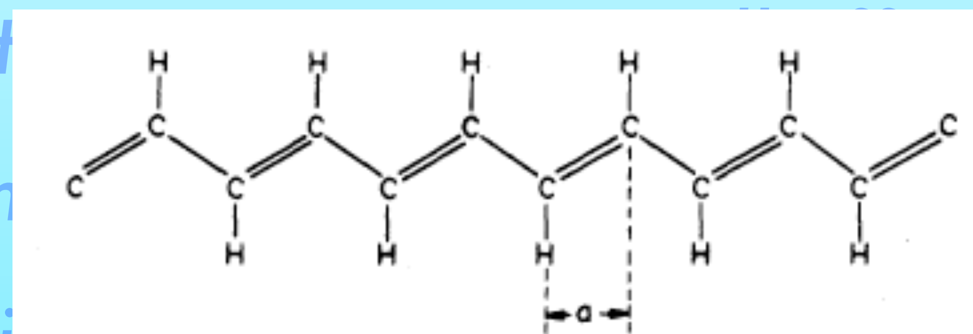
☆ Solitons in polyacetylene

VOLUME 42, NUMBER 25 PHYSICAL REVIEW LETTERS 18 JUNE 1979

Solitons in Polyacetylene

W. P. Su, J. R. Schrieffer, and A. J. Heeger
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 15 March 1979)

- Edge st... s
- Local m... ins near the impurities
- Zero bi... e d-wave superconductors
- Z... ene
- Q...
- E... al lattice
- C... ic photonic crystals
- S...



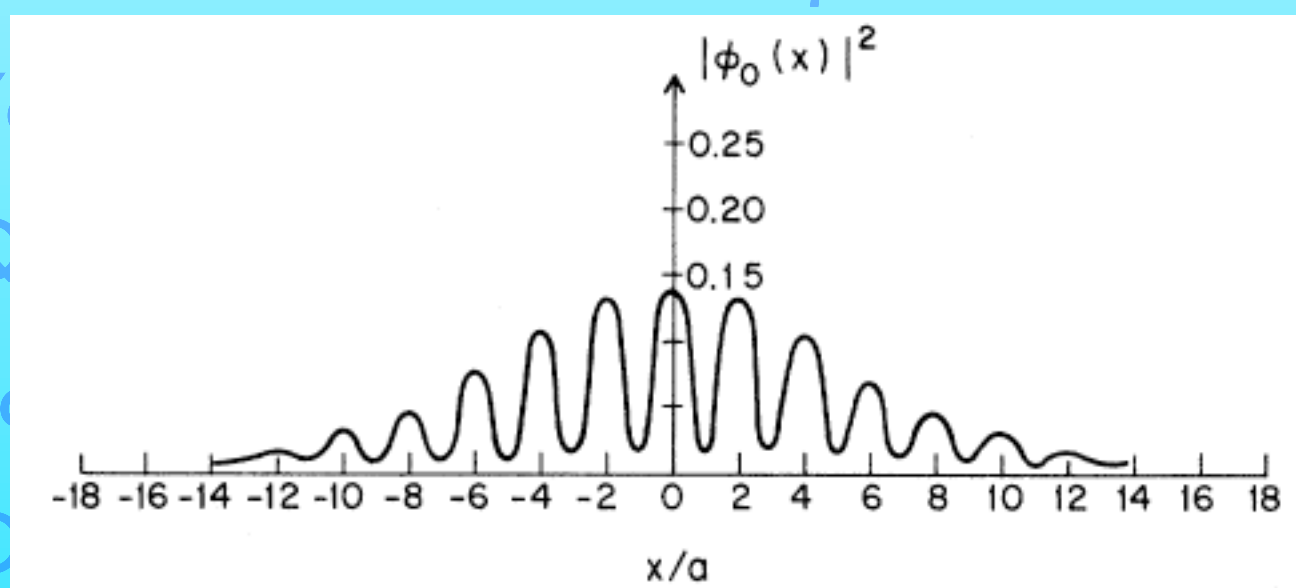
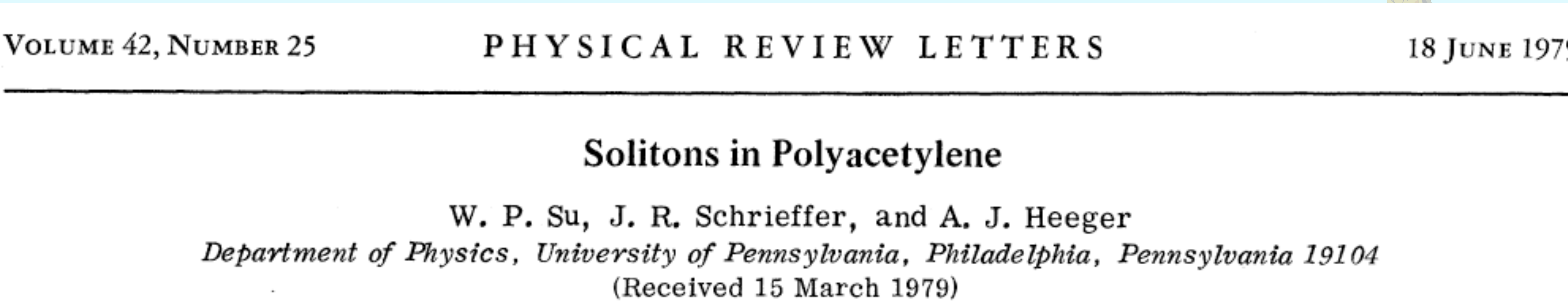


FIG. 3. Probability distribution of the localized electronic state at the center of the gap.

☆ Solitons in polyacetylene

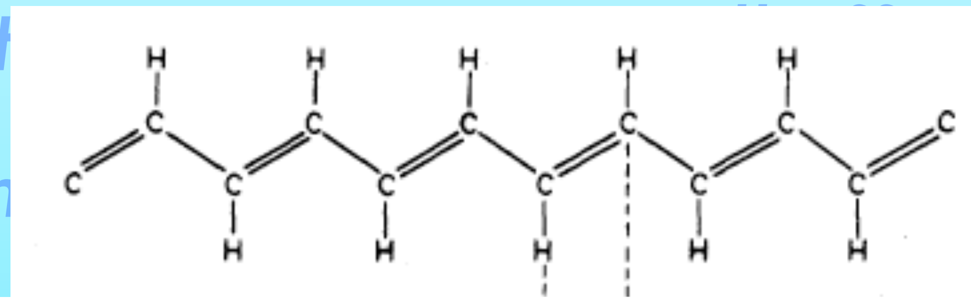

VOLUME 42, NUMBER 25 PHYSICAL REVIEW LETTERS 18 JUNE 1979

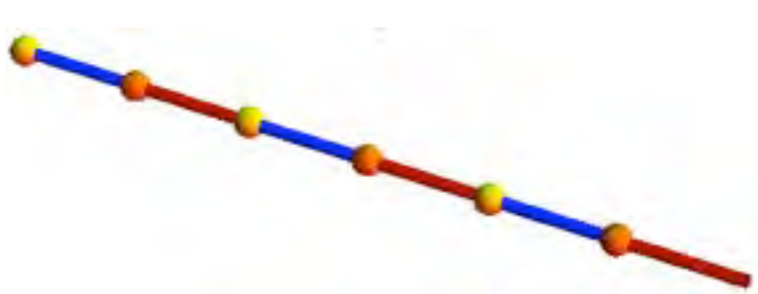
Solitons in Polyacetylene

W. P. Su, J. R. Schrieffer, and A. J. Heeger
Department of Physics, University of Pensylvania, Philadelphia, Pennsylvania 19104
(Received 15 March 1979)

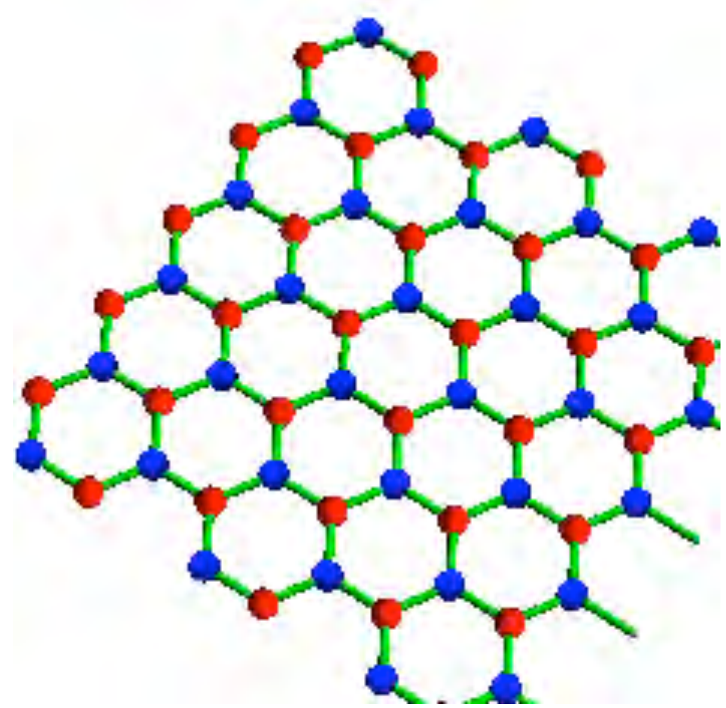
☆ Edge st… s

☆ Local m… ins near the impurities

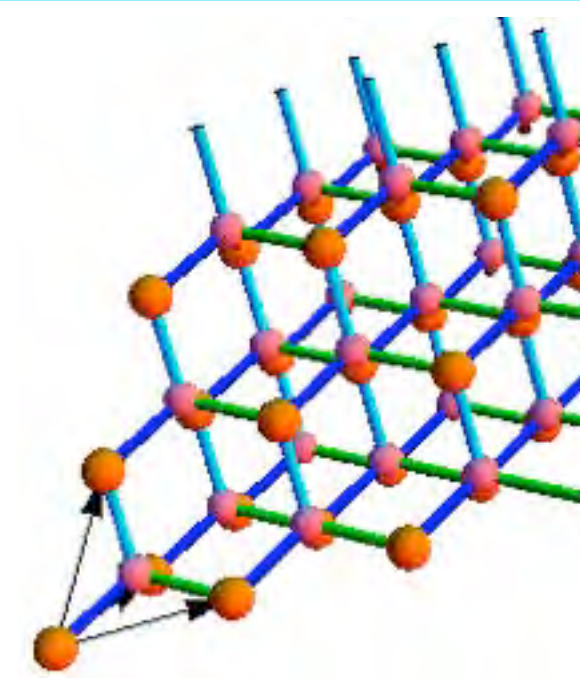




polyacetylene



graphene

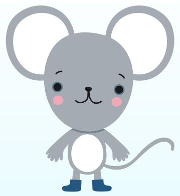


diamond

- Edge states in quantum Hall effects

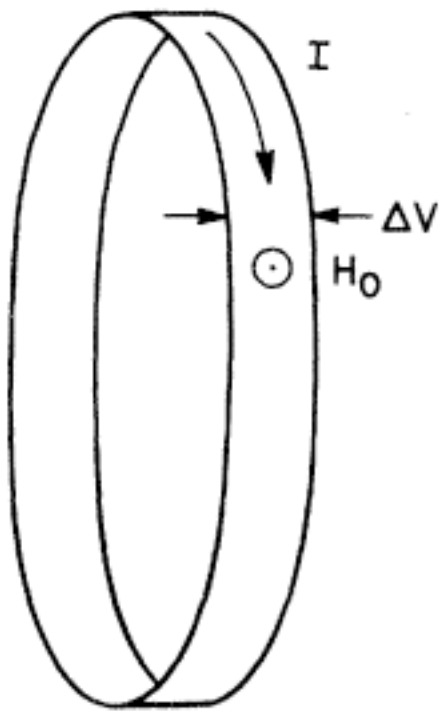
- Levinson's theorem to the Friedel's sum rule
- Surface states of Semiconductors & polarization
- Solitons in polyacetylene

- Local moments in integer spin chains near the impurities
- Zero bias conductance peaks of the d-wave superconductors
- Zero energy localized states of graphene
- Quantum Spin Hall Edge states
- Edge states in 2D cold atoms in optical lattice
- One-way edge modes in gyromagnetic photonic crystals
- Spin Ladder with ring exchanges



☆ *Edge states in quantum Hall effects*

PHYSICAL REVIEW B VOLUME 23, NUMBER 10 15 MAY 1981

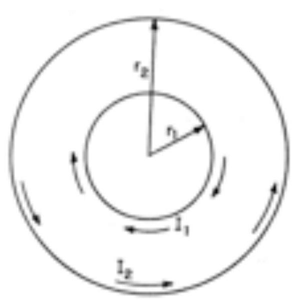


Quantized Hall conductivity in two dimensions

R. B. Laughlin
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 20 January 1981)

PHYSICAL REVIEW B VOLUME 25, NUMBER 4 15 FEBRUARY 1982

Quantized Hall conductance, current-carrying edge states, and the existence of extended states in a two-dimensional disordered potential



B. I. Halperin

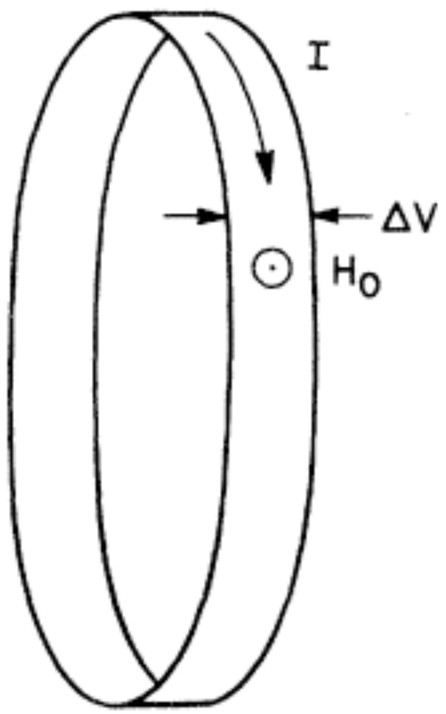
PHYSICAL REVIEW B VOLUME 48, NUMBER 16 15 OCTOBER 1993-II

Edge states in the integer quantum Hall effect and the Riemann surface of the Bloch function

Yasuhiro Hatsugai*

☆ Edge states in quantum Hall effects

PHYSICAL REVIEW B VOLUME 23, NUMBER 10 15 MAY 1981



Quantized Hall conductivity in two dimensions

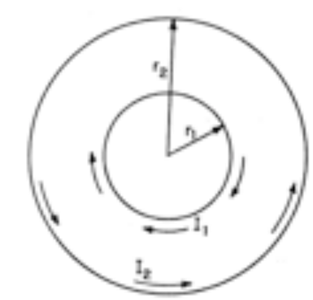
R. B. Laughlin
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 20 January 1981)

Everything started from here
discuss later

PHYSICAL REVIEW B VOLUME 25, NUMBER 4 15 FEBRUARY 1982

Quantized Hall conductance, current-carrying edge states, and the existence of extended states in a two-dimensional disordered potential

B. I. Halperin



PHYSICAL REVIEW B VOLUME 48, NUMBER 16 15 OCTOBER 1993-II

Edge states in the integer quantum Hall effect and the Riemann surface of the Bloch function

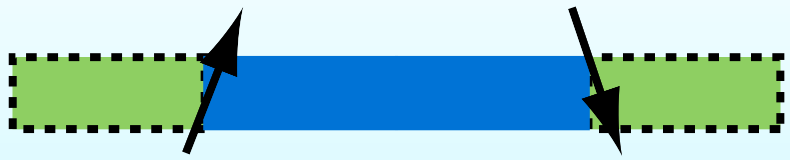
Yasuhiro Hatsugai*

☆ *Local moments in integer spin chains near the impurities*

- ☆ *Levinson's theorem to the Friedel's sum rule*
- ☆ *Surface states of Semiconductors & polarization*
- ☆ *Solitons in polyacetylene*
- ☆ *Edge states in quantum Hall effects*

- ☆ *Zero bias conductance peaks of the d-wave superconductors*
- ☆ *Zero energy localized states of graphene*
- ☆ *Quantum Spin Hall Edge states*
- ☆ *Edge states in 2D cold atoms in optical lattice*
- ☆ *One-way edge modes in gyromagnetic photonic crystals*
- ☆ *Spin Ladder with ring exchanges*

☆ *Local moments in integer spin chains near the impurities*



Edge states of the Haldane phase

☆ *Levinson's theorem to the Friedel's sum rule*

Exact diagonalisations of open spin-1 chains

Tom Kennedy

Department of Mathematics, University of Arizona, Tucson, AZ 85721, USA

J. Phys.: Condens. Matter 2 (1990) 5737–5745. Printed in the UK

VOLUME 65, NUMBER 25 PHYSICAL REVIEW LETTERS 17 DECEMBER 1990

Observation of $S = \frac{1}{2}$ Degrees of Freedom in an $S = 1$ Linear-Chain Heisenberg Antiferromagnet

M. Hagiwara and K. Katsumata

The Institute of Physical and Chemical Research (RIKEN), Wako, Saitama 351-01, Japan

Ian Affleck

Canadian Institute for Advanced Research and Physics Department, University of British Columbia, Vancouver, British Columbia, Canada V6T 2A6

B. I. Halperin

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

J. P. Renard

Institut d'Electronique Fondamentale, Bâtiment 220, Université Paris-Sud, 91405 Orsay CEDEX, France

(Received 31 July 1990)

☆ Spin Ladder with ring exchanges

- ☆ Levinson's theorem to the Friedel's sum rule
- ☆ Surface states of Semiconductors & polarization
- ☆ Solitons in polyacetylene
- ☆ Edge states in quantum Hall effects
- ☆ Local moments in integer spin chains near the impurities
- ☆ Zero bias conductance peaks of the d-wave superconductors
- ☆ Zero energy localized states of graphene
- ☆ Quantum Spin Hall Edge states
- ☆ Edge states in 2D cold atoms in optical lattice
- ☆ One-way edge modes in gyromagnetic photonic crystals

☆ *Spin Ladder with ring exchanges*

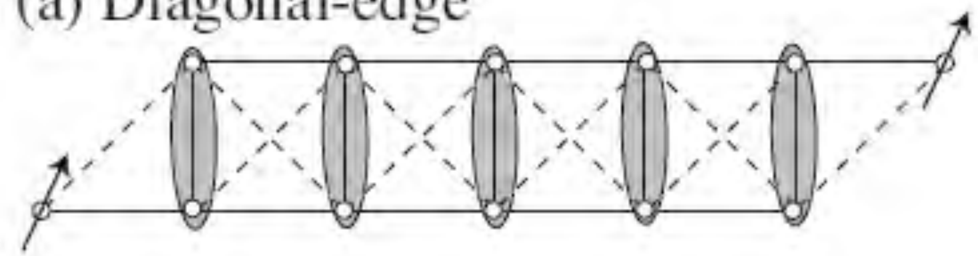
Spin edge states in gapped spin systems

PHYSICAL REVIEW B **79**, 205107 (2009)

Edge states of a spin-$\frac{1}{2}$ two-leg ladder with four-spin ring exchange

Mitsuhiro Arikawa,[1] Shou Tanaya,[1] Isao Maruyama,[2] and Yasuhiro Hatsugai[1,3,*]

(a) Diagonal-edge



PHYSICAL REVIEW B **76**, 012401 (2007)

Exact analysis of entanglement in gapped quantum spin chains

Hosho Katsura,[1,*] Takaaki Hirano,[1,†] and Yasuhiro Hatsugai[1,2,‡]

☆ *Zero bias conductance peaks of the d-wave superconductors*

- ☆ *Levinson's theorem to the Friedel's sum rule*
- ☆ *Surface states of Semiconductors & polarization*
- ☆ *Solitons in polyacetylene*
- ☆ *Edge states in quantum Hall effects*
- ☆ *Local moments in integer spin chains near the impurities*

- ☆ *Zero energy localized states of graphene*
- ☆ *Quantum Spin Hall Edge states*
- ☆ *Edge states in 2D cold atoms in optical lattice*
- ☆ *One-way edge modes in gyromagnetic photonic crystals*
- ☆ *Spin Ladder with ring exchanges*

☆ Zero bias conductance peaks of the d-wave superconductors

VOLUME 72, NUMBER 10 PHYSICAL REVIEW LETTERS 7 MARCH 1994

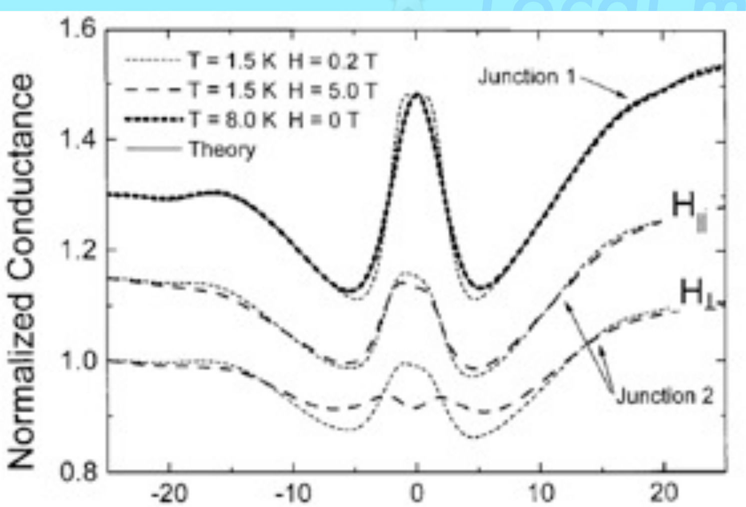
Midgap Surface States as a Novel Signature for $d_{x_a^2-x_b^2}$-Wave Superconductivity

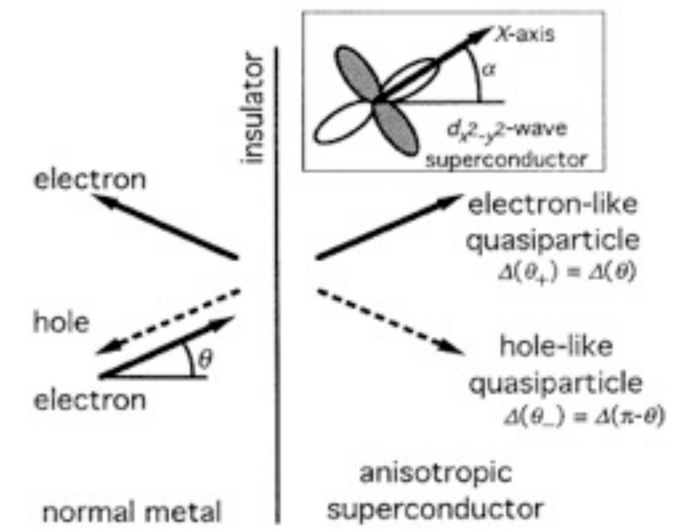
Chia-Ren Hu

VOLUME 74, NUMBER 17 PHYSICAL REVIEW LETTERS 24 APRIL 1995

Theory of Tunneling Spectroscopy of *d*-Wave Superconductors

Yukio Tanaka[1] and Satoshi Kashiwaya[2]



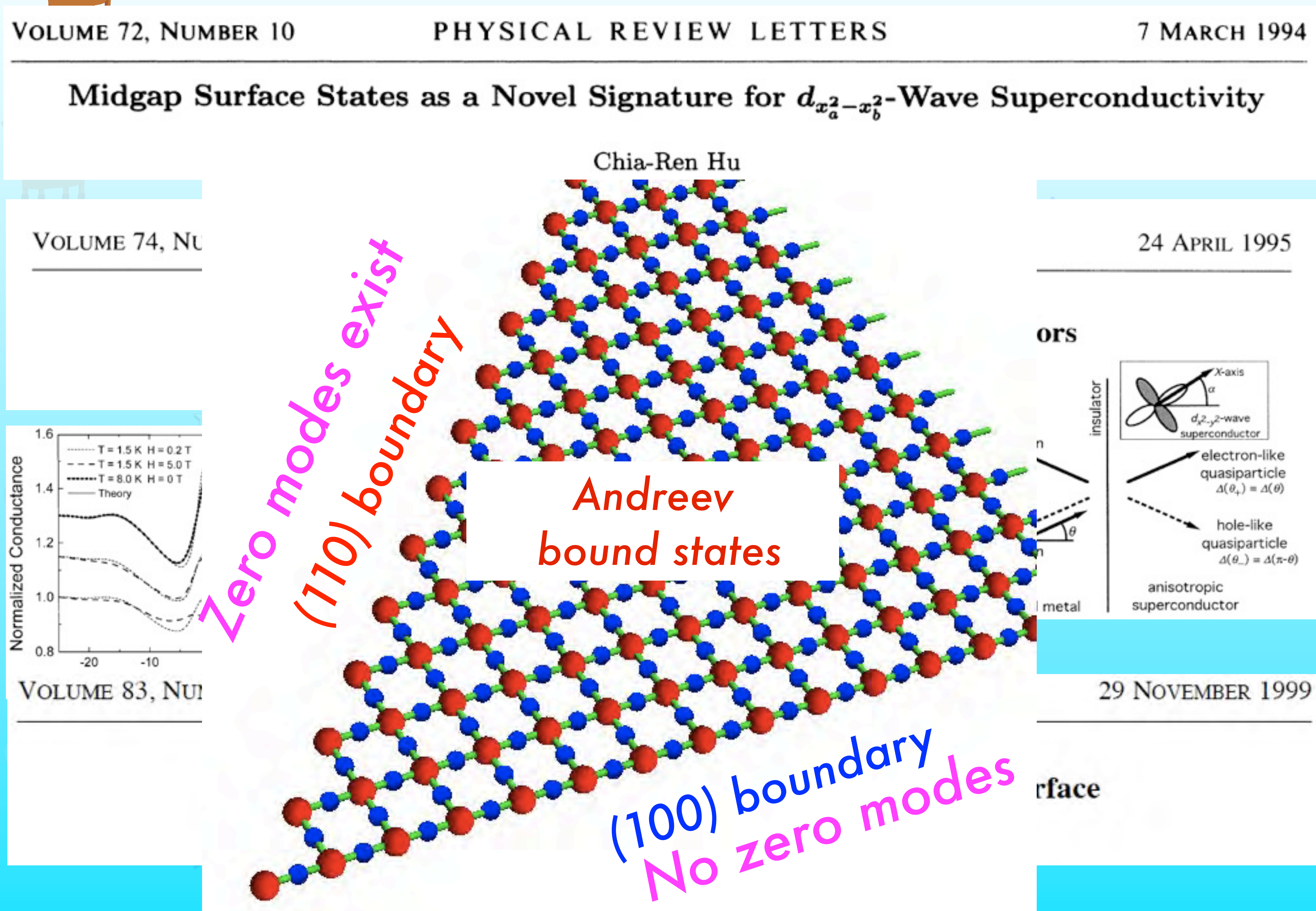


VOLUME 83, NUMBER 22 PHYSICAL REVIEW LETTERS 29 NOVEMBER 1999

Doppler Shift of the Andreev Bound States at the YBCO Surface

M. Aprili,* E. Badica, and L. H. Greene

☆ *Zero bias conductance peaks of the d-wave superconductors*



☆ *Zero energy localized states of graphene*

- ☆ *Levinson's theorem to the Friedel's sum rule*
- ☆ *Surface states of Semiconductors & polarization*
- ☆ *Solitons in polyacetylene*
- ☆ *Edge states in quantum Hall effects*
- ☆ *Local moments in integer spin chains near the impurities*
- ☆ *Zero bias conductance peaks of the d-wave superconductors*

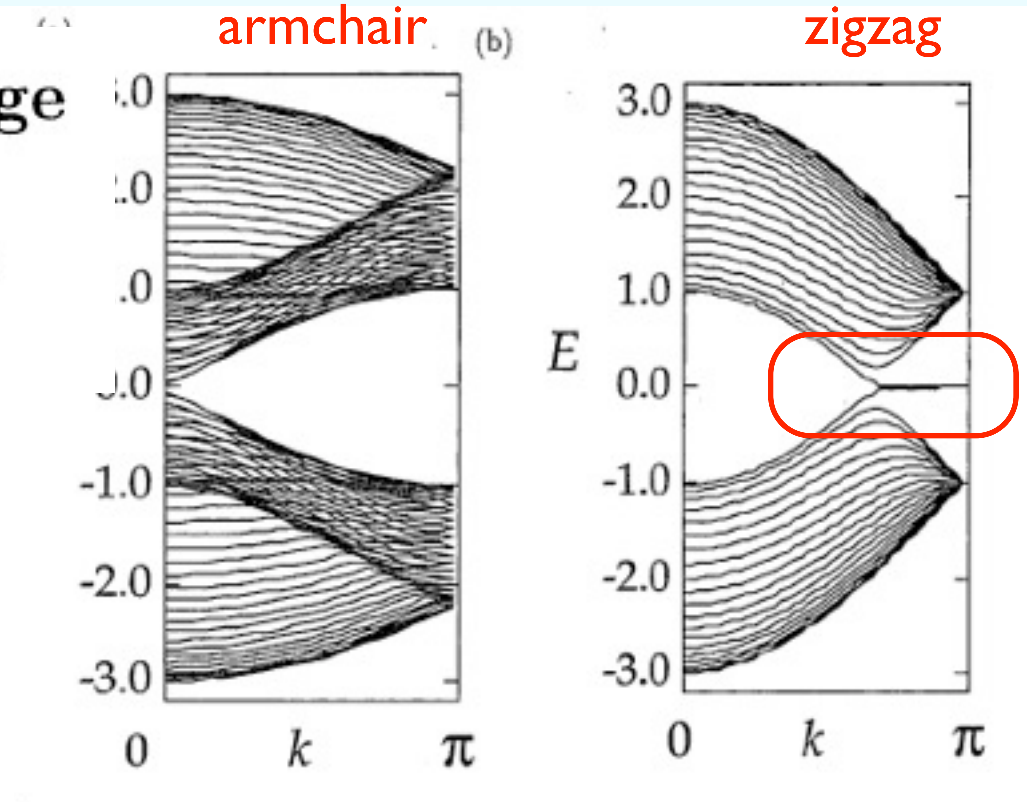
- ☆ *Quantum Spin Hall Edge states*
- ☆ *Edge states in 2D cold atoms in optical lattice*
- ☆ *One-way edge modes in gyromagnetic photonic crystals*
- ☆ *Spin Ladder with ring exchanges*

☆ *Zero energy localized states of graphene*

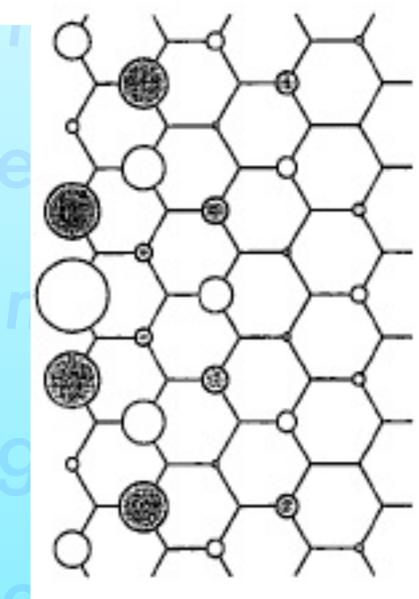
Peculiar Localized State at Zigzag Graphite Edge

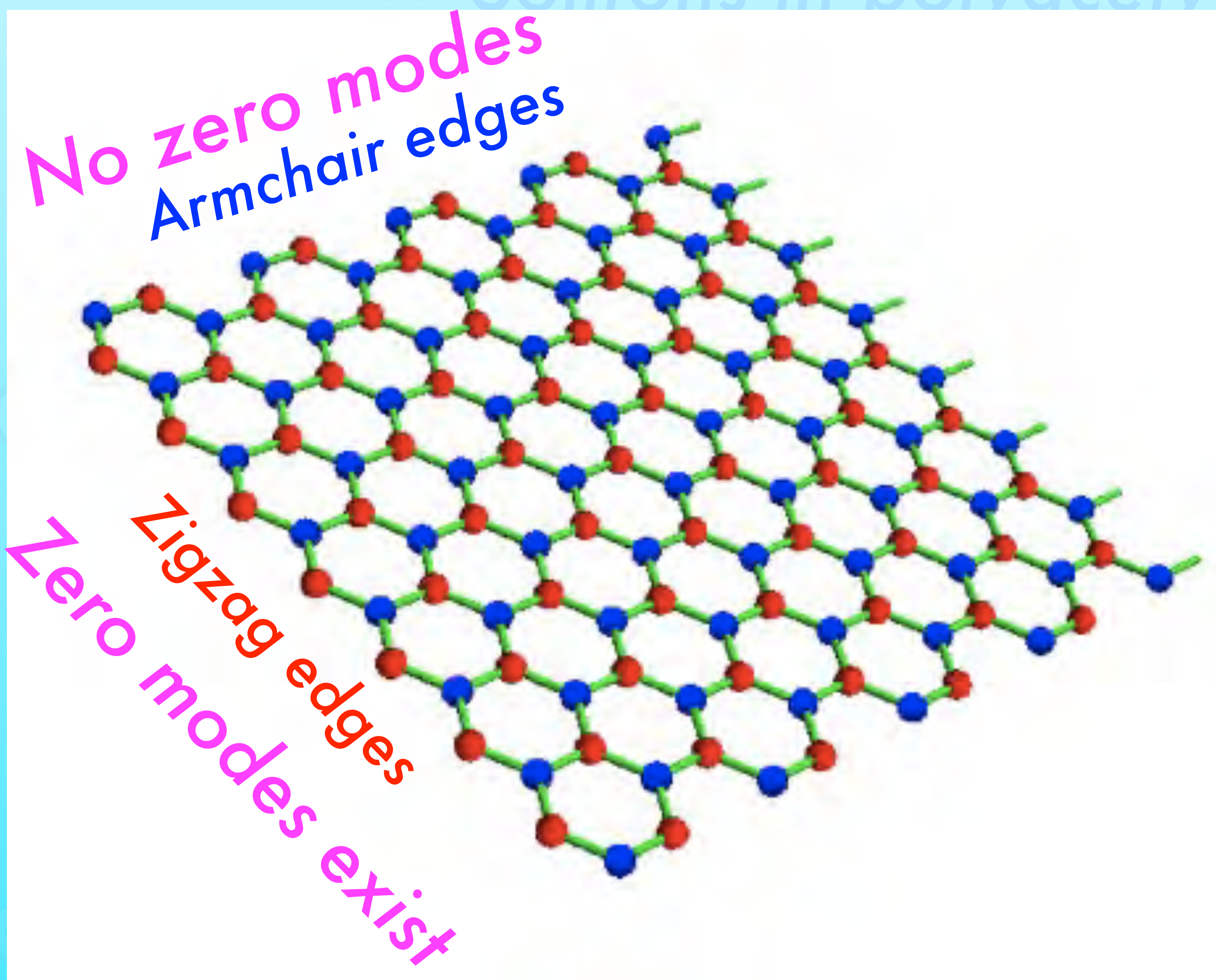
Mitsutaka FUJITA, Katsunori WAKABAYASHI, Kyoko NAKADA and Koichi KUSAKABE[1]

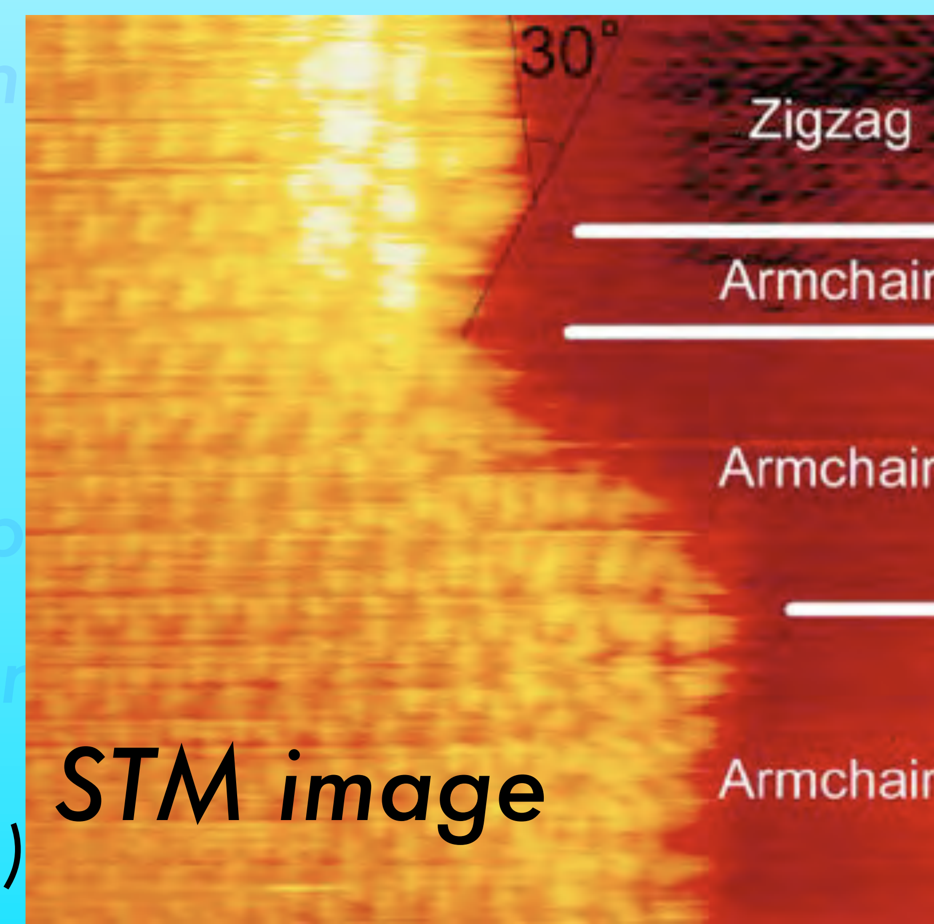
Journal of the Physical Society of Japan
Vol. 65, No. 7, July, 1996, pp. 1920-1923



Band structure of (a) armchair and (b) zigzag ri







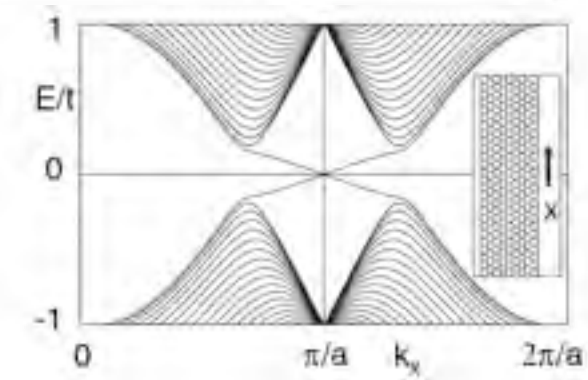
Kobayashi et al,
Phys. Rev. B71, 193406 (2005)

☆ *Quantum Spin Hall Edge states*

PRL **95,** 226801 (2005) PHYSICAL REVIEW LETTERS week ending 25 NOVEMBER 2005

Quantum Spin Hall Effect in Graphene

C. L. Kane and E. J. Mele



- ☆ *Solitons in polyacetylene*
- ☆ *Edge states in quantum Hall effects*
- ☆ *Local moments in integer spin chains near the impurities*
- ☆ *Zero bias conductance peaks of the d-wave superconductors*
- ☆ *Zero energy localized states of graphene*

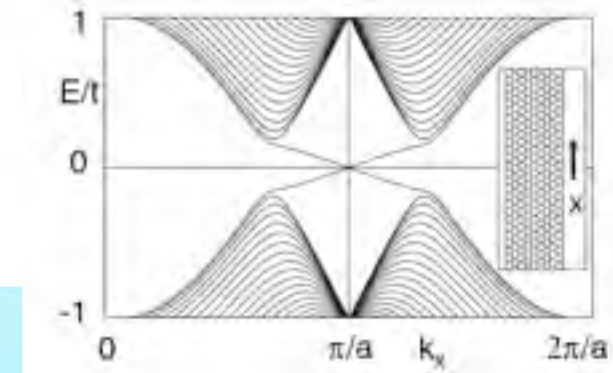
- ☆ *Edge states in 2D cold atoms in optical lattice*
- ☆ *One-way edge modes in gyromagnetic photonic crystals*
- ☆ *Spin Ladder with ring exchanges*

☆ Quantum Spin Hall Edge states

PRL 95, 226801 (2005) PHYSICAL REVIEW LETTERS week ending 25 NOVEMBER 2005

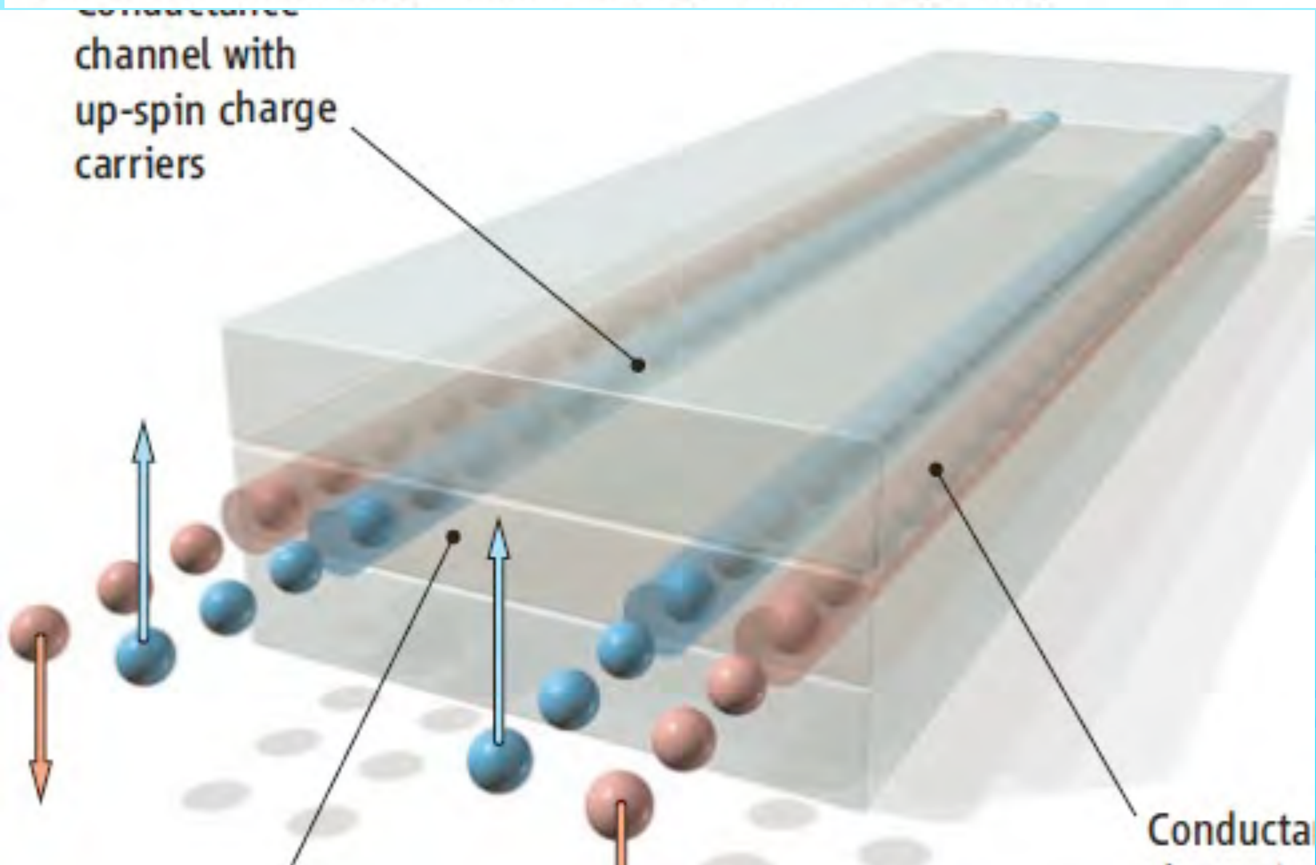
Quantum Spin Hall Effect in Graphene

C. L. Kane and E. J. Mele



Quantum Spin Hall Insulator State in HgTe Quantum Wells

Markus König,[1] Steffen Wiedmann,[1] Christoph Brüne,[1] Andreas Roth,[1] Hartmut Buhmann,[1] Laurens W. Molenkamp,[1*] Xiao-Liang Qi,[2] Shou-Cheng Zhang[2]



☆ Quantum Spin Hall Edge states

TOPOLOGICAL INSULATORS

The next generation

Spin-orbit coupling in some materials leads to the formation of surface states that are topologically protected from scattering. Theory and experiments have found an important new family of such materials.

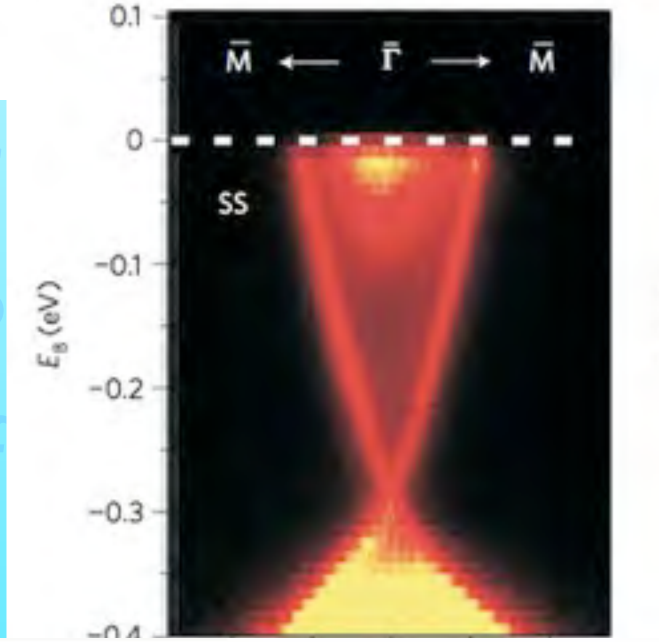
Joel Moore

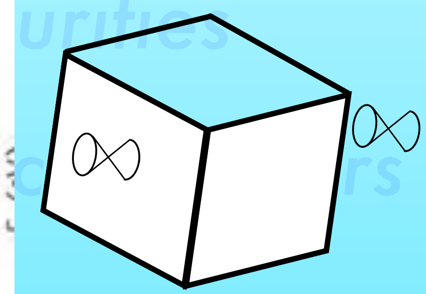
NATURE PHYSICS | VOL 5 | JUNE 2009 | www.nature.com/naturephysics

☆ Local moments in integer spin c... urities

☆ Zero bias conductance peaks o... rs

☆ Zero energy localized states of





☆ Edge states in 2D c

☆ One-way edge mo

☆ Spin Ladder with ri

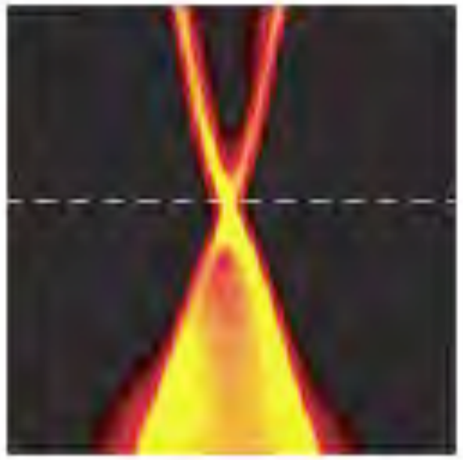
LETTERS
PUBLISHED ONLINE: 10 MAY 2009 | DOI: 10.1038/NPHYS1274
nature physics

Observation of a large-gap topological-insulator class with a single Dirac cone on the surface

Y. Xia[1,2], D. Qian[1,3], D. Hsieh[1,2], L. Wray[1], A. Pal[1], H. Lin[4], A. Bansil[4], D. Grauer[5], Y. S. Hor[5], R. J. Cava[5] and M. Z. Hasan[1,2,6]★

One more thing :Ternary

Synopsis: A more perfect Dirac cone



Credit: K. Kuroda *et al.*, Phys. Rev. Lett. (2010)

Experimental Realization of a Three-Dimensional Topological Insulator Phase in Ternary Chalcogenide $TlBiSe_2$

K. Kuroda, M. Ye, A. Kimura, S. V. Eremeev, E. E. Krasovskii, E. V. Chulkov, Y. Ueda, K. Miyamoto, T. Okuda, K. Shimada, H. Namatame, and M. Taniguchi

Phys. Rev. Lett. **105**, 146801 (2010)

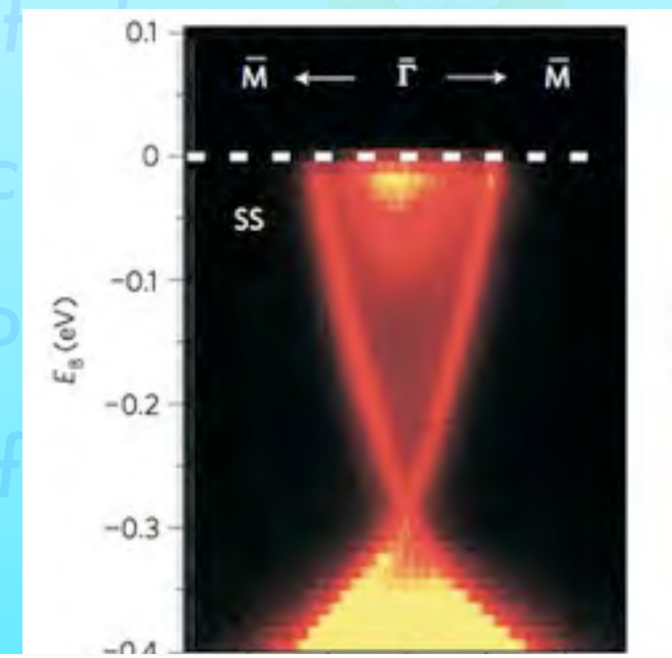
Published September 28, 2010

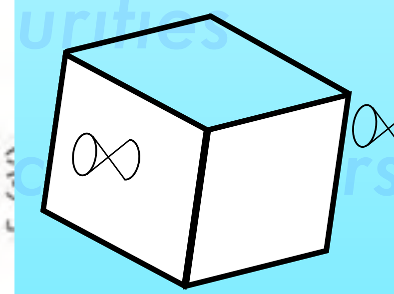
☆ Edge states in quantum Hall eff

☆ Local moments in integer spin c ... urities

☆ Zero bias conductance peaks o ... rs

☆ Zero energy localized states of





☆ Edge states in 2D c

☆ One-way edge mo

☆ Spin Ladder with ri

LETTERS

PUBLISHED ONLINE: 10 MAY 2009 | DOI: 10.1038/NPHYS1274

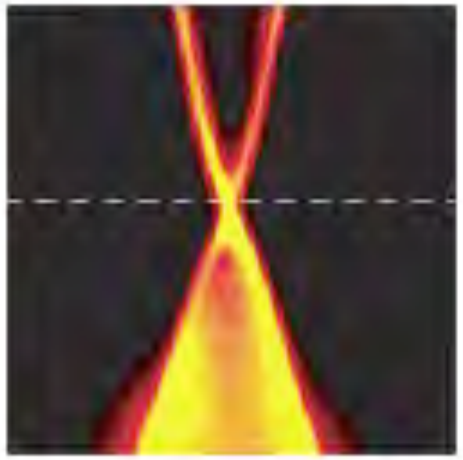
nature physics

Observation of a large-gap topological-insulator class with a single Dirac cone on the surface

Y. Xia[1,2], D. Qian[1,3], D. Hsieh[1,2], L. Wray[1], A. Pal[1], H. Lin[4], A. Bansil[4], D. Grauer[5], Y. S. Hor[5], R. J. Cava[5] and M. Z. Hasan[1,2,6]★

One more thing :Ternary

Synopsis: A more perfect Dirac cone



Experimental Realization of a Three-Dimensional Topological Insulator Phase in Ternary Chalcogenide $TlBiSe_2$

K. Kuroda, M. Ye, A. Kimura, S. V. Eremeev, E. E. Krasovskii, E. V. Chulkov, Y. Ueda, K. Miyamoto, T. Okuda, K. Shimada, H. Namatame, and M. Taniguchi

Phys. Rev. Lett. **105**, 146801 (2010)

Published September 28, 2010

REVIEWS OF MODERN PHYSICS, VOLUME 82, OCTOBER–DECEMBER 2010

Colloquium: Topological insulators

M. Z. Hasan*

Joseph Henry Laboratories, Department of Physics, Princeton University, Princeton, New Jersey 08544, USA

C. L. Kane†

Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

REVIEWS OF MODERN PHYSICS, VOLUME 83, OCTOBER–DECEMBER 2011

Topological insulators and superconductors

Xiao-Liang Qi

Microsoft Research, Station Q, Elings Hall, University of California, Santa Barbara, California 93106, USA and Department of Physics, Stanford University, Stanford, California 94305, USA

Shou-Cheng Zhang

Department of Physics, Stanford University, Stanford, California 94305, USA

☆ *Edge states in 2D cold atoms in optical lattice*

- ☆ *Levinson's theorem to the Friedel's sum rule*
- ☆ *Surface states of Semiconductors & polarization*
- ☆ *Solitons in polyacetylene*
- ☆ *Edge states in quantum Hall effects*
- ☆ *Local moments in integer spin chains near the impurities*
- ☆ *Zero bias conductance peaks of the d-wave superconductors*
- ☆ *Zero energy localized states of graphene*
- ☆ *Quantum Spin Hall Edge states*

- ☆ *One-way edge modes in gyromagnetic photonic crystals*
- ☆ *Spin Ladder with ring exchanges*

☆ *Edge states in 2D cold atoms in optical lattice*

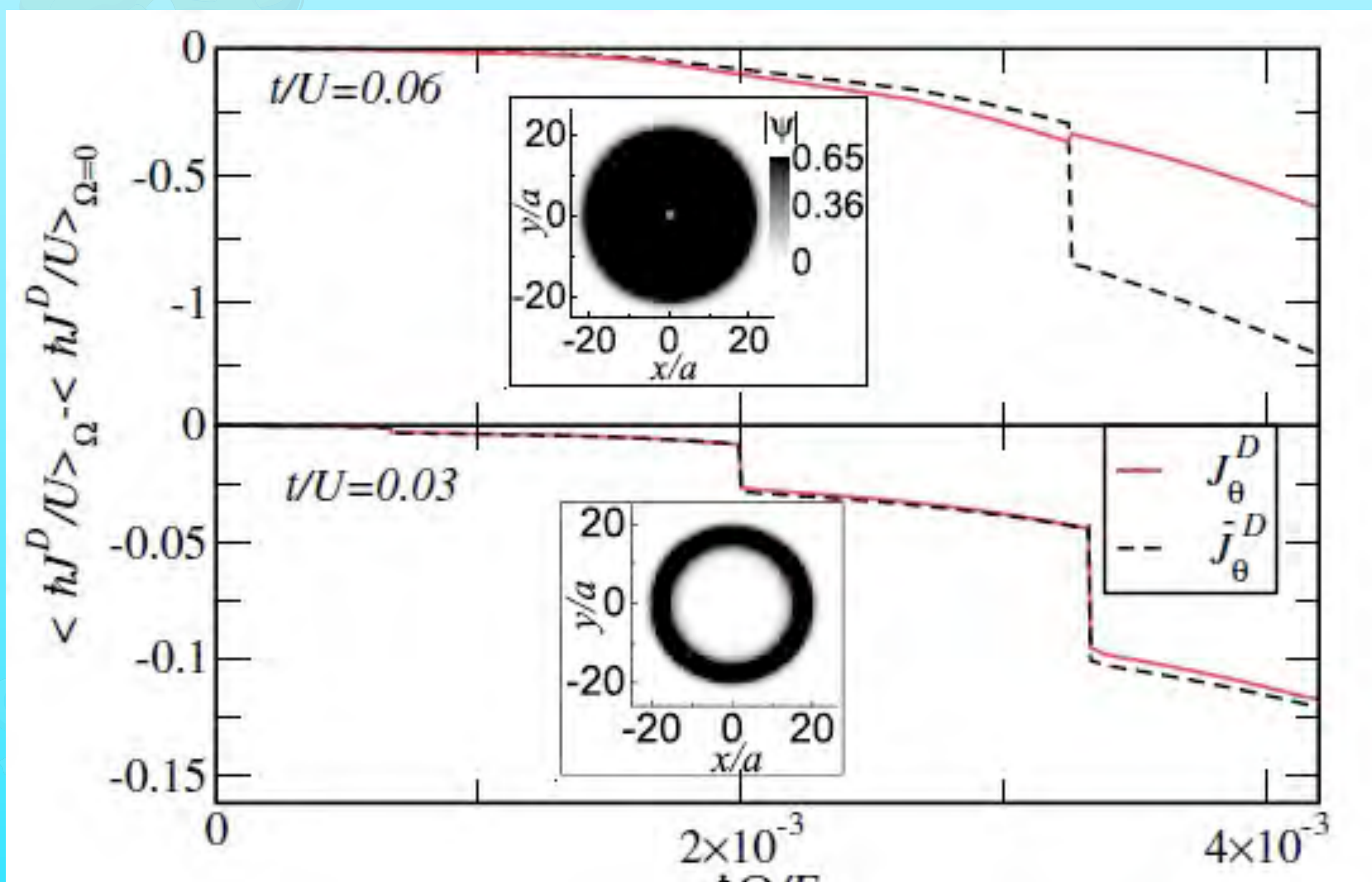
PRL **98,** 210403 (2007) PHYSICAL REVIEW LETTERS week ending 25 MAY 2007

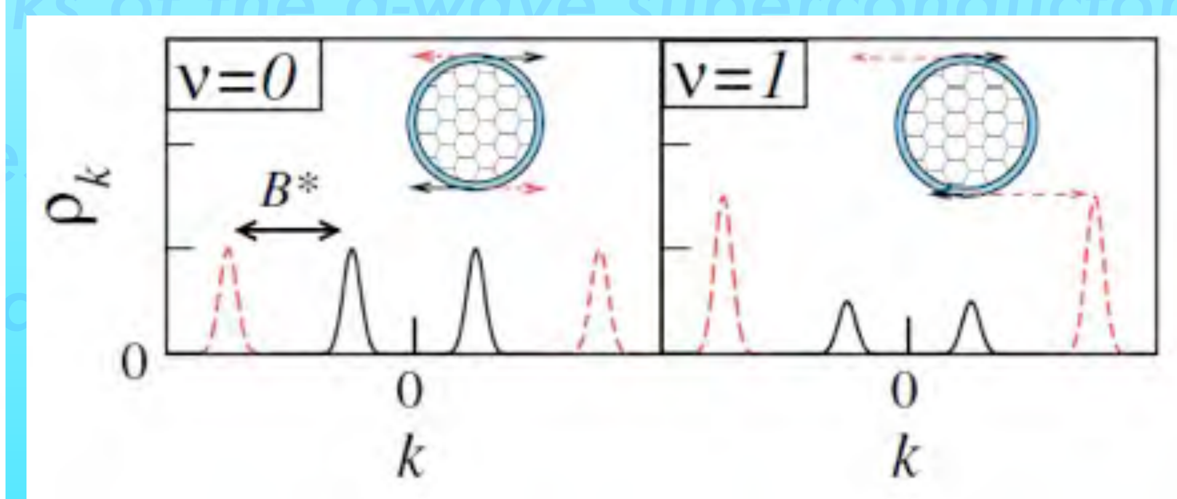
Edge Transport in 2D Cold Atom Optical Lattices

V. W. Scarola and S. Das Sarma

Condensed Matter Theory Center, Department of Physics, University of Maryland, College Park, Maryland 20742, USA

(Received 4 January 2007; published 24 May 2007)





☆ *Edge states in 2D cold atoms in optical lattice*

PRL **108**, 255303 (2012) PHYSICAL REVIEW LETTERS week ending 22 JUNE 20

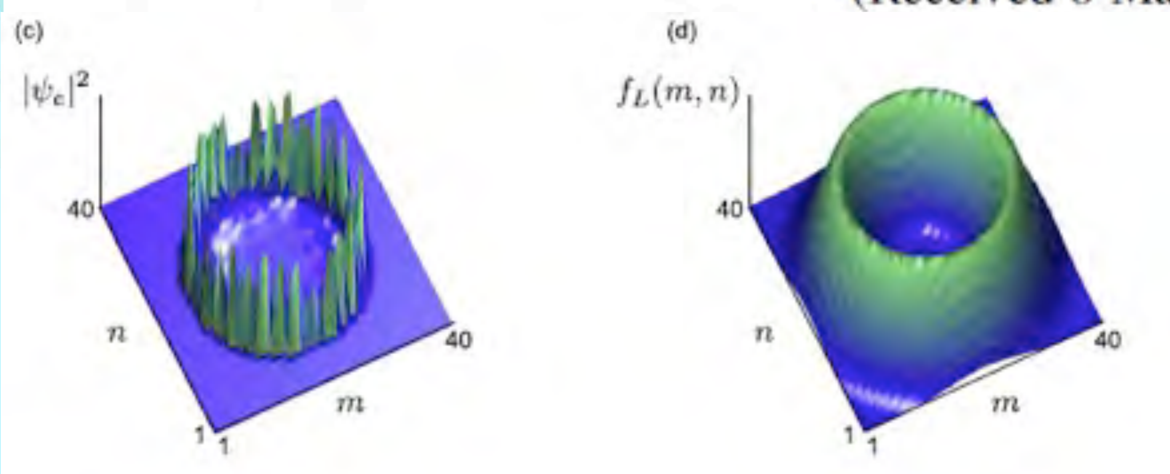
Detecting Chiral Edge States in the Hofstadter Optical Lattice

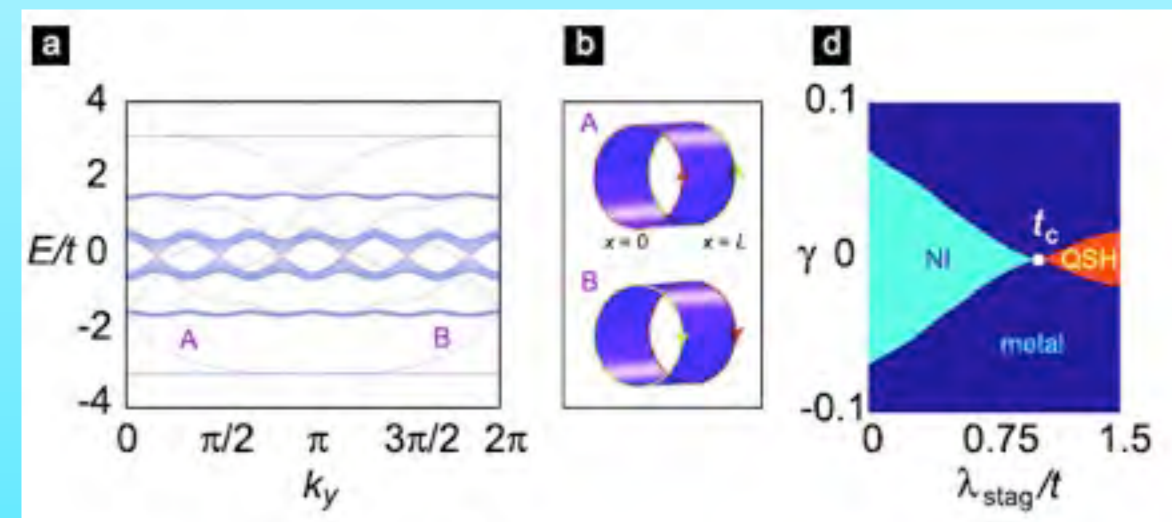
Nathan Goldman,[1,*] Jérôme Beugnon,[2] and Fabrice Gerbier[2]

[1] *Center for Nonlinear Phenomena and Complex Systems-Université Libre de Bruxelles (U.L.B.), B-1050 Brussels, Belgium*

[2] *Laboratoire Kastler Brossel, CNRS, ENS, UPMC, 24 rue Lhomond, 75005 Paris*

(Received 6 March 2012; published 19 June 2012)





PRL **105**, 255302 (2010) PHYSICAL REVIEW LETTERS week ending 17 DECEMBER 2010

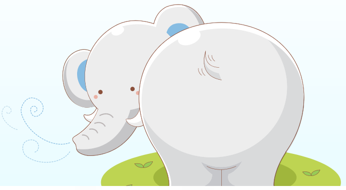
Realistic Time-Reversal Invariant Topological Insulators with Neutral Atoms

N. Goldman,[1] I. Satija,[2,3] P. Nikolic,[2,3] A. Bermudez,[4] M. A. Martin-Delgado,[4] M. Lewenstein,[5,6] and I. B. Spielman[7]

☆ One-way edge modes in gyromagnetic photonic crystals

- ☆ Levinson's theorem to the Friedel's sum rule
- ☆ Surface states of Semiconductors & polarization
- ☆ Solitons in polyacetylene
- ☆ Edge states in quantum Hall effects
- ☆ Local moments in integer spin chains near the impurities
- ☆ Zero bias conductance peaks of the d-wave superconductors
- ☆ Zero energy localized states of graphene
- ☆ Quantum Spin Hall Edge states
- ☆ Edge states in 2D cold atoms in optical lattice

- ☆ Spin Ladder with ring exchanges

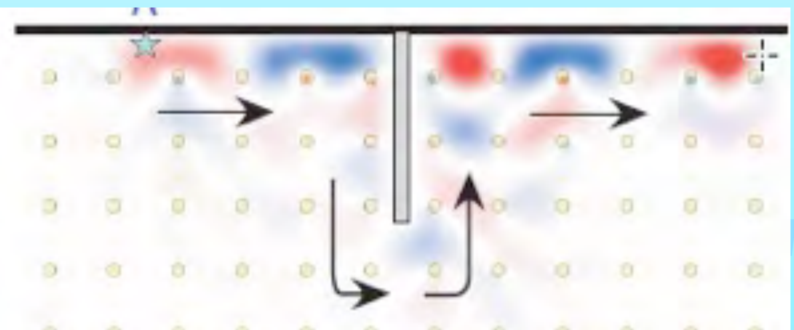


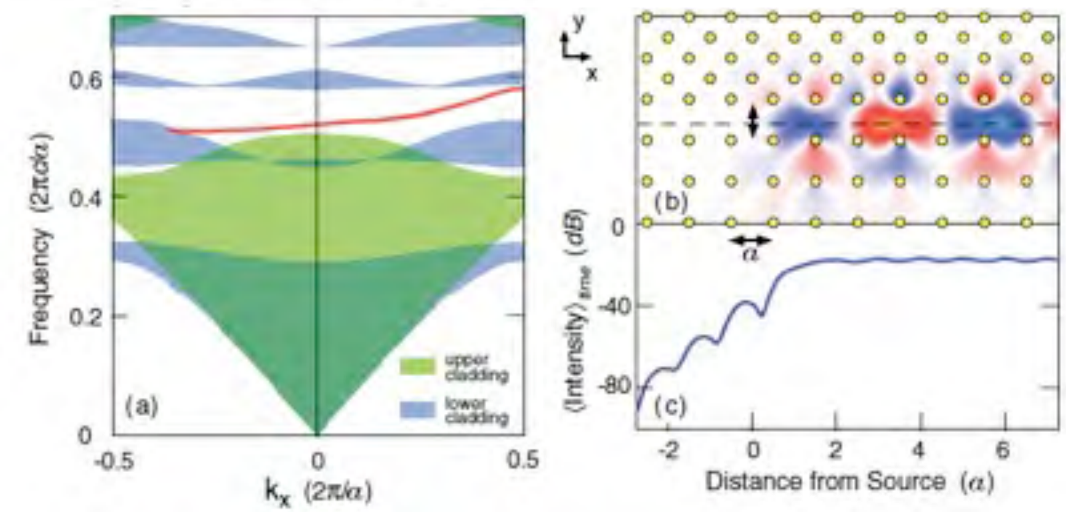
☆ One-way edge modes in gyromagnetic photonic crystals

PRL **100**, 013905 (2008) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2008

Reflection-Free One-Way Edge Modes in a Gyromagnetic Photonic Crystal

Zheng Wang, Y. D. Chong, John D. Joannopoulos, and Marin Soljačić





Vol 461 | 8 October 2009 | doi:10.1038/nature08293

Observation of unidirectional backscattering-immune topological electromagnetic states

Zheng Wang[1]*, Yidong Chong[1]†*, J. D. Joannopoulos[1] & Marin Soljačić[1]

PRL **100**, 013904 (2008) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2008

Possible Realization of Directional Optical Waveguides in Photonic Crystals with Broken Time-Reversal Symmetry

F. D. M. Haldane and S. Raghu*

Why do we care edge states?

Why the Edge States are there??

Accidental ?

NO !

Y.Hatsugai, PRL 71,3697 (1993)

Inevitable reasons

Physical Structures behind:

"Bulk determines the edges"

"Edge determines the bulk"

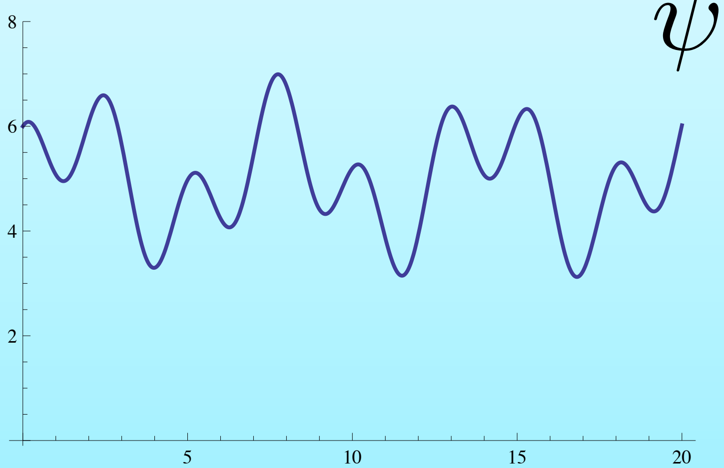
Bulk-Edge Correspondence

Protected by Topological constraints

Right / left to the symmetry

Extended states

unnormalizable

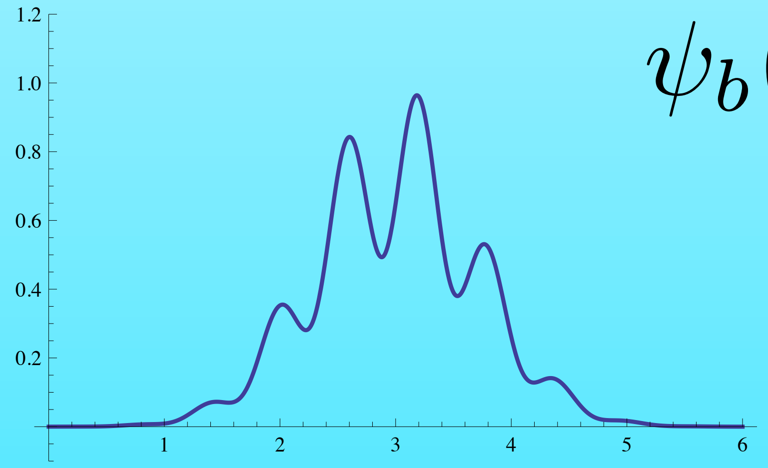


$$\psi_e(r) \sim \frac{1}{\sqrt{V}} e^{ikr} \longrightarrow 0 \quad (V \to \infty)$$

V: Volume

Bound states / Edge states

normalizable



$$\psi_b(r) \sim \frac{1}{\sqrt{a_0^3}} e^{-r/a_0}$$

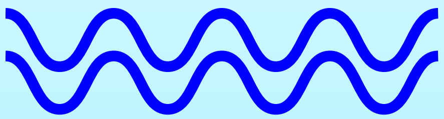
a_0 :size of the bound state

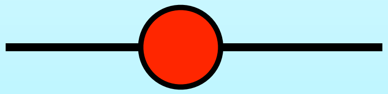
Edge state is topological

☑ Clear difference only in the infinite system with boundaries

Bulk-Edge correspondence

Universality



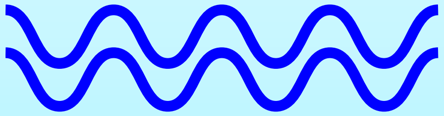


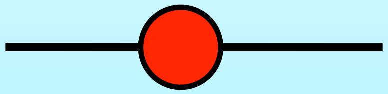
Bulk state (scattering state) Bulk Gap Non trivial Vacuum	Control with each other ⟷	Edge state (Bound state) Particles in the gap

Y.Hatsugai, PRL 71,3697 (1993)

Bulk-Edge correspondence

Universality





Bulk state (scattering state) Bulk Gap Non trivial Vacuum	Control with each other ⟷	Edge state (Bound state) Particles in the gap

Y.Hatsugai, PRL 71,3697 (1993)

Let's discuss 2 lucky examples

QHE & Dirac fermions

Bulk-Edge correspondence in QHE

QHE

Bulk-Edge correspondence

Common property of topological ordered states

Bulk

↓↑ YH

Edge

classically featureless

1-st Chern number for QHE

Thouless-Kohmoto-Nightingale-den Nijs (TKNN)

Niu-Thouless-Wu

low energy localized modes in the gap

edge states for QHE

Laughlin, Halperin, Wen, YH

☑ Edge states

Hall Conductance has a Topological meaning

When E_F is in the j-th gap   Two topological quantities

★ $$\sigma_{xy}^{\text{bulk}} = \frac{e^2}{h} \sum_{\ell:\epsilon_\ell(k)<E_F} C_\ell$$

Sum of the First Chern numbers below E_F
Thouless-Kohmoto-Nightingale-den Nijs '82

★ $$\sigma_{xy}^{\text{edge}} = \frac{e^2}{h} I(\alpha_j, C^j)$$

Winding number of the edge state on the complex energy surface   YH '93a

Bulk — Edge Correspondence

Y.Hatsugai, PRL 71,3697 (1993)

$$\sigma_{xy}{}^{\text{bulk}} = \sigma_{xy}{}^{\text{edge}}$$

Manybody

$$C_j = I_j - I_{j-1}$$

As for the Chern # of the single band

Chern # of j-th band

Difference in edge states just above and below the band

Bulk - Edge Correspondence

VOLUME 71, NUMBER 22 PHYSICAL REVIEW LETTERS 29 NOVEMBER 1993

Chern Number and Edge States in the Integer Quantum Hall Effect

Yasuhiro Hatsugai
Department of Physics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139 and Institute for Solid State Physics, University of Tokyo, 7-22-1 Roppongi Minato-ku, Tokyo 106, Japan
(Received 12 July 1993)

We consider the integer quantum Hall effect on a square lattice in a uniform rational magnetic field. The relation between two different interpretations of the Hall conductance as topological invariants is clarified. One is the Thouless–Kohmoto–Nightingale–den Nijs (TKNN) integer in the infinite system and the other is a winding number of the edge state. In the TKNN form of the Hall conductance, a phase of the Bloch wave function defines U(1) vortices on the magnetic Brillouin zone and the total vorticity gives σ_{xy}. We find that these vortices are given by the edge states when they are degenerate with the bulk states.

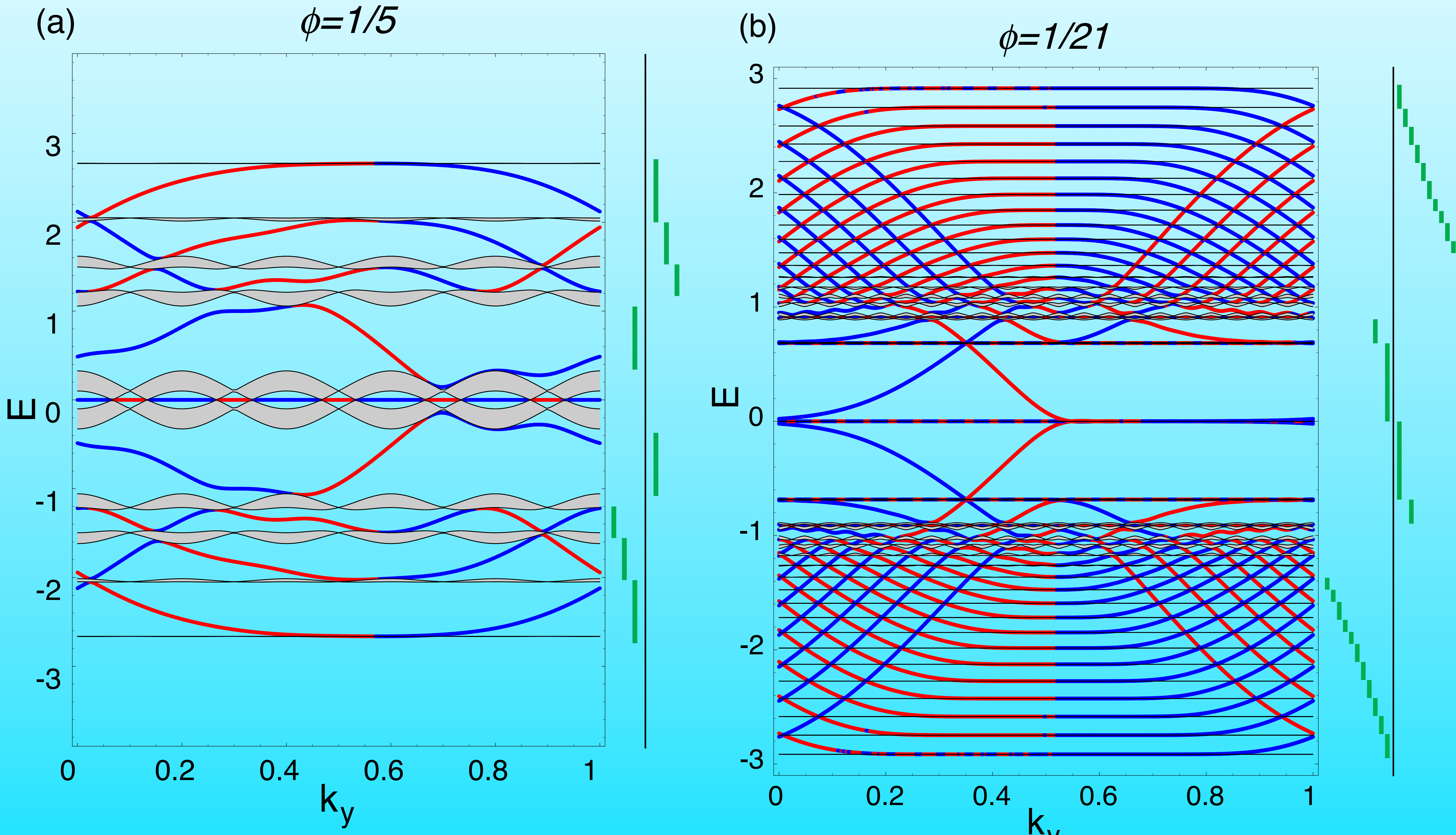
$$\sigma_{xy}{}^{\text{bulk}} = \sigma_{xy}{}^{\text{edge}}$$

Manybody

Bulk – Edge Correspondence

$$\sigma_{xy}^{\text{bulk}} = \sigma_{xy}^{\text{edge}}$$

Graphene

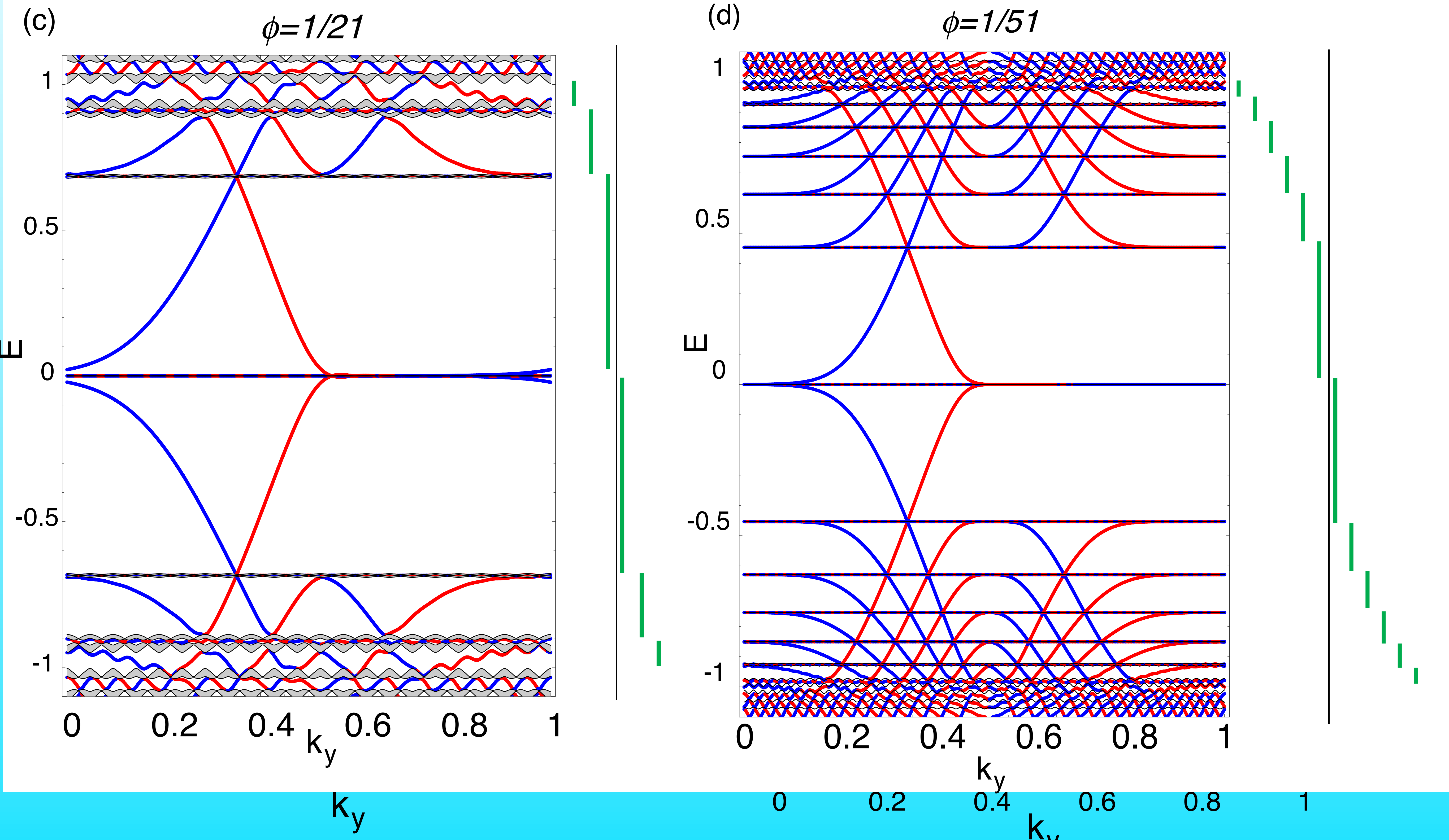


Bulk – Edge Correspondence

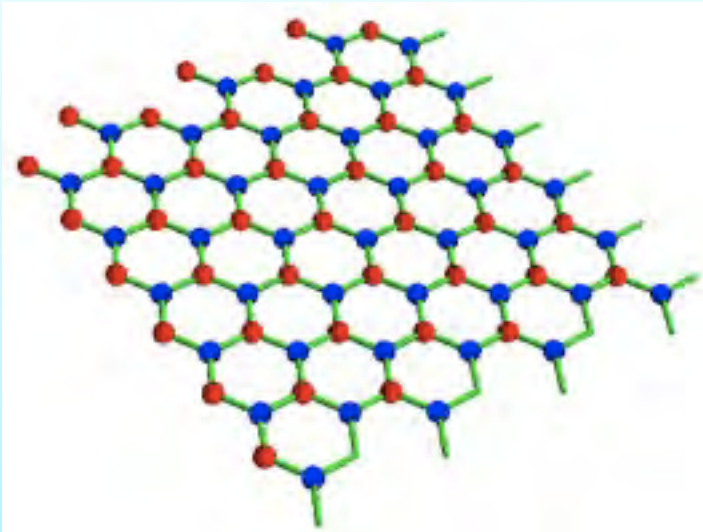
$$\sigma_{xy}^{\text{bulk}} = \sigma_{xy}^{\text{edge}}$$

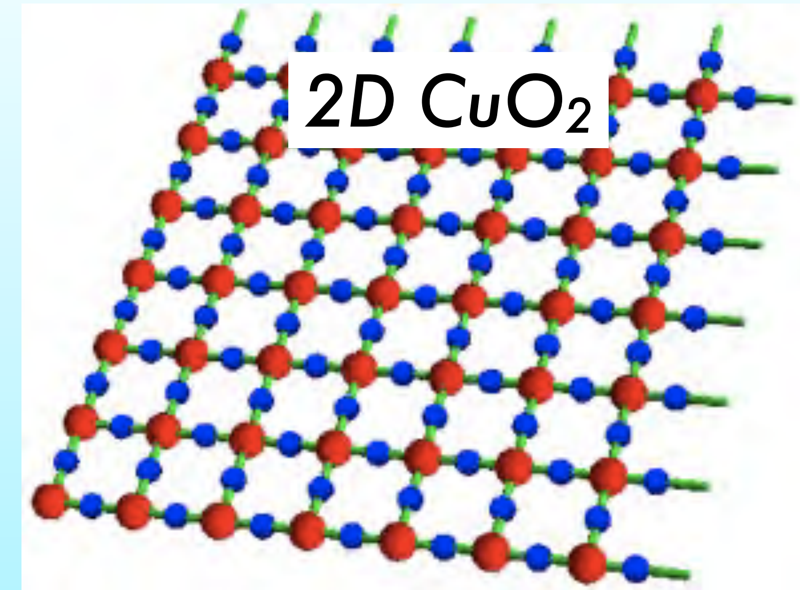
Graphene

Near Zero



Universality in the zero modes of Dirac Fermions





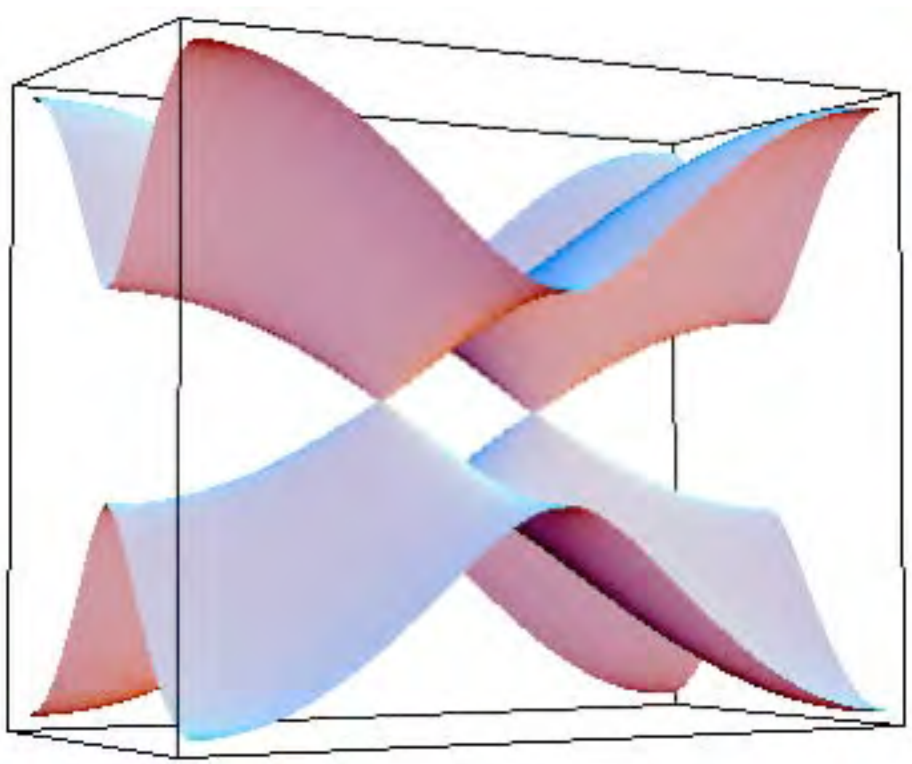
2D Dirac fermions :

Edge States

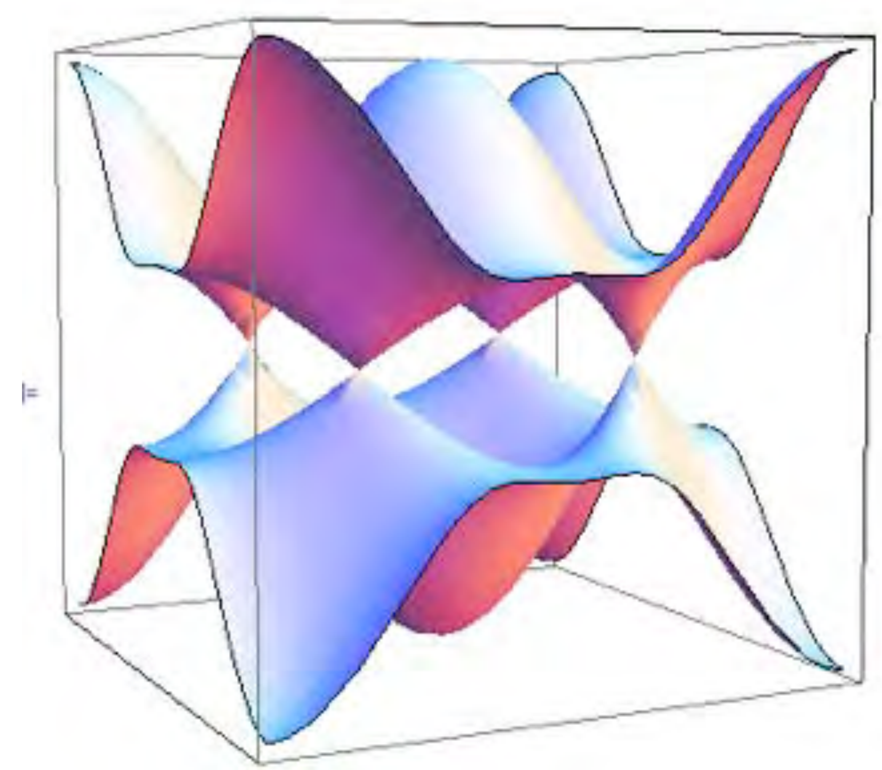
Zero mode localized states ??

YH, '09 (review)

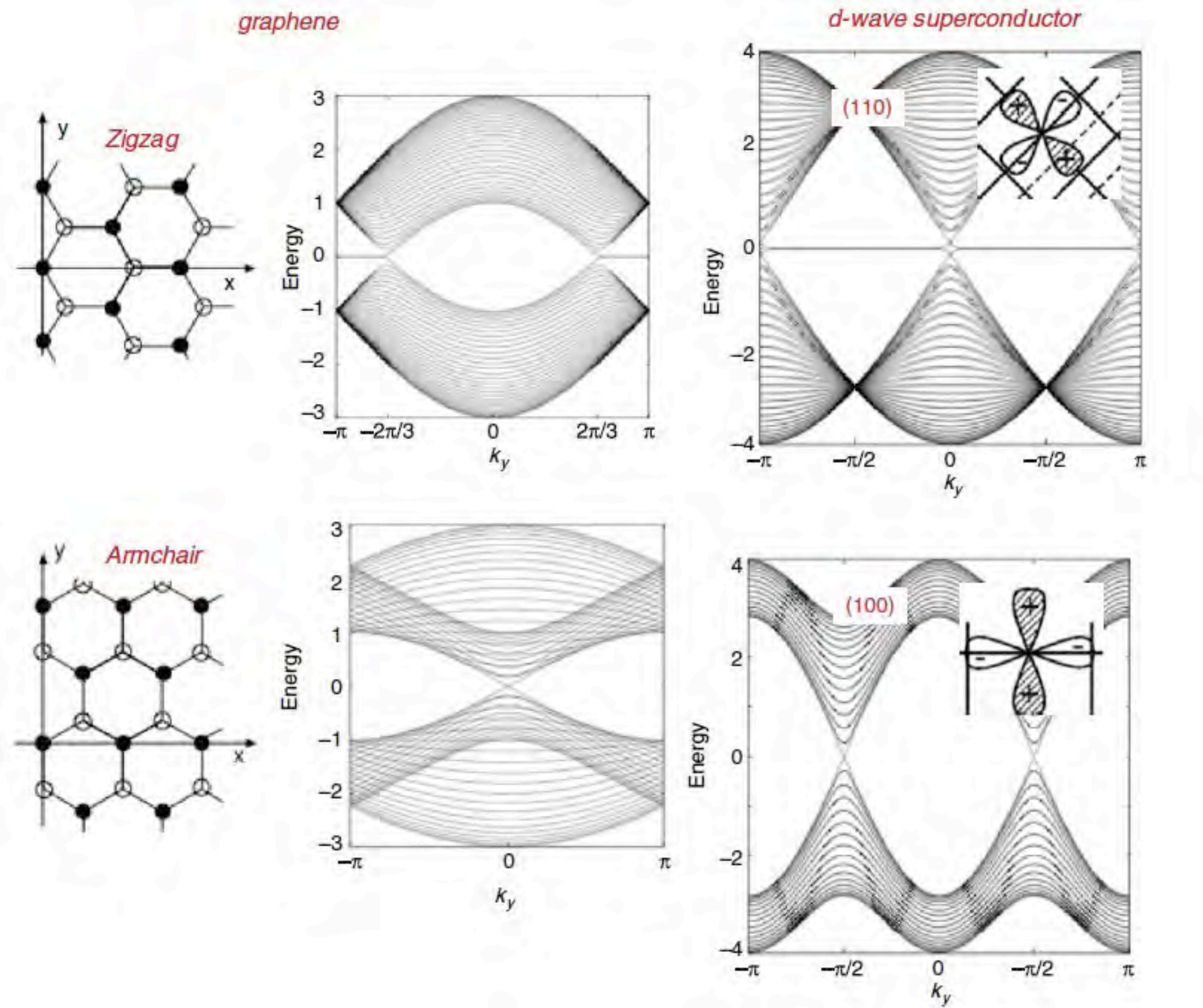
Graphene



d-wave superconductor



Analogy between graphene & d-wave superconductor



Universality of Zero Energy Edge States

'02–'04 S. Ryu & YH

Universality of Zero Energy Edge States

'02–'04 S. Ryu & YH

1. Zero energy edge states of graphene

- Boundary Magnetic moments of graphene

Universality of Zero Energy Edge States

'02–'04 S. Ryu & YH

1. Zero energy edge states of graphene
   - Boundary Magnetic moments of graphene
2. Andreev bound states of d-wave superconductors
   - Zero bias conductance peak

Universality of Zero Energy Edge States

'02–'04 S. Ryu & YH

1. Zero energy edge states of graphene
    - Boundary Magnetic moments of graphene
2. Andreev bound states of d-wave superconductors
    - Zero bias conductance peak

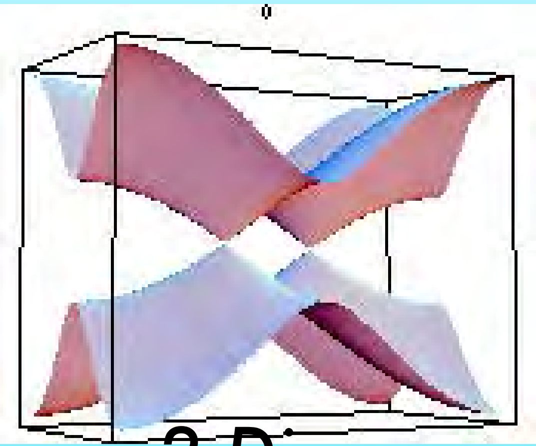
These 2 systems are topologically equivalent

Universality of Zero Energy Edge States

'02–'04 S. Ryu & YH

1. Zero energy edge states of graphene
   - Boundary Magnetic moments of graphene
2. Andreev bound states of d-wave superconductors
   - Zero bias conductance peak

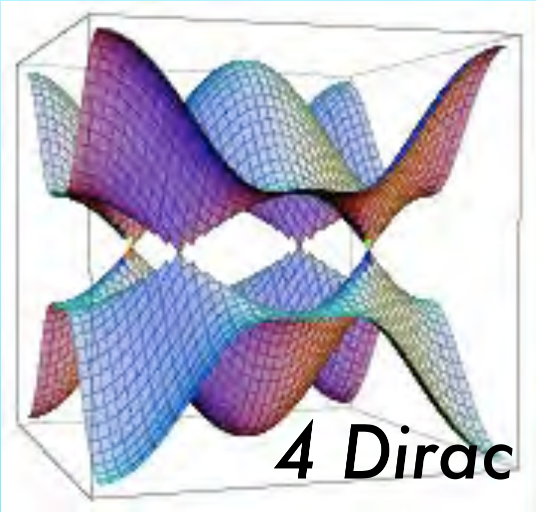
graphene



2 Dirac cones

These 2 systems are topologically equivalent

d-wave superconductor



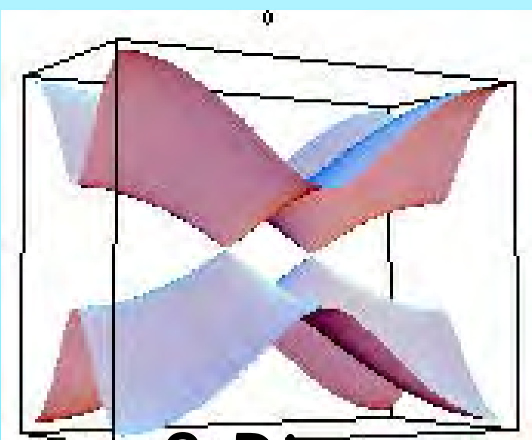
4 Dirac cones

Universality of Zero Energy Edge States

'02–'04 S. Ryu & YH

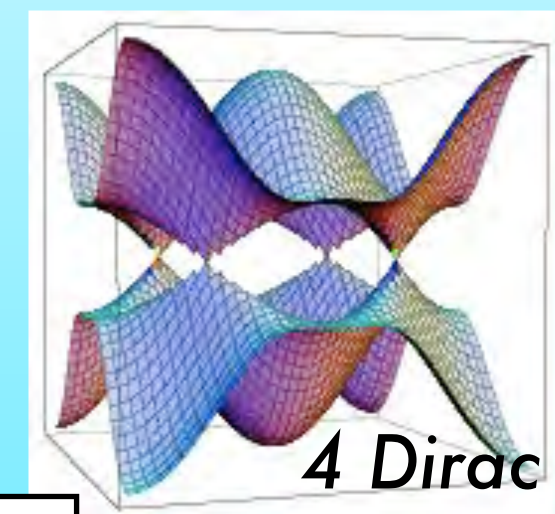
1. Zero energy edge states of graphene
   - Boundary Magnetic moments of graphene
2. Andreev bound states of d-wave superconductors
   - Zero bias conductance peak

graphene



2 Dirac cones

d-wave superconductor



4 Dirac cones

These 2 systems are topologically equivalent

Symmetry protected Zero modes of Dirac fermions :1D Flat Band of edge states

$^{\exists}\Gamma$ chiral symmetry

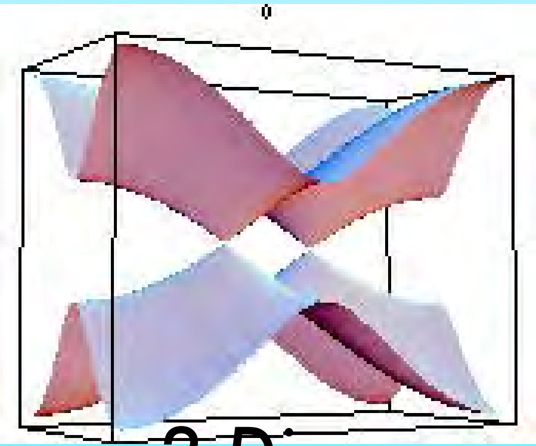
$$\{\Gamma, H\} = \Gamma H + H\Gamma = 0\ ,\ \ \Gamma^2 = 1$$

Universality of Zero Energy Edge States

'02–'04 S. Ryu & YH

1. Zero energy edge states of graphene
   - Boundary Magnetic moments of graphene
2. Andreev bound states of d-wave superconductors
   - Zero bias conductance peak

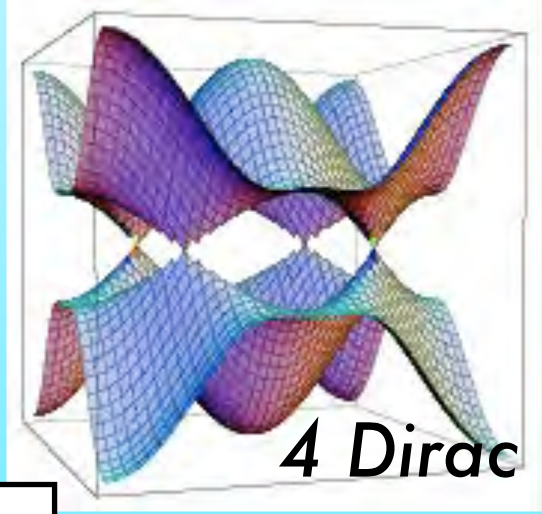
graphene



2 Dirac cones

These 2 systems are topologically equivalent

d-wave superconductor



4 Dirac cones

Symmetry protected
Zero modes of Dirac fermions
:1D Flat Band of edge states

Γ :Bipartite

(A-B sublattice symmetry)

$^\exists\Gamma$ chiral symmetry

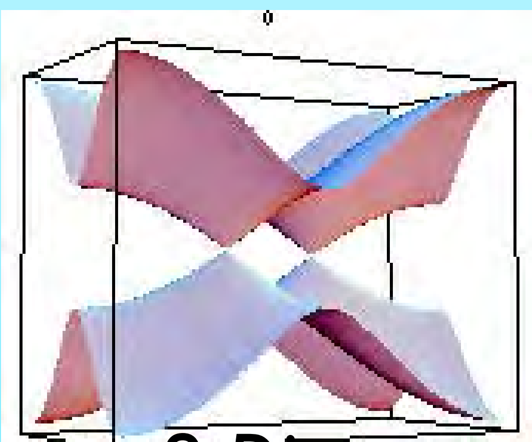
$$\{\Gamma, H\} = \Gamma H + H\Gamma = 0\ ,\ \Gamma^2 = 1$$

Universality of Zero Energy Edge States

'02–'04 S. Ryu & YH

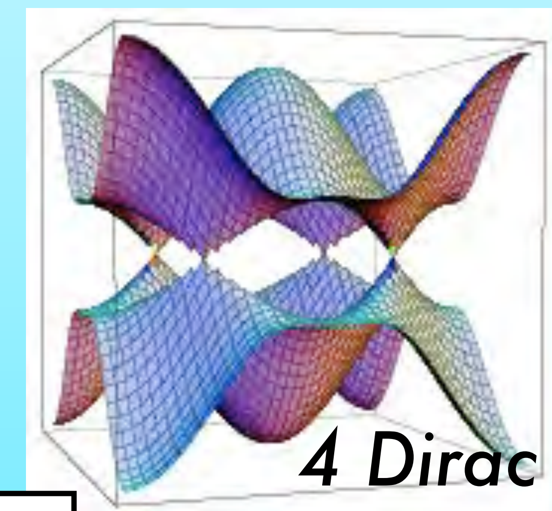
1. Zero energy edge states of graphene
   - Boundary Magnetic moments of graphene
2. Andreev bound states of d-wave superconductors
   - Zero bias conductance peak

graphene



2 Dirac cones

d-wave superconductor



4 Dirac cones

These 2 systems are topologically equivalent

Symmetry protected
Zero modes of Dirac fermions
:1D Flat Band of edge states

Γ :Bipartite

(A-B sublattice symmetry)

Γ:Time Reversal

(Real Order parameter)

$^\exists\Gamma$ chiral symmetry

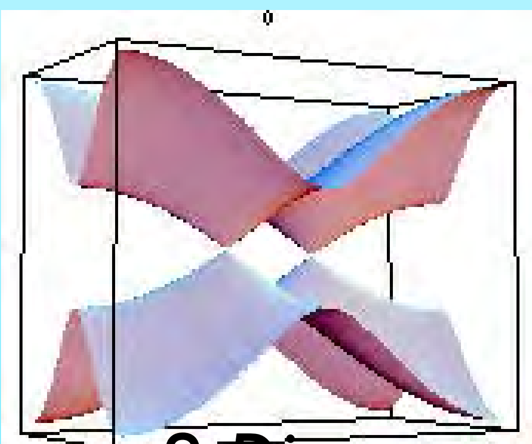
$\{\Gamma, H\} = \Gamma H + H\Gamma = 0\ ,\ \Gamma^2 = 1$

Universality of Zero Energy Edge States

'02–'04 S. Ryu & YH

Spontaneous breaking of these chiral symmetries : Peierls instabilities of Flat (edge) bands

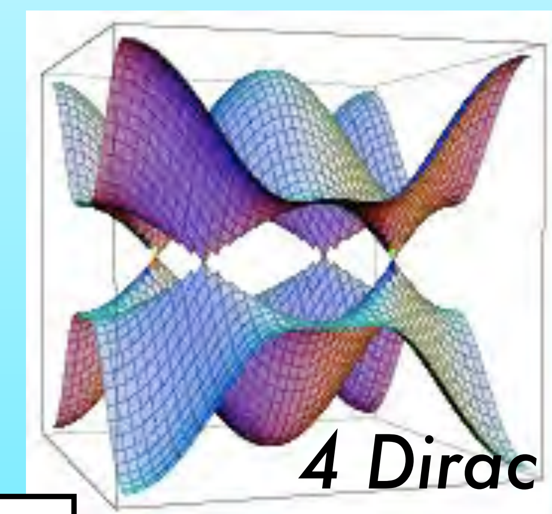
graphene



2 Dirac cones

These 2 systems are topologically equivalent

d-wave superconductor



4 Dirac cones

Symmetry protected Zero modes of Dirac fermions :1D Flat Band of edge states

Γ :Bipartite

(A-B sublattice symmetry)

Γ:Time Reversal

(Real Order parameter)

$^{\exists}\Gamma$ chiral symmetry

$$\{\Gamma, H\} = \Gamma H + H\Gamma = 0\ ,\ \Gamma^2 = 1$$

Universality of Zero Energy Edge States

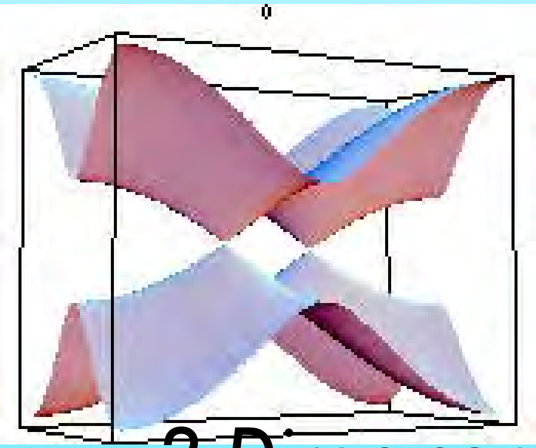
'02–'04 S. Ryu & YH

Boundary magnetic moments

Spontaneous breaking of these chiral symmetries : Peierls instabilities of Flat (edge) bands

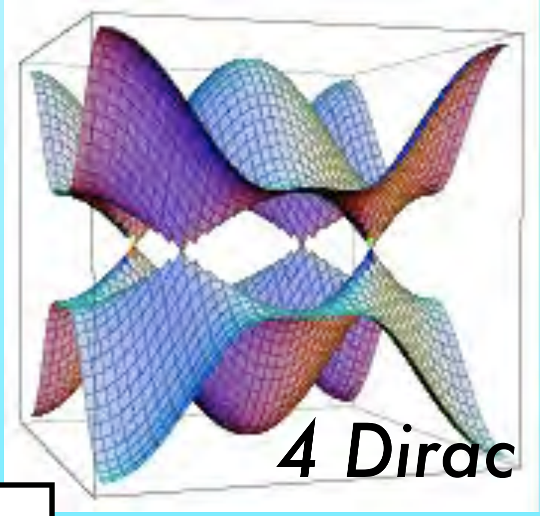
graphene

d-wave superconductor



2 Dirac cones

These 2 systems are topologically equivalent



4 Dirac cones

Symmetry protected
Zero modes of Dirac fermions
:1D Flat Band of edge states

Γ :Bipartite

(A-B sublattice symmetry)

Γ:Time Reversal

(Real Order parameter)

$^{\exists}\Gamma$ chiral symmetry

$\{\Gamma, H\} = \Gamma H + H\Gamma = 0\ ,\ \Gamma^2 = 1$

Universality of Zero Energy Edge States

'02–'04 S. Ryu & YH

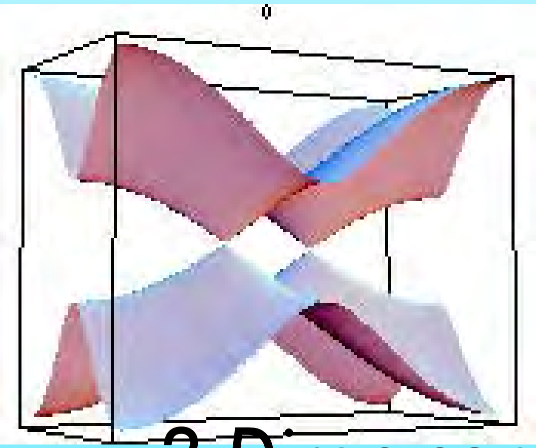
Boundary magnetic moments

Spontaneous breaking of these chiral symmetries : Peierls instabilities of Flat (edge) bands

Spontaneous local flux generation near defects

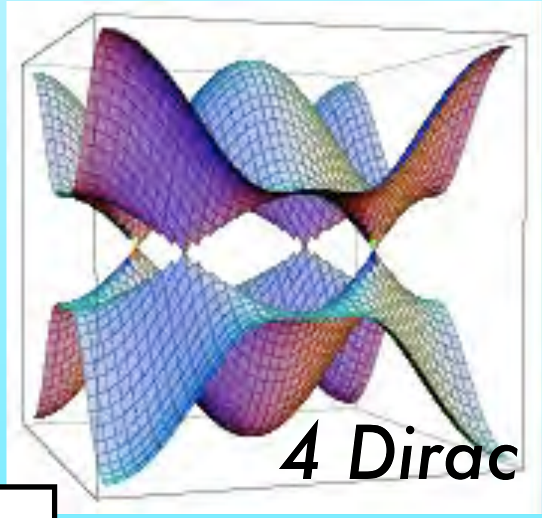
graphene

d-wave superconductor



2 Dirac cones

These 2 systems are topologically equivalent



4 Dirac cones

Symmetry protected Zero modes of Dirac fermions :1D Flat Band of edge states

Γ :Bipartite

(A-B sublattice symmetry)

Γ:Time Reversal

(Real Order parameter)

$^\exists\Gamma$ chiral symmetry

$$\{\Gamma, H\} = \Gamma H + H\Gamma = 0\ ,\ \Gamma^2 = 1$$

When the zero modes exist ?

Lattice analogue of
Witten's SUSY QM

S.Ryu & Y.Hatsugai, Phys. Rev. Lett. 89, 077002 (2002)
Y.Hatsugai., J. Phys. Soc. Jpn. 75 123601 (2006)
Kuge, Maruyama, Y. Hatsugai, arXiv:0802.2425

Edge states with boundaries
Determined by the Berry phase of the bulk (without boundaries)

Zak $\gamma = \int A = \int d\vec{k}\cdot\vec{A}$ $\quad \vec{A} = \langle\psi(k)|\vec{\nabla}_k\psi(k)\rangle$

Require Local Chiral Symmetry
(ex. bipartite)
$\{\Gamma, H\} = \Gamma H + H\Gamma = 0$

⇒

Quantized
$$\gamma = \int A = \begin{cases} \pi \\ 0 \end{cases}$$

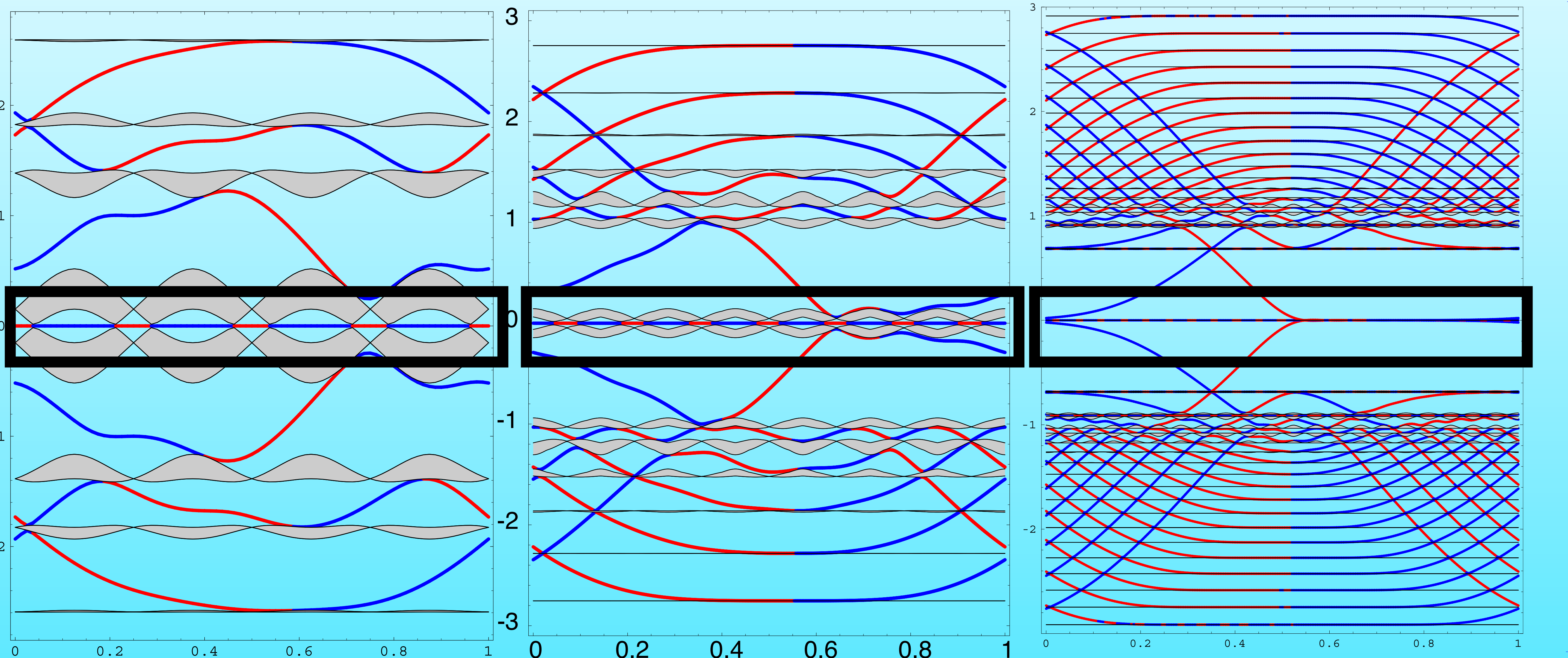
$\gamma = \pi$ ⇒ **Zero energy localized states EXIST**

: There exists odd number of zero modes

Bulk-edge correspondence: "Bulk determines the edges"

Close look at E=0

☆ Graphene



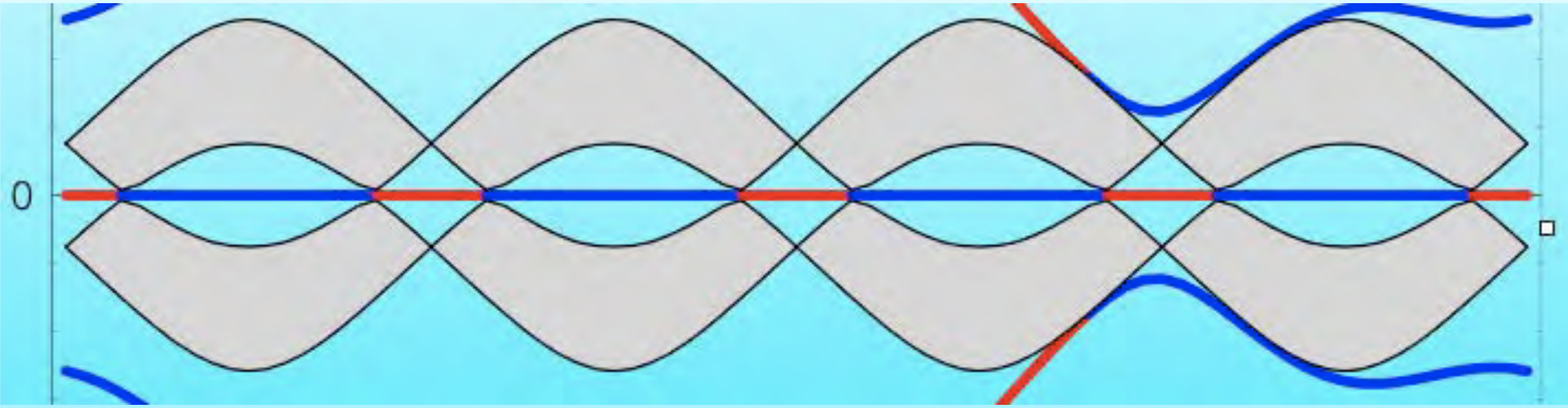
$\phi = 1/4$ $\phi = 1/7$ $\phi = 1/21$

strong ⟷ weak

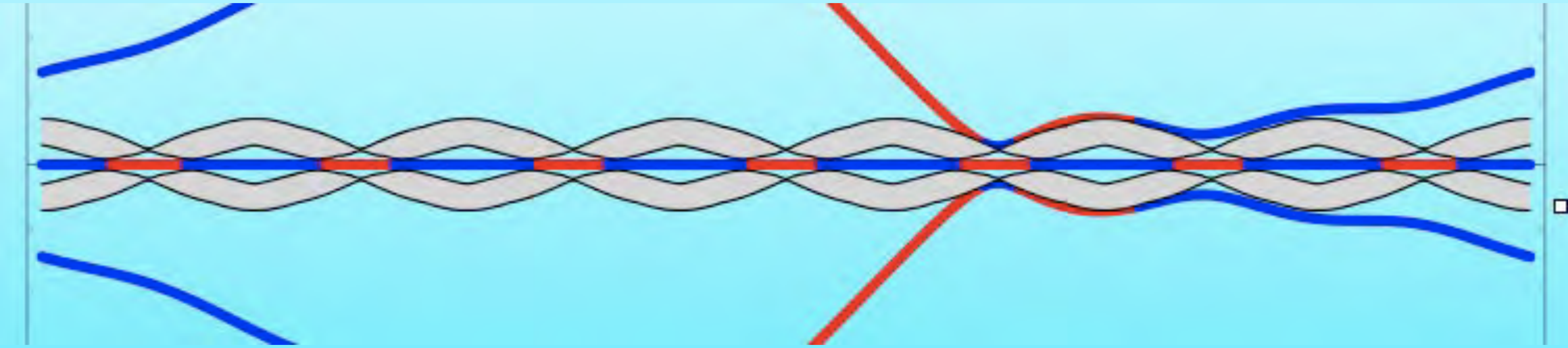
Close look at E=0

☆ *n=0 Landau Level*

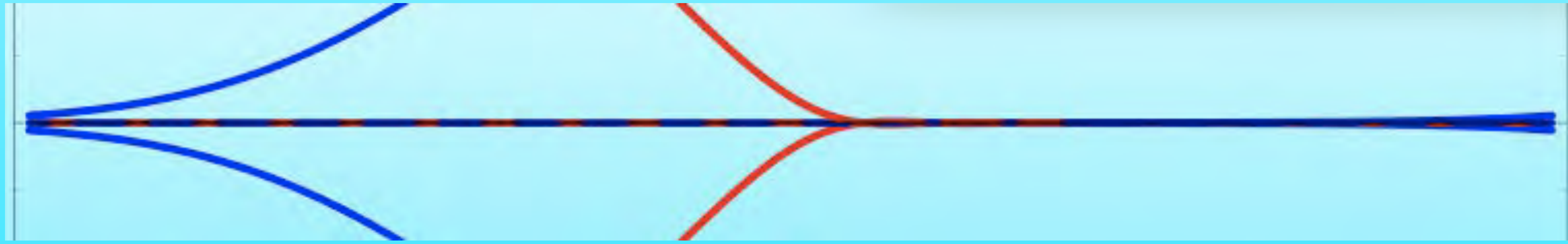
$\phi = 1/4$



$\phi = 1/7$



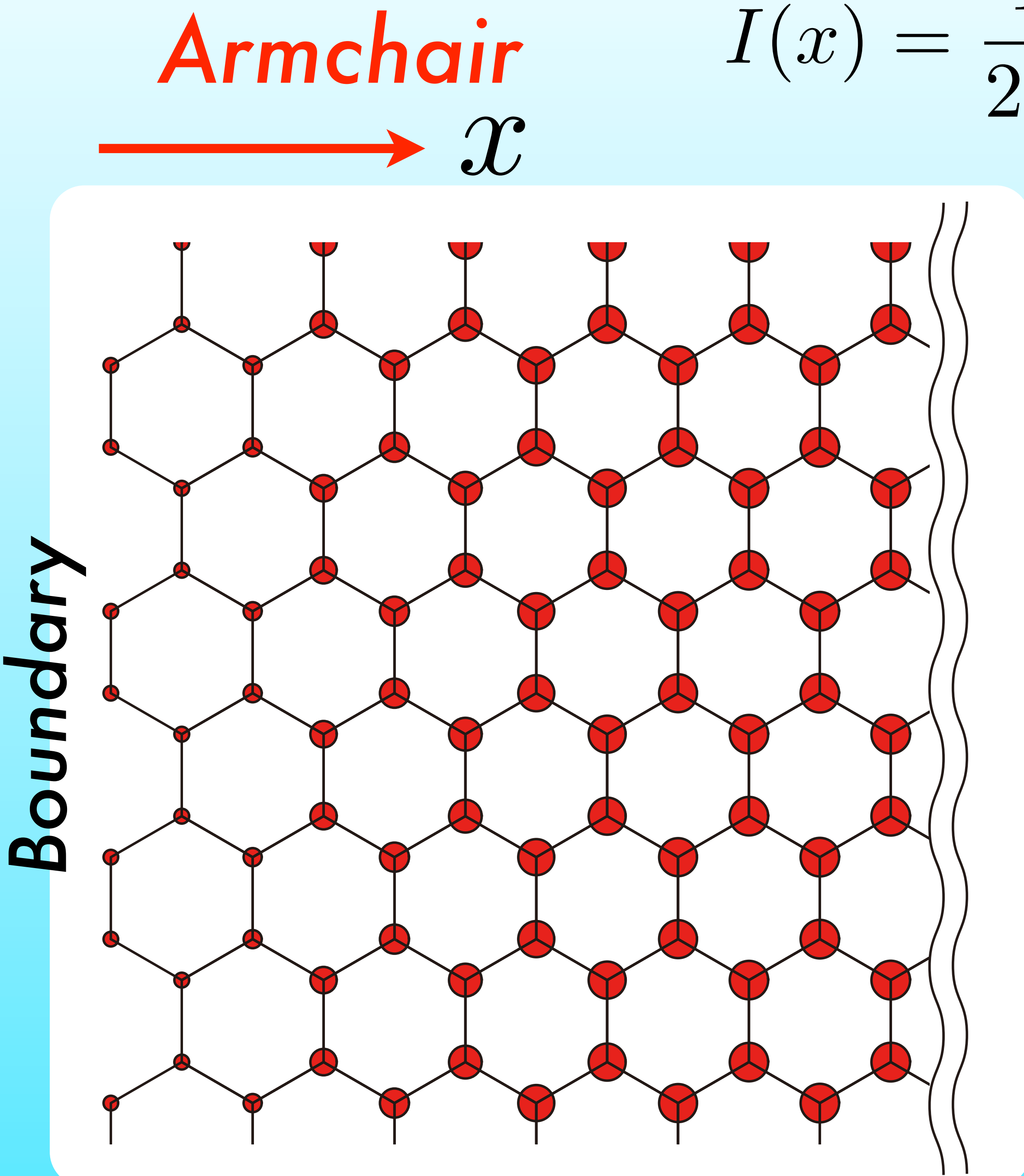
$\phi = 1/21$

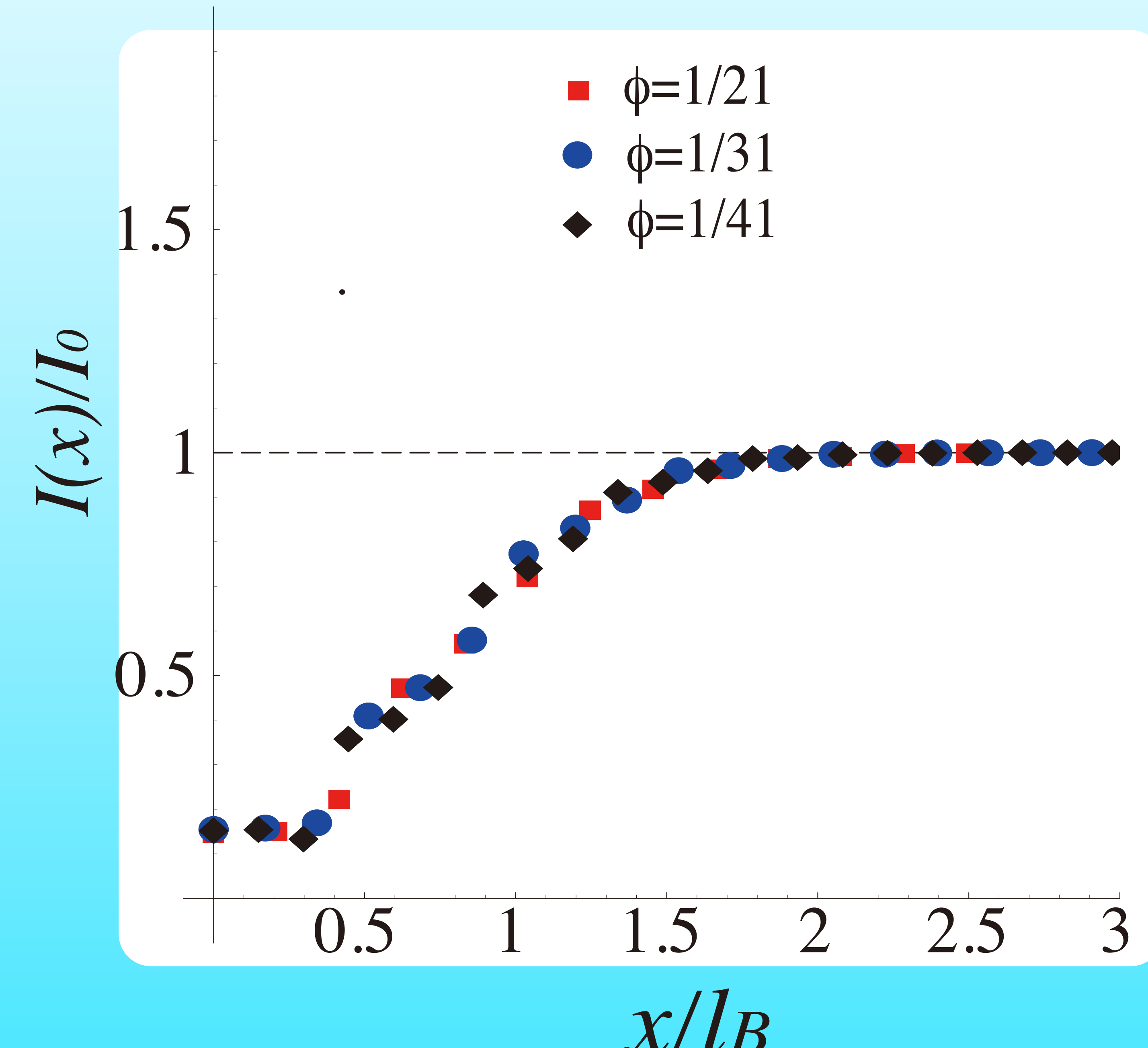


Bulk Landau Level and the zero mode edge states coexist

LDOS around E=0 with Landau Level

$$I(x) = \frac{1}{2\pi}\int_{-E_C}^{E_C} dE \int_{-\pi}^{\pi} dk_y \, |\Psi(x, k_y, E)|^2$$





Suppression near the edge

Standard behavior due to edge potential

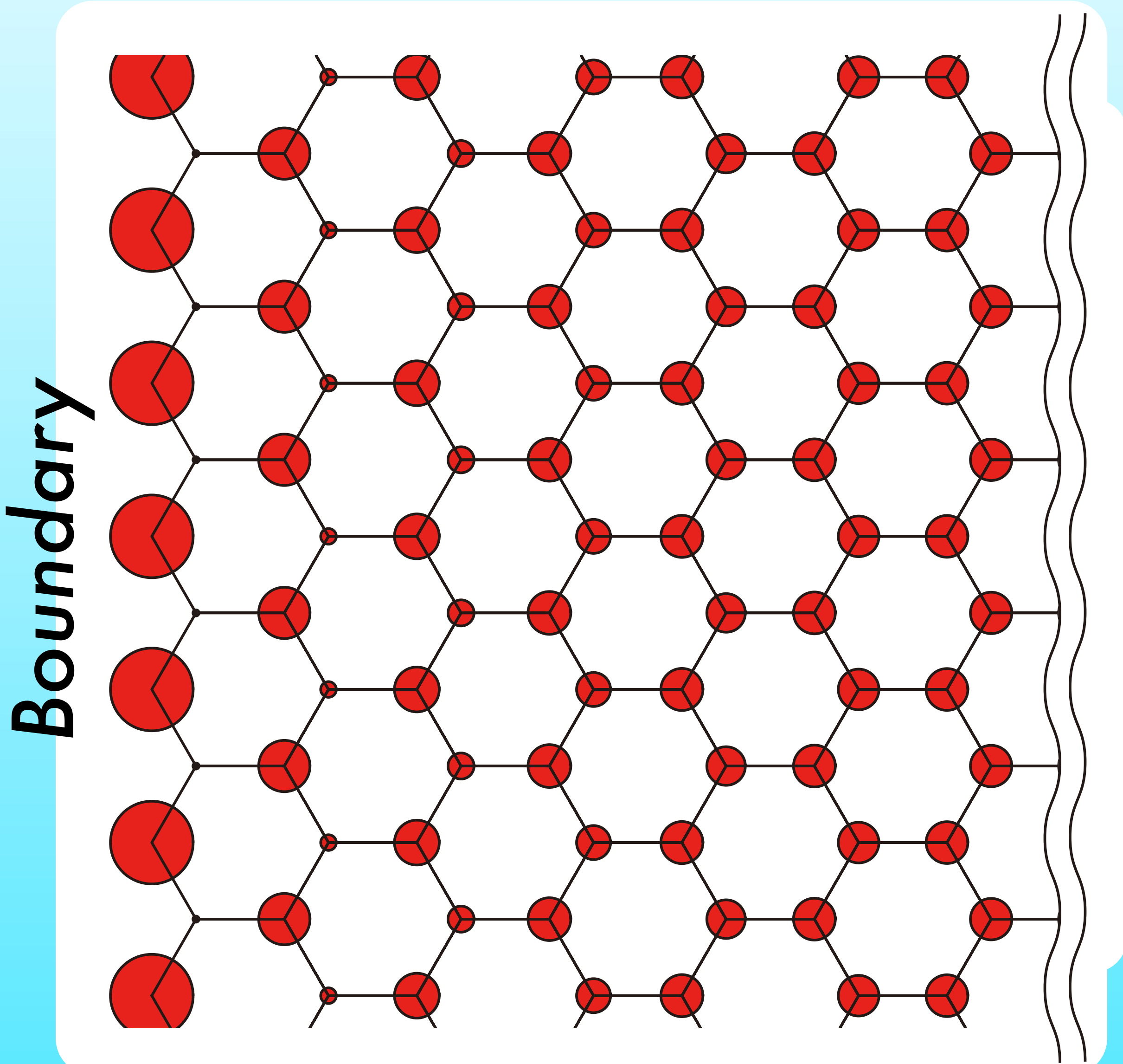
LDOS around E=0 with Landau Level

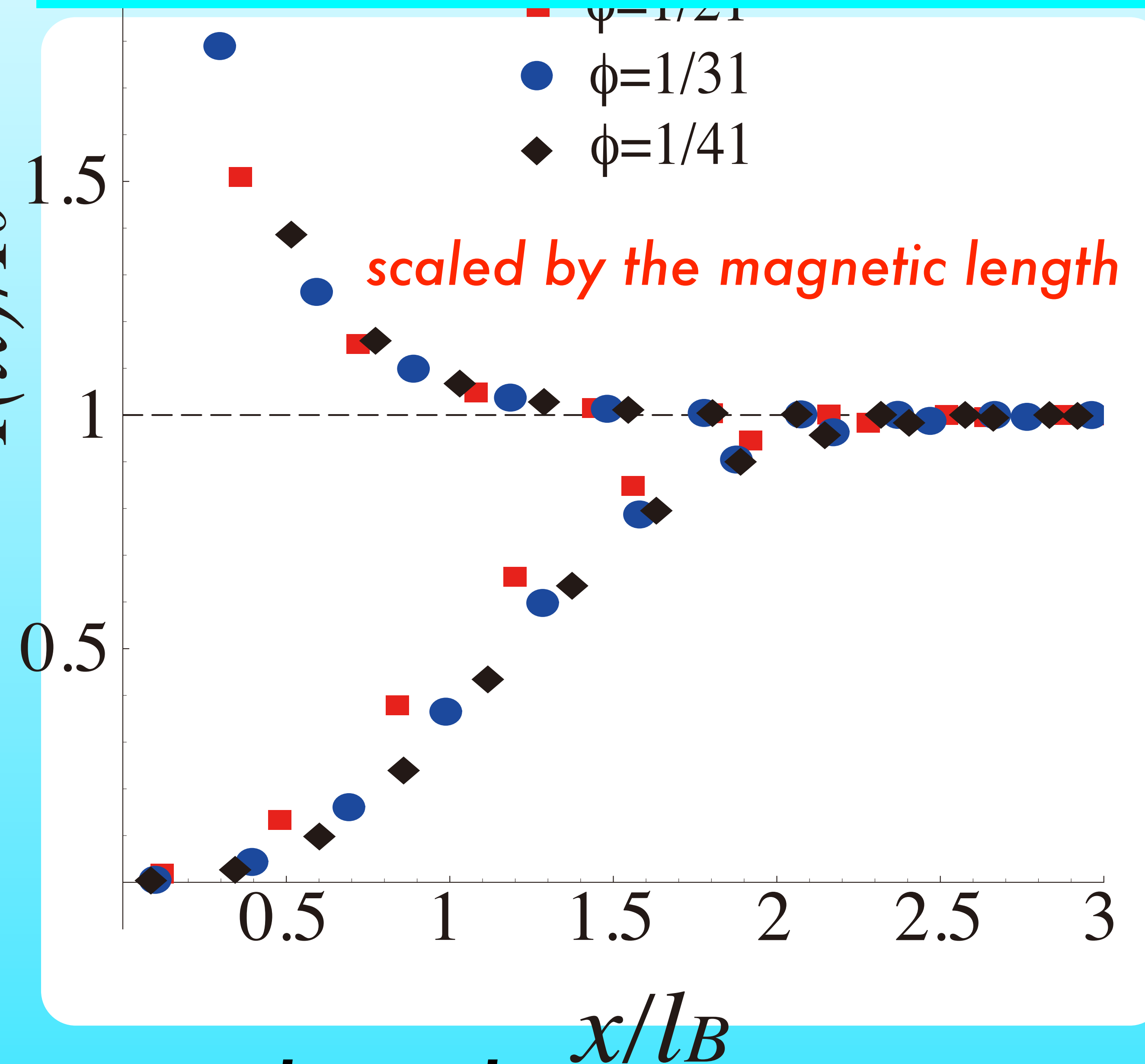
Zigzag x

$$I(x) = \frac{1}{2\pi}\int_{-E_C}^{E_C} dE \int_{-\pi}^{\pi} dk_y \,|\Psi(x,k_y,E)|^2$$

M. Arikawa, H. Aoki & YH

Phys. Rev. B79, 075429 (2009),arXiv: & 0806.2429





STM observable

Strong enhancement near the edge

Characteristic feature of the Graphene Zigzag edges!

Bulk-Edge correspondence: Dirac fermions

2D Dirac fermions

Graphene d-wave superconductor surfaces of 3D TI ...

Edge states: quantum Hall edge state $B \neq 0$

zero mode localized states $B = 0$

Universality in the zero modes
of Dirac Fermions

Applications

Use of the edge states

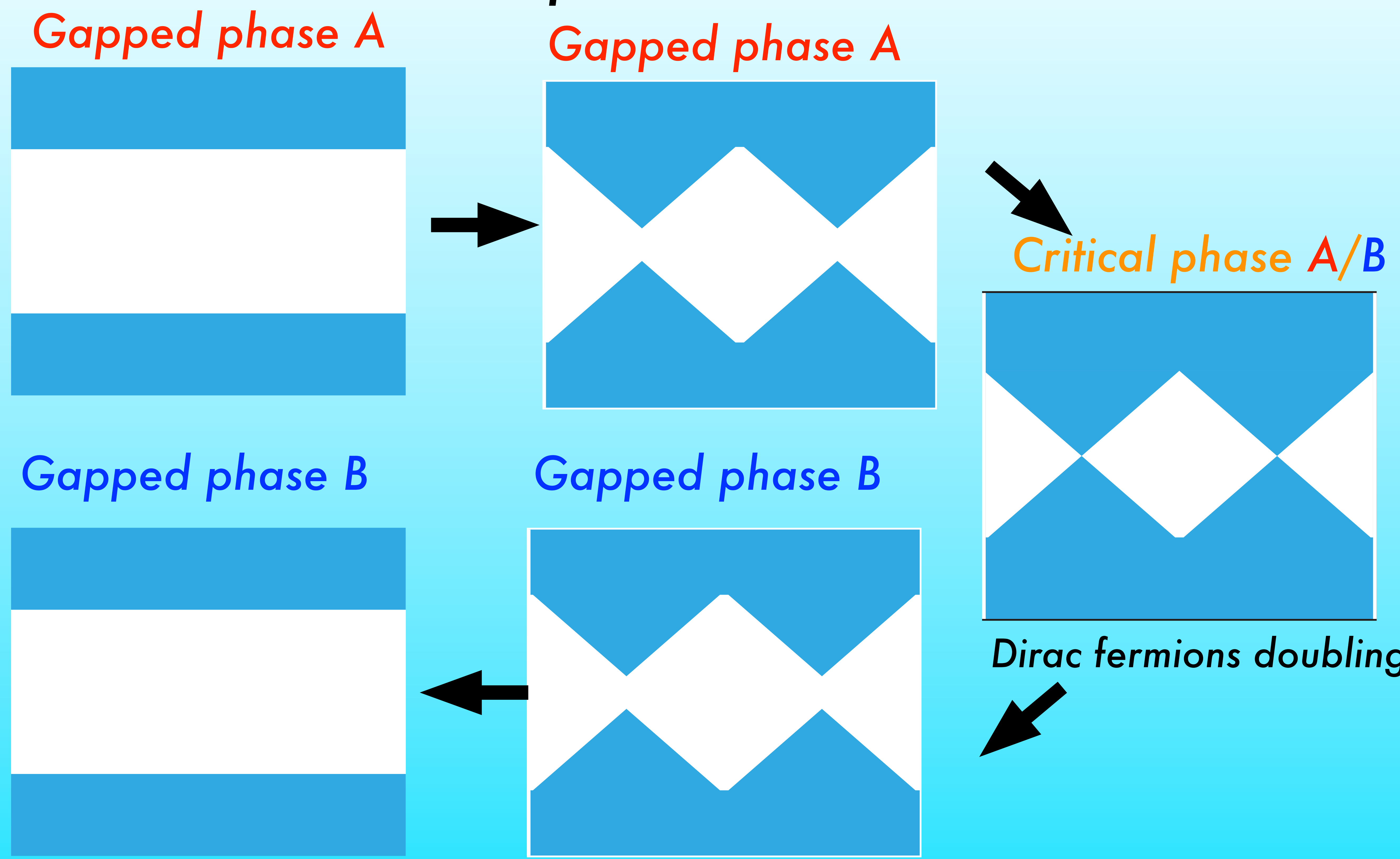
- *Edge states in the topological quantum phase transition*

- *Edge states to see 1/2 Hall conductance of Dirac fermions*

- *Adiabatic transport among edge states*

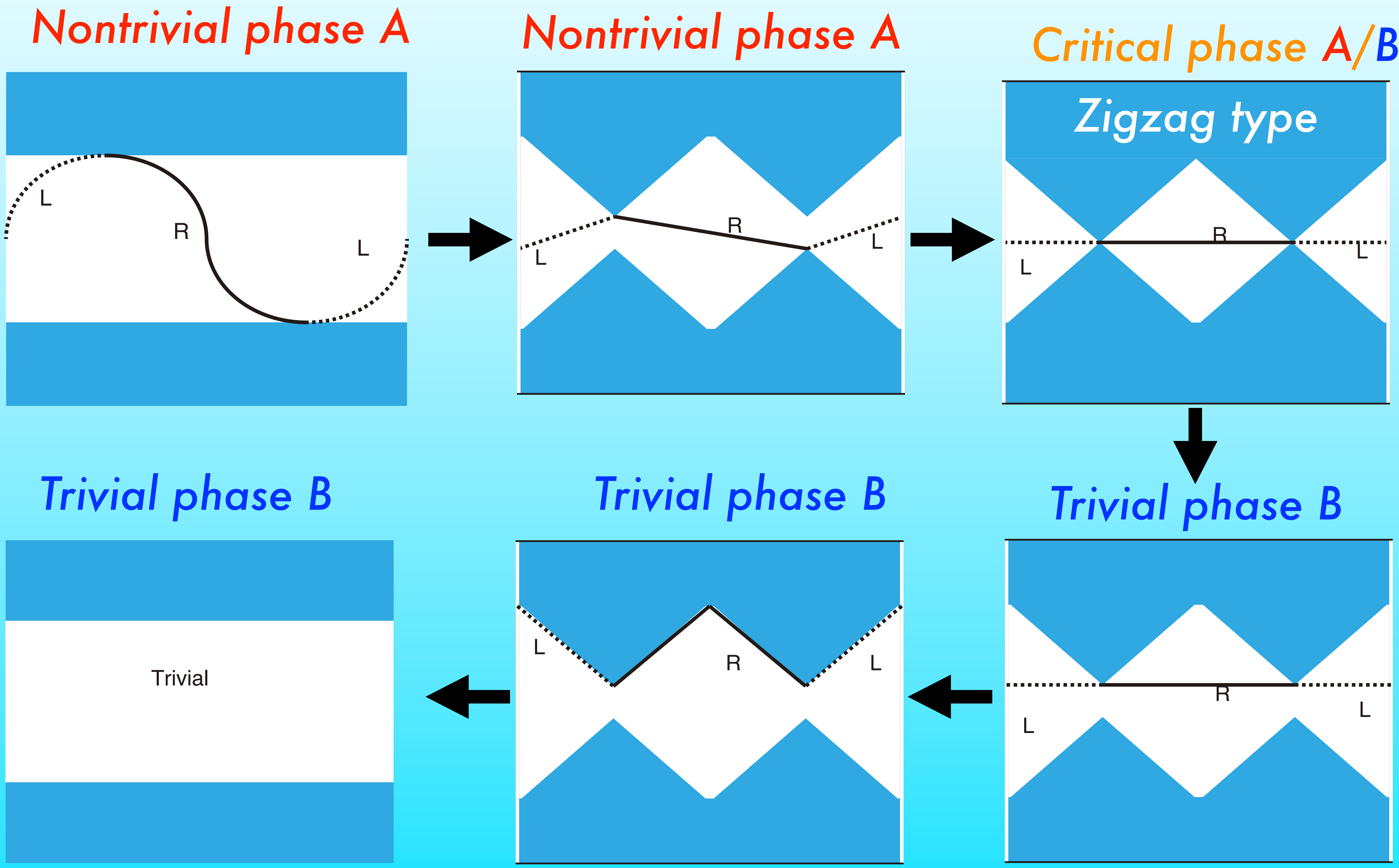
Zero modes as critical edge states

Quantum phase transition: Bulk



Zero modes as critical edge states : Zigzag type

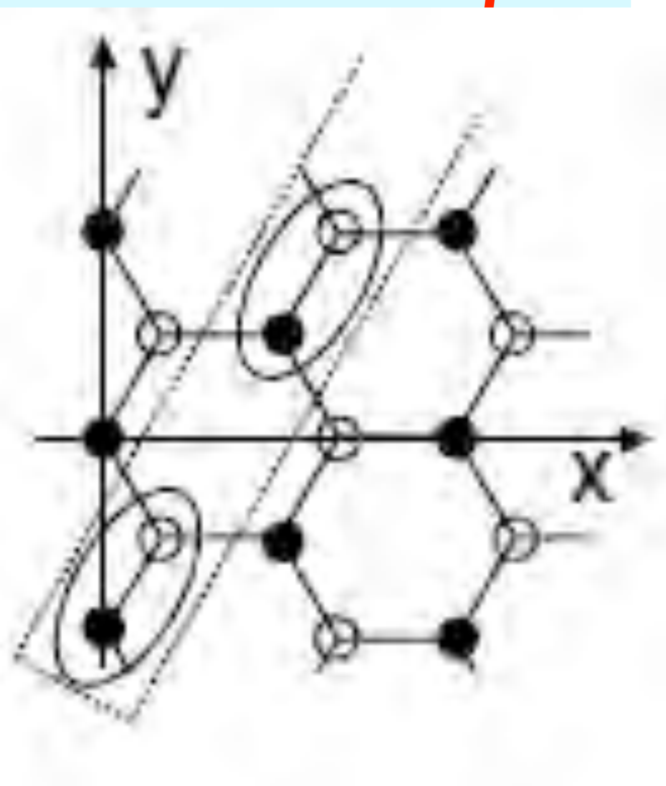
Topological quantum phase transition: Edge



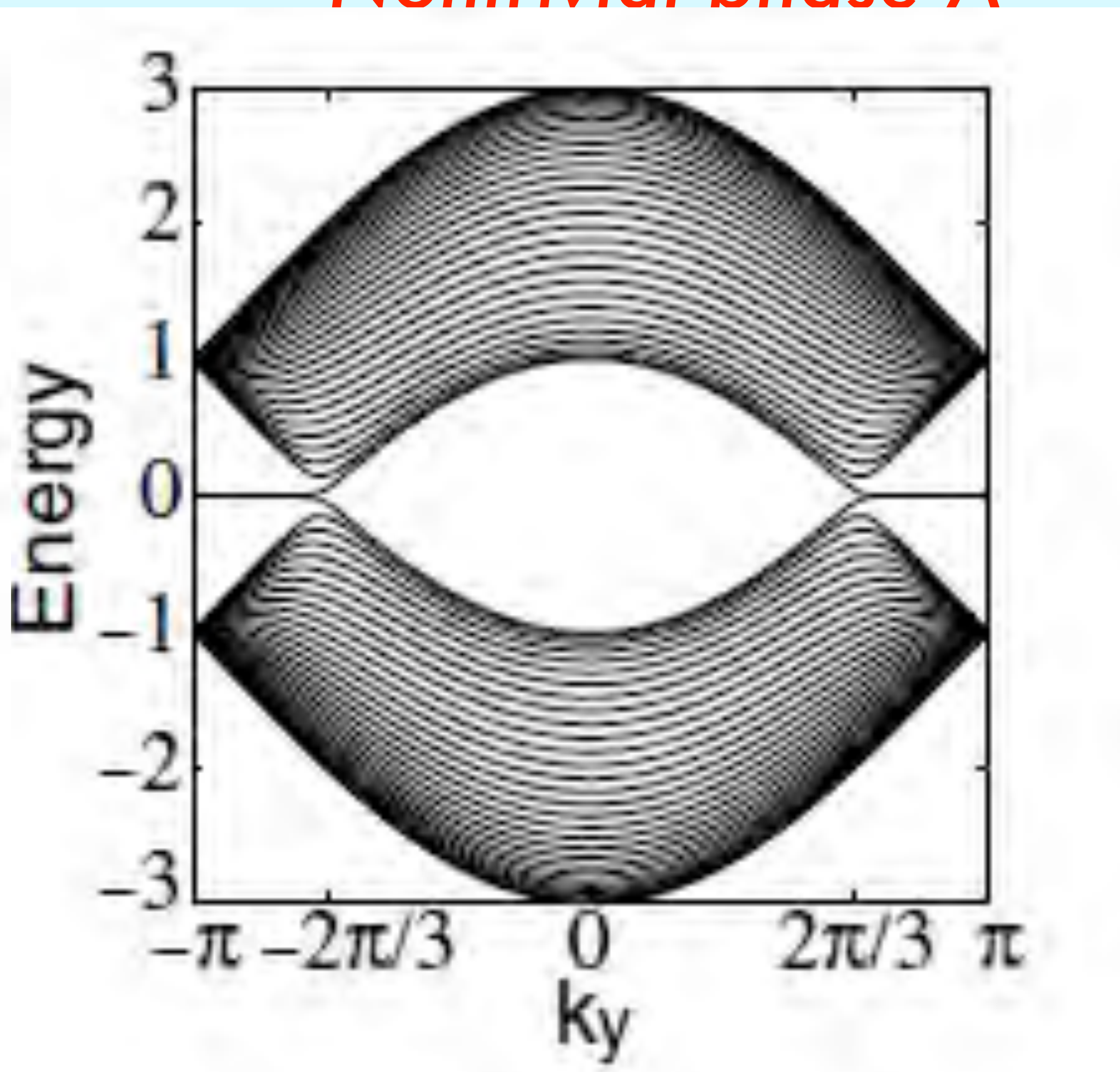
Zero modes as critical edge states : Zigzag type

Topological quantum phase transition: Edge

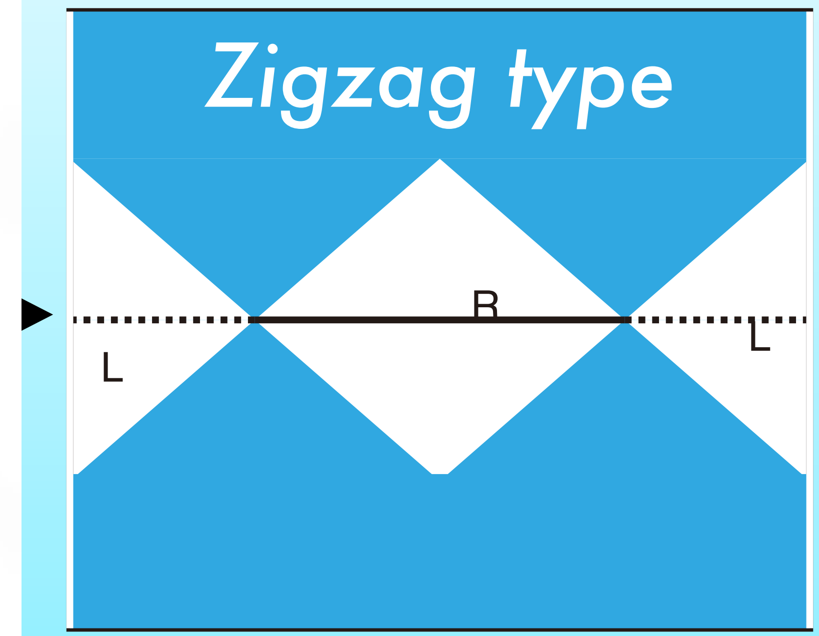
Nontrivial phase A



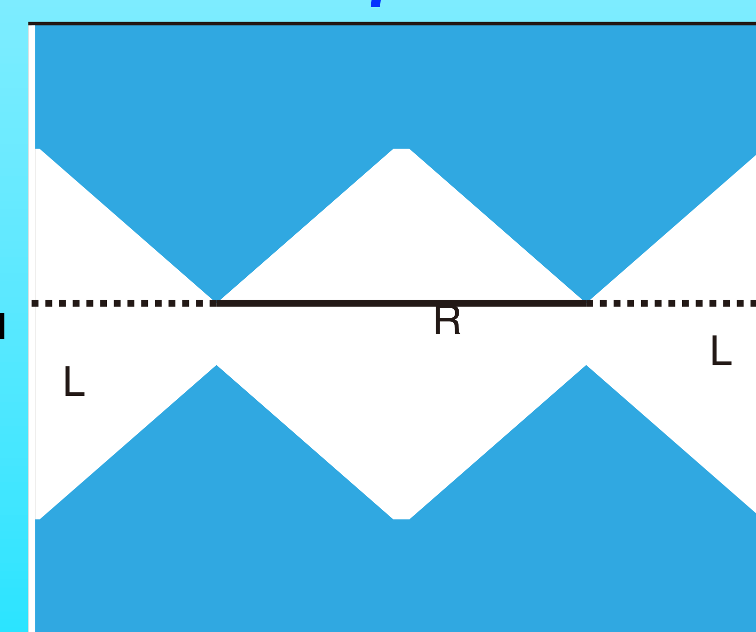
Nontrivial phase A

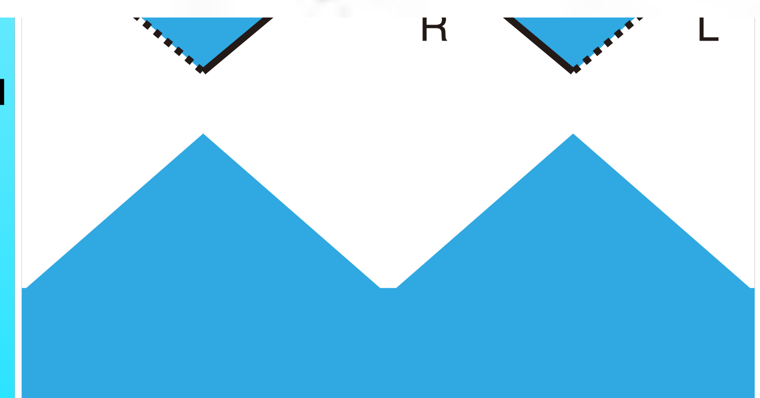


Critical phase A/B

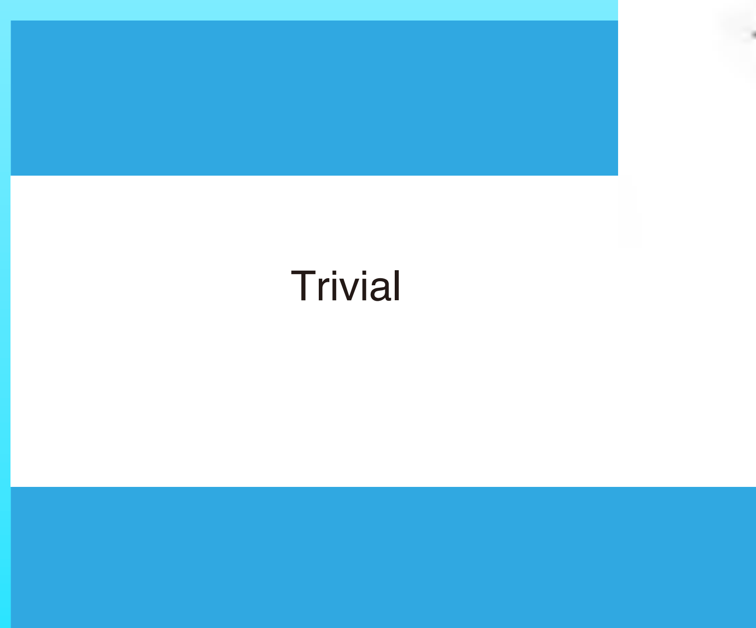


Trivial phase B



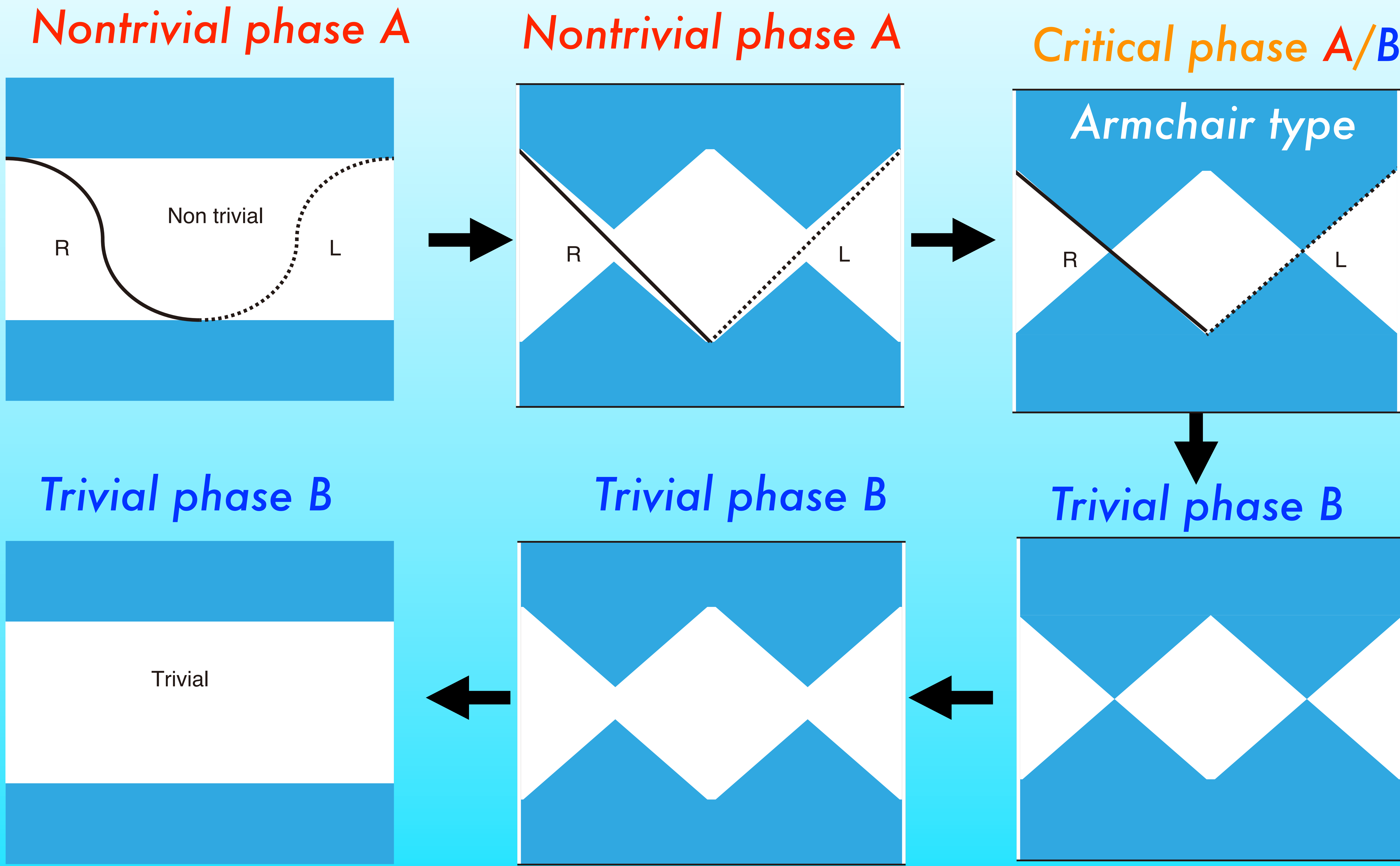


Trivial phase B



Zero modes as critical edge states: Armchair type

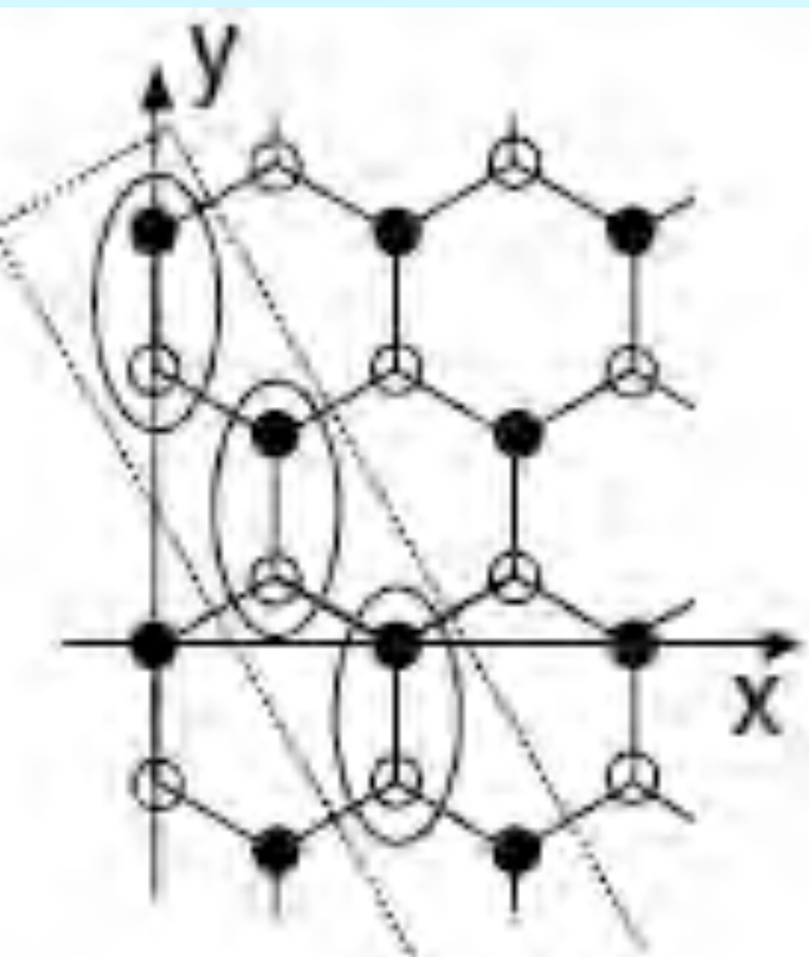
Topological quantum phase transition: Edge



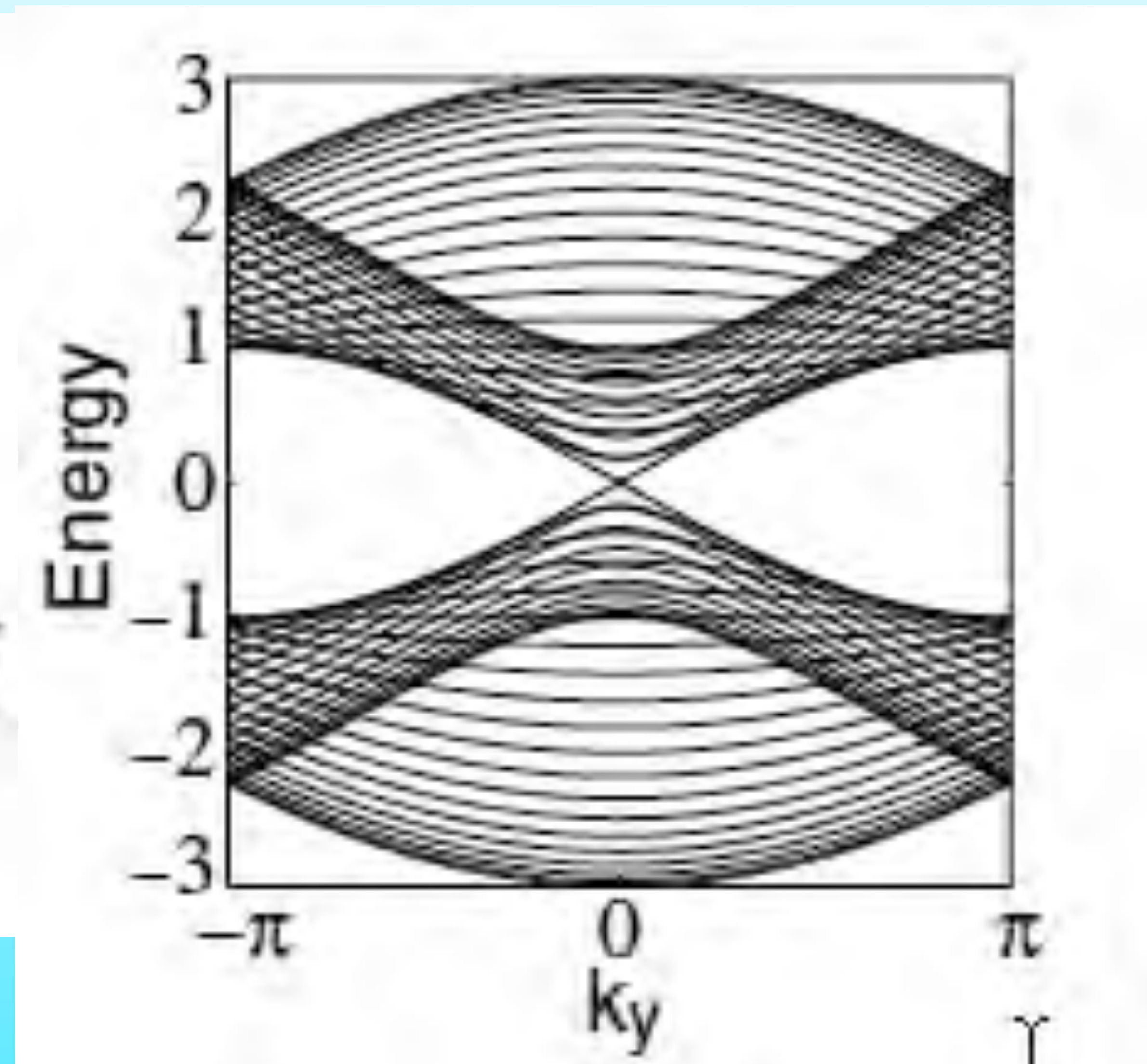
Zero modes as critical edge states: Armchair type

Topological quantum phase transition: Edge

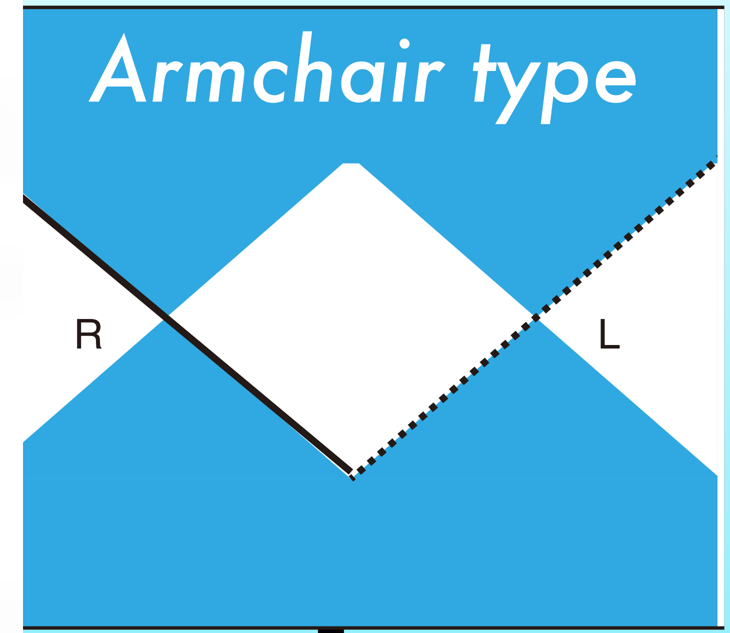
Nontrivial phase A



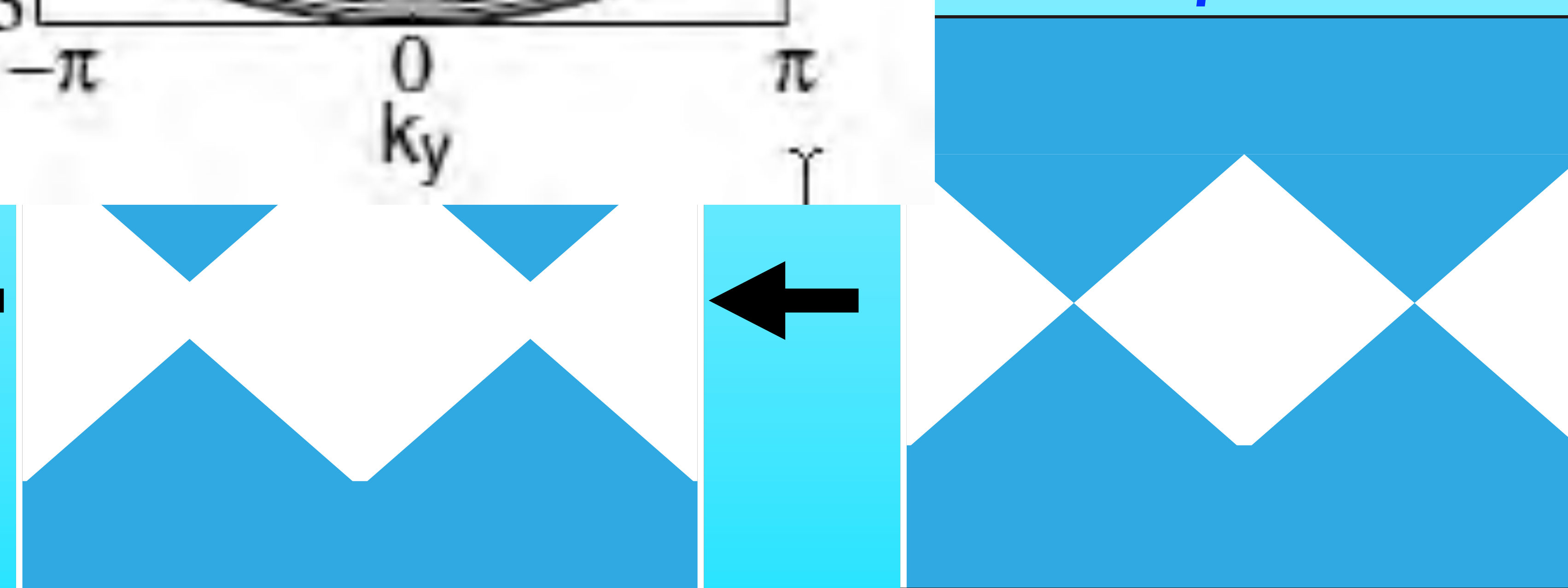
Nontrivial phase A

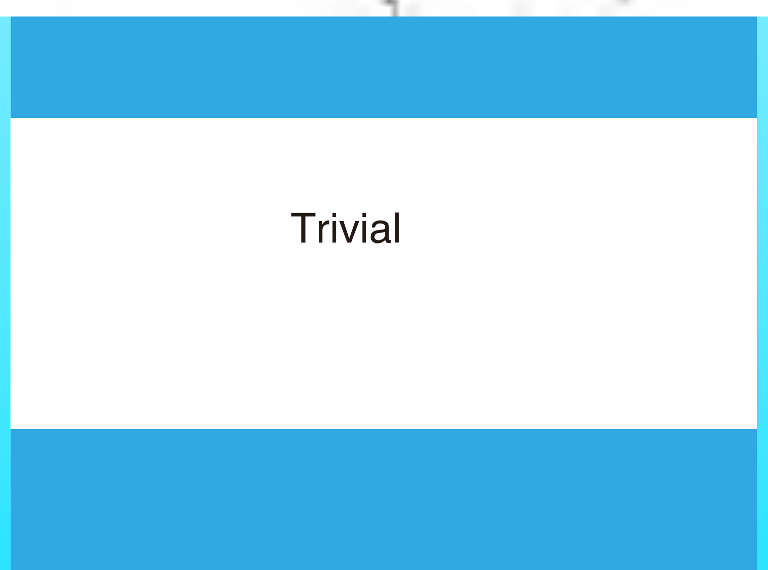


Critical phase A/B



Trivial phase B





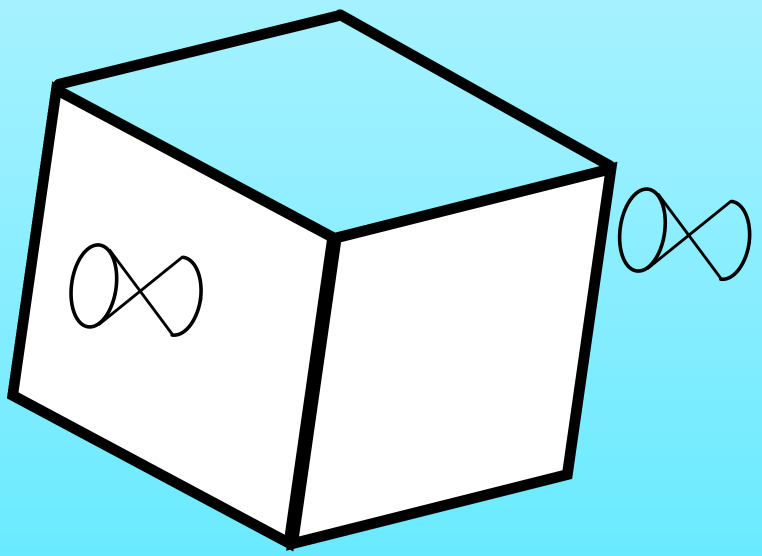
How to see 1/2 Hall conductance of Dirac fermions

$$\sigma_{xy} = \frac{e^2}{h}(n + \frac{1}{2}), n = 0, 1, 2, \cdots$$

with inevitable (topological) fermion doubling

$$\sigma_{xy} = 2\frac{e^2}{h}(n + \frac{1}{2}) = \frac{e^2}{h}(2n + 1)$$: *integer*

It can not be odd/2 since the Chern number is integer



surfaces of the TI

This is also doubling

c.f. domain wall fermions in lattice gauge theory

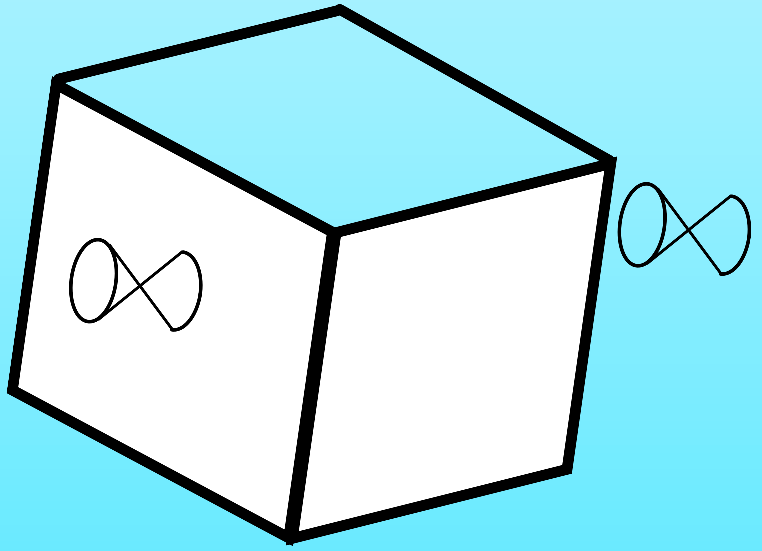
How to see 1/2 Hall conductance of Dirac fermions

$$\sigma_{xy} = \frac{e^2}{h}(n + \frac{1}{2}), n = 0, 1, 2, \cdots$$

with inevitable (topological) fermion doubling

$$\sigma_{xy} = 2\frac{e^2}{h}(n + \frac{1}{2}) = \frac{e^2}{h}(2n + 1)$$: integer

It can not be odd/2 since the Chern number is integer



surfaces of the TI

This is also doubling

c.f. domain wall fermions in lattice gauge theory

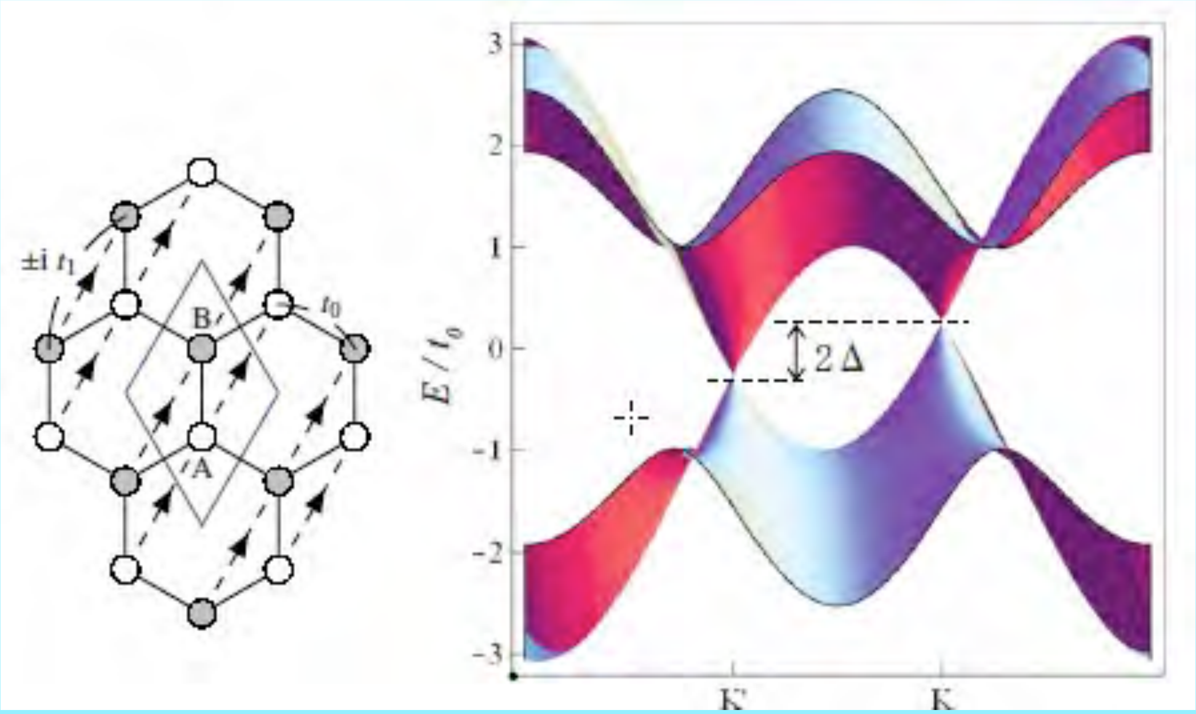
How to observe ??

Continuously shifts the Dirac cones !

H. Watanabe, YH, H.Aoki, Phys.Rev.B82, 24140(R), (2010)

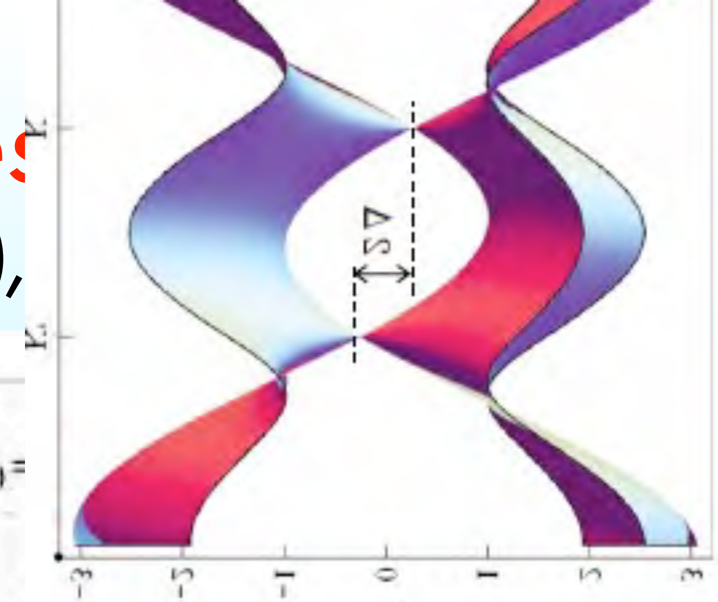
Continuously shifts the Dirac cones !

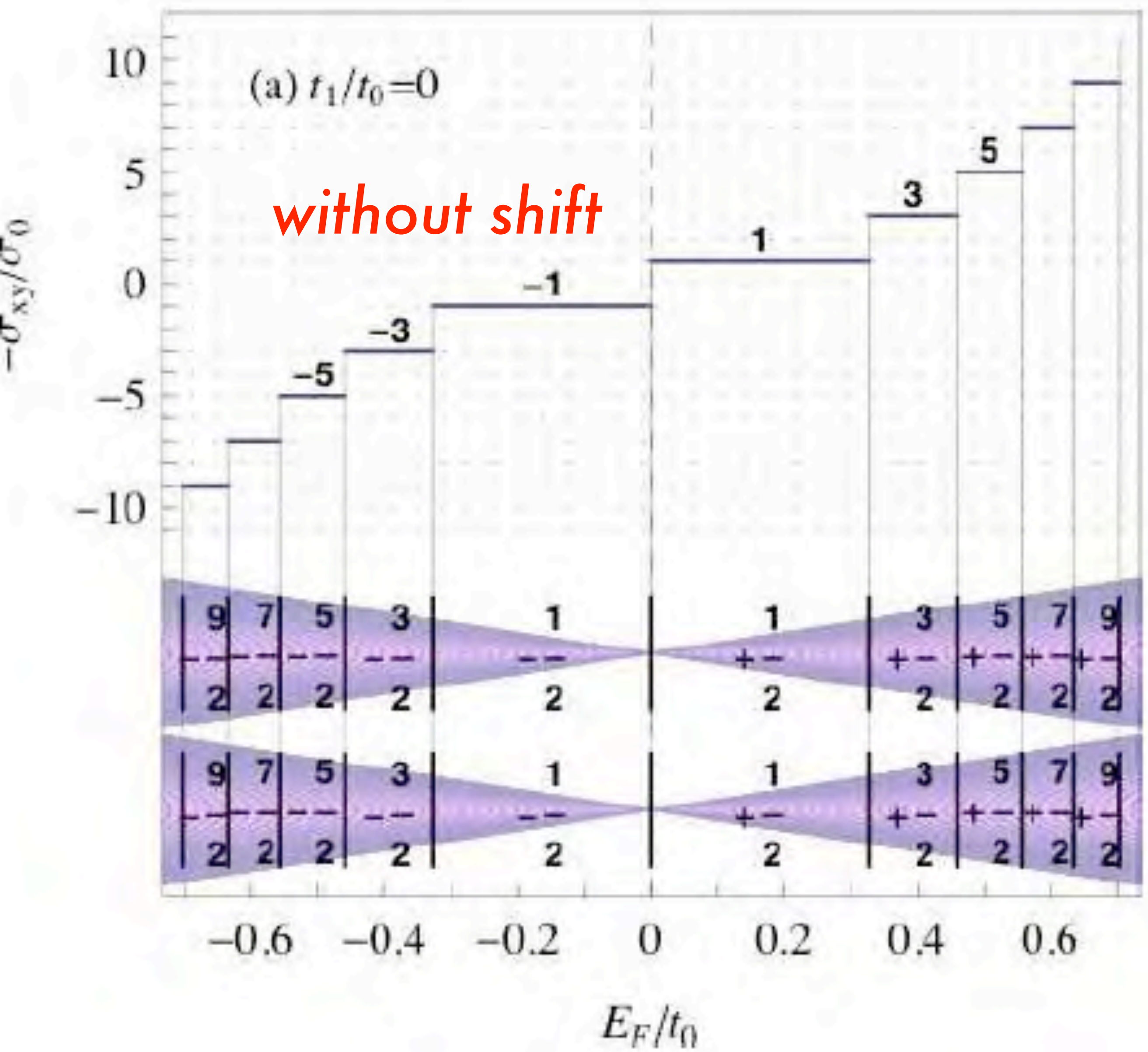
H. Watanabe, YH, H.Aoki, Phys.Rev.B82, 24140(R), (2010)

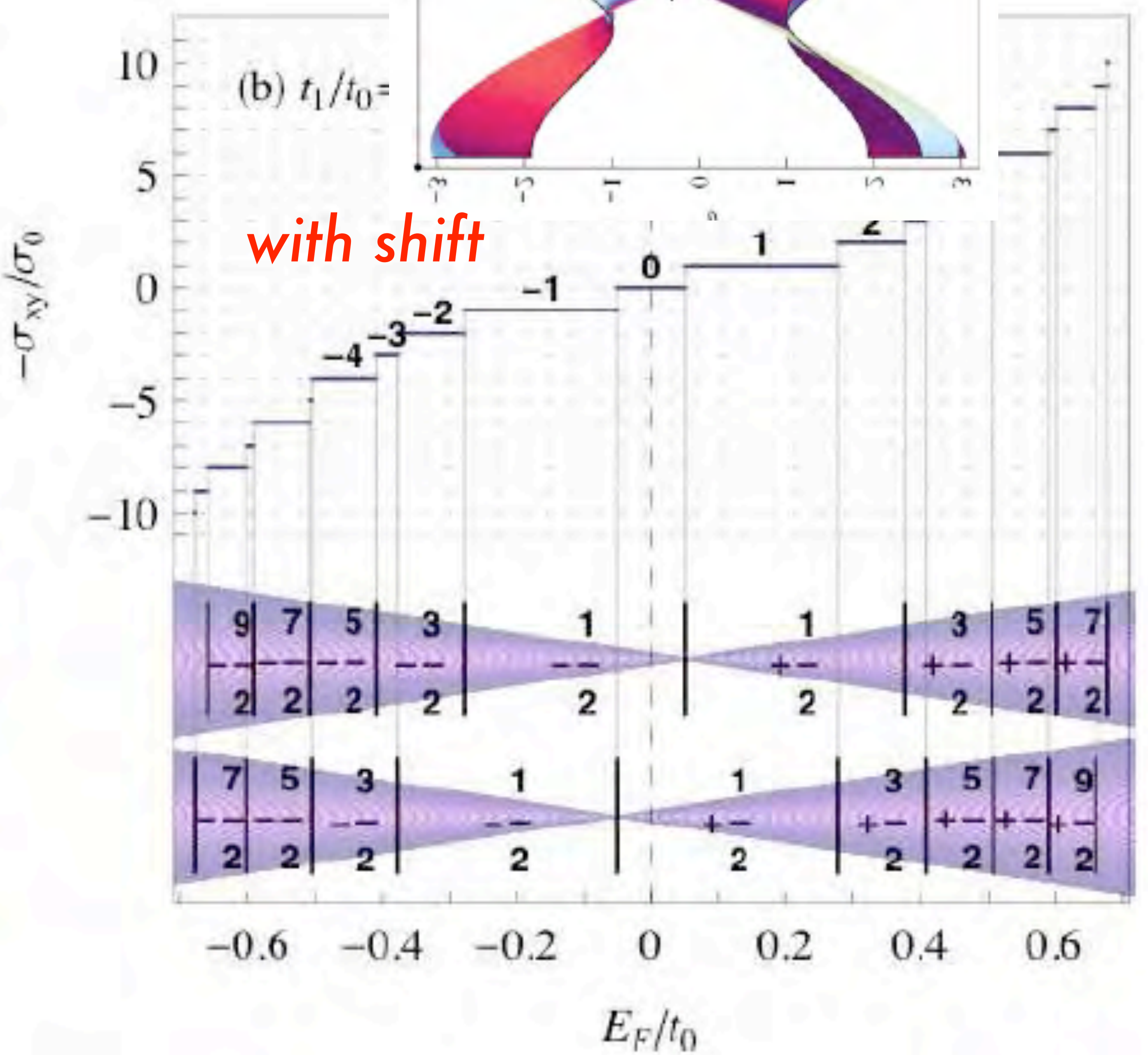


Continuously shifts the Dirac cones

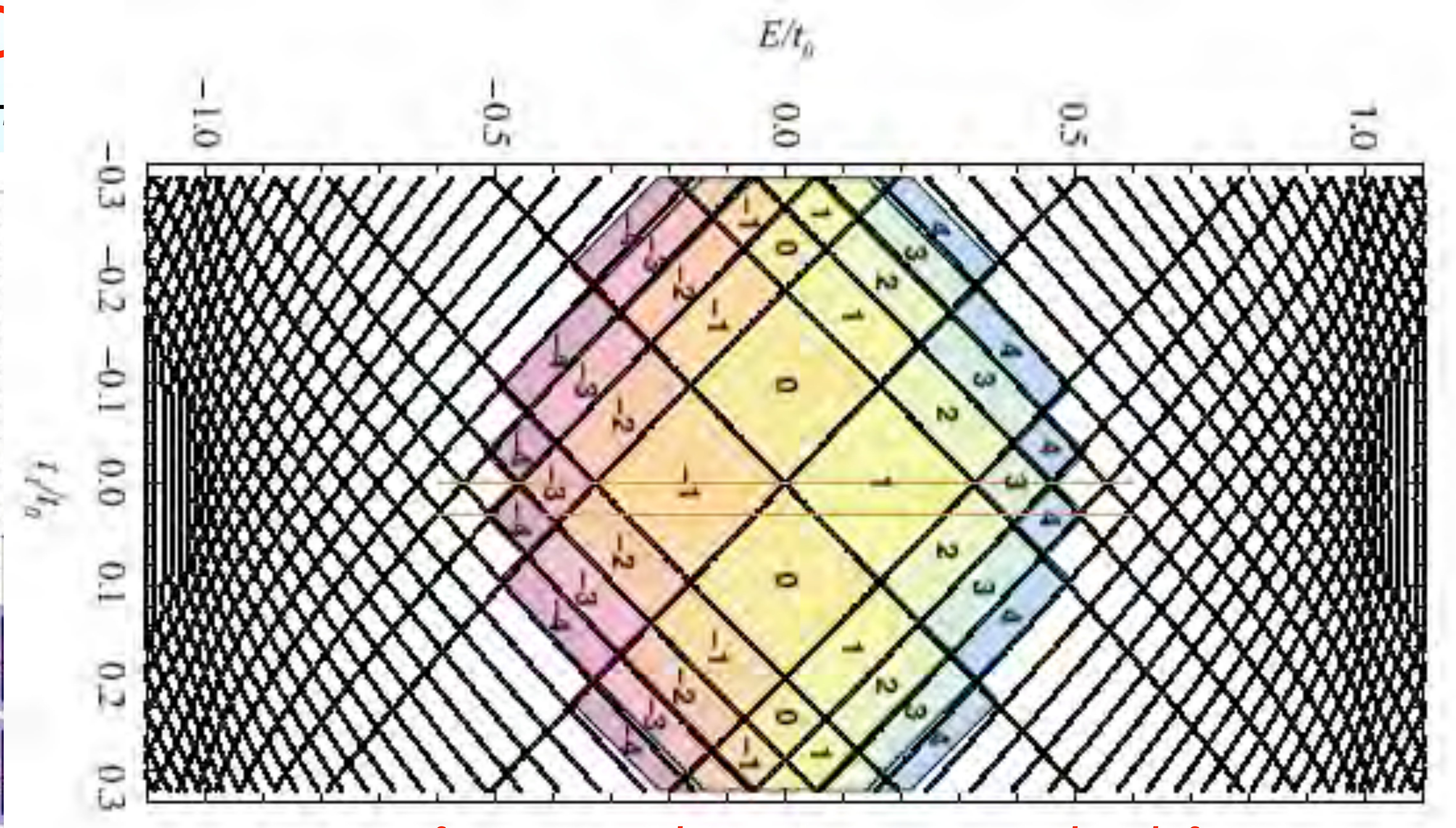
H. Watanabe, YH, H.Aoki, Phys.Rev.B82, 24140(R),





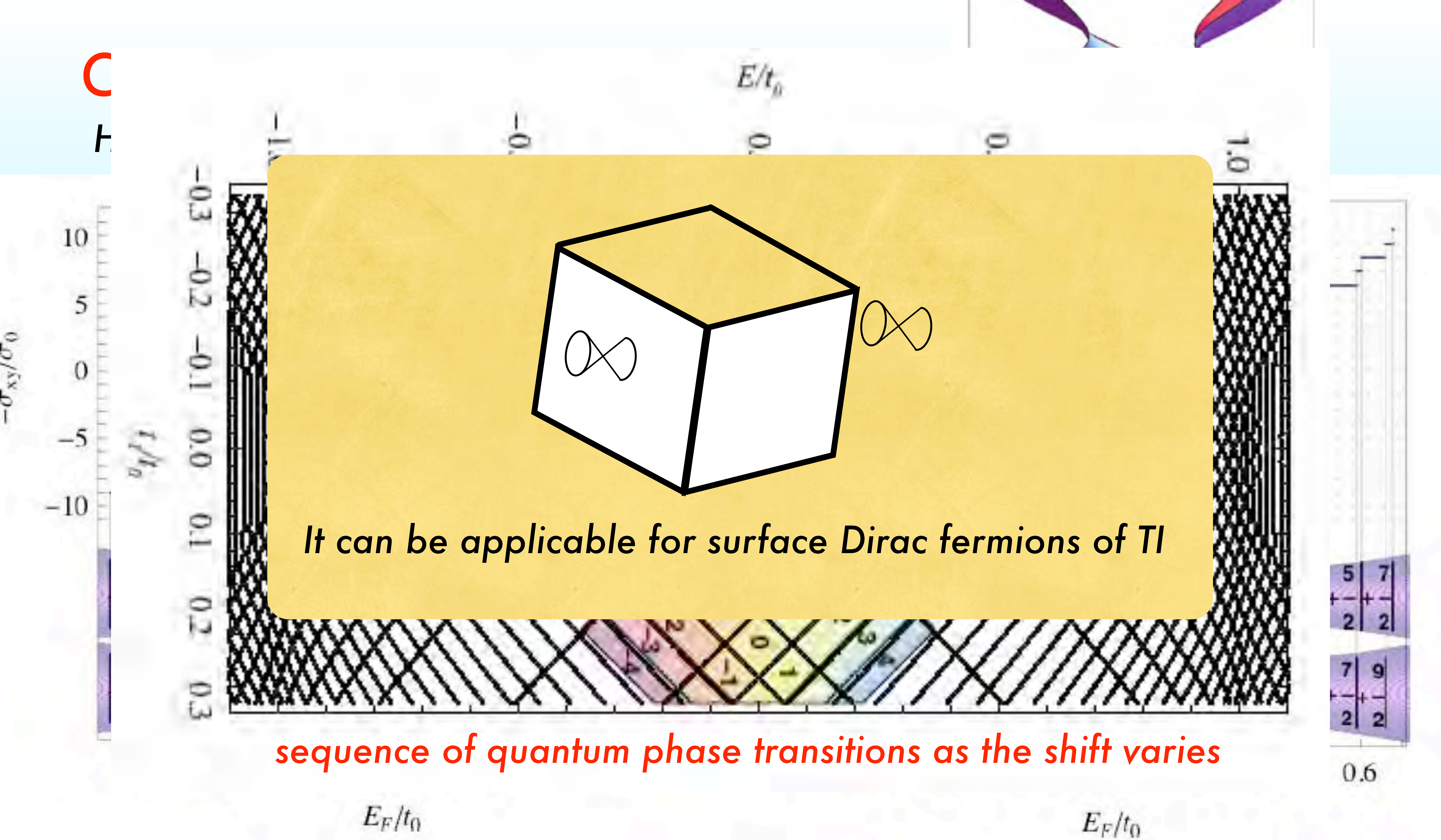


Integers to integers transitions as governed
by the Dirac fermion's $\sqrt{n}$ *rule of the Landau levels*



sequence of quantum phase transitions as the shift varies

Integers to integers transitions as governed by the Dirac fermion's $\sqrt{n}$ rule of the Landau levels

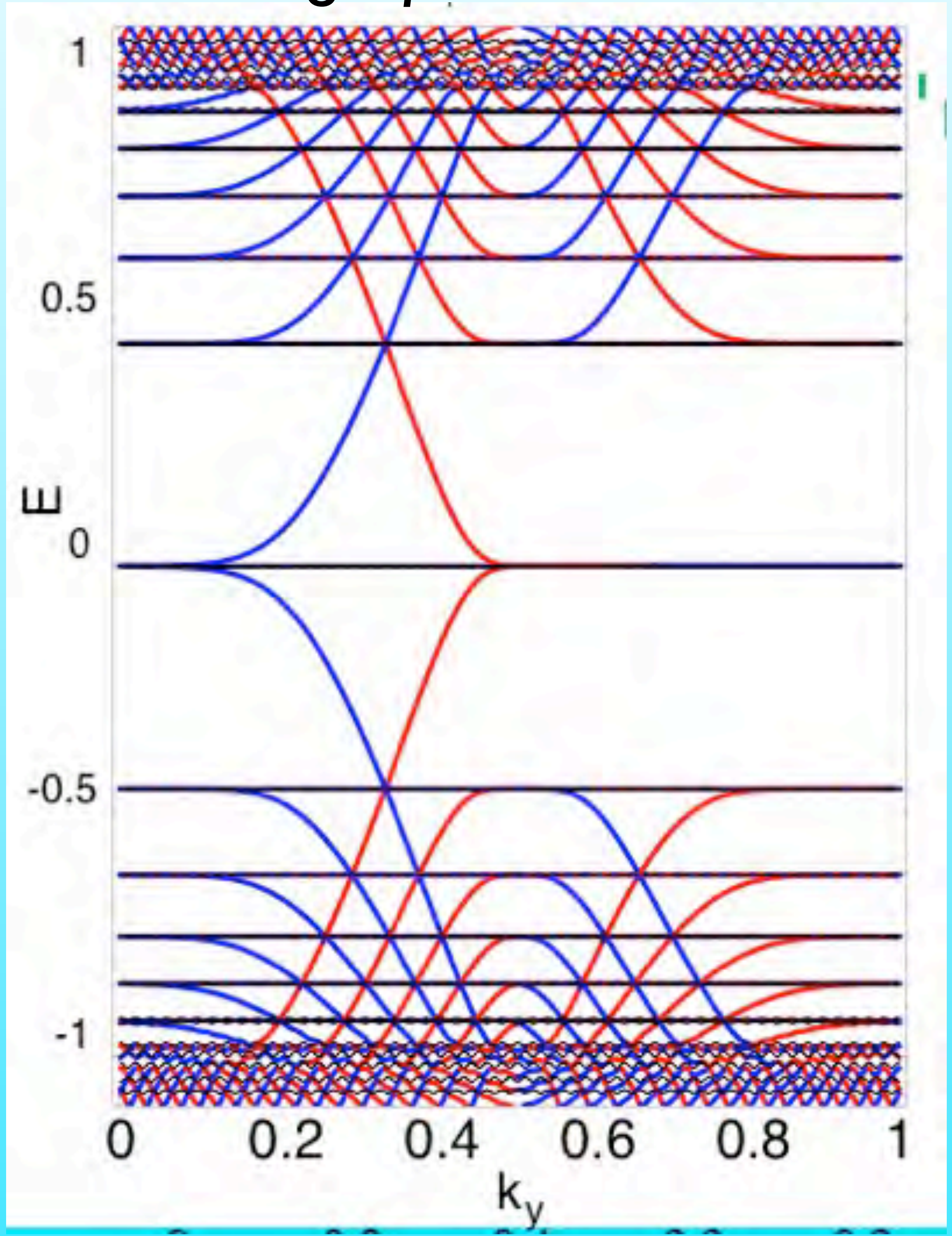


Integers to integers transitions as governed
by the Dirac fermion's $\sqrt{n}$ *rule of the Landau levels*

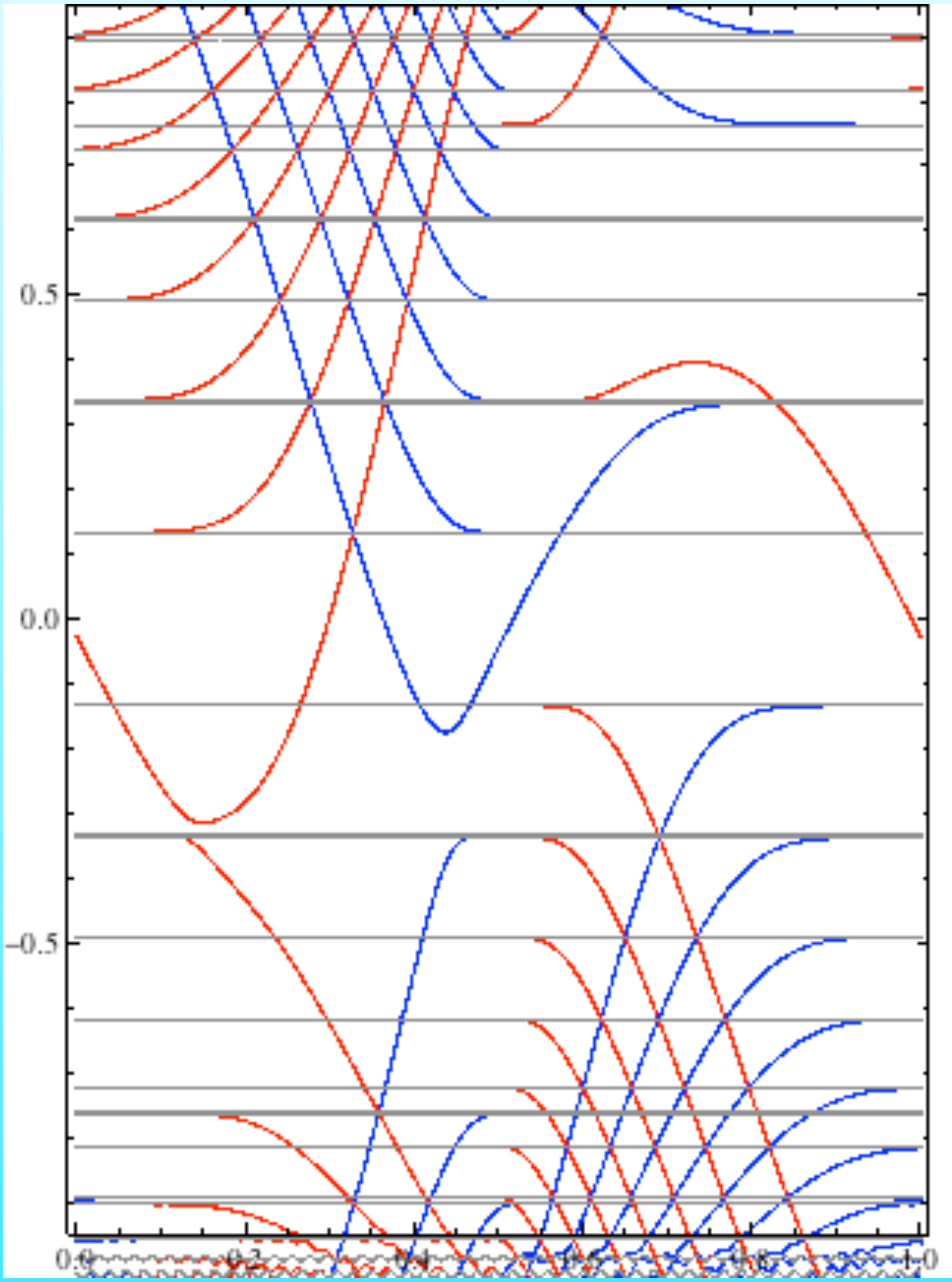
Edge states are also governed by the rule

H. Watanabe, YH, H.Aoki, Phys.Rev.B82, 24140(R), (2010)

graphene



Our model with continuous shift



Integers to integers transitions as governed by the Dirac fermion's $\sqrt{n}$ rule

Also some experiments in cold atoms in a non-Abelian optical lattice

PHYSICAL REVIEW A **84**, 023622 (2011)

Probing a half-odd topological number sequence with cold atoms in a non-Abelian optical lattice

Feng Mei,[1,3] Shi-Liang Zhu,[2] Xun-Li Feng,[1,3] Zhi-Ming Zhang,[1,*] and C. H. Oh[3,†]

[1]*Laboratory of Photonic Information Technology, LQIT & SIPSE, South China Normal University, Guangzhou 510006, China*

[2]*Laboratory of Quantum Information Technology and SPTE, South China Normal University, Guangzhou, China*

[3]*Centre for Quantum Technologies and Department of Physics, National University of Singapore, 3 Science Drive 2, Singapore 117543, Singapore*

(Received 27 April 2011; published 16 August 2011)

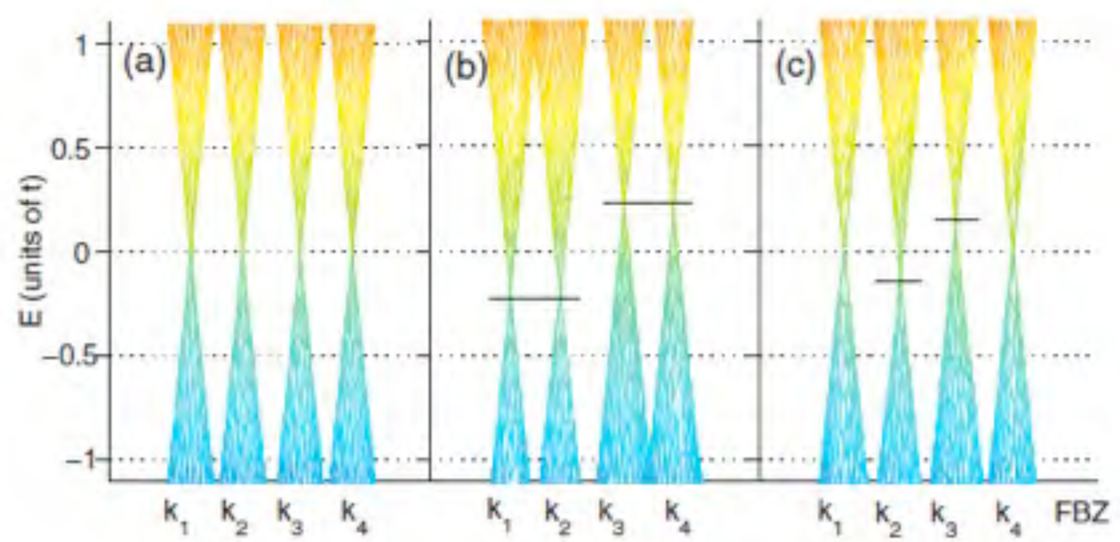


FIG. 1. (Color online) The dispersion relations in the vicinity of the four Dirac cones in the FBZ with different parameters α and β. (a) $\alpha = \beta = \pi/2$; (b) $\alpha = \pi/2 + 0.1$, $\beta = \pi/2$; (c) $\alpha = \pi/2 + 0.05$, $\beta = \pi/2 - 0.05$.

find that the energies of the Dirac points become

$$\delta E_{1(4)} = \pm 2t(\cos\alpha + \cos\beta),$$
$$\delta E_{2(3)} = \pm 2t(\cos\alpha - \cos\beta). \quad (2)$$

Use of the edge states

Edge states

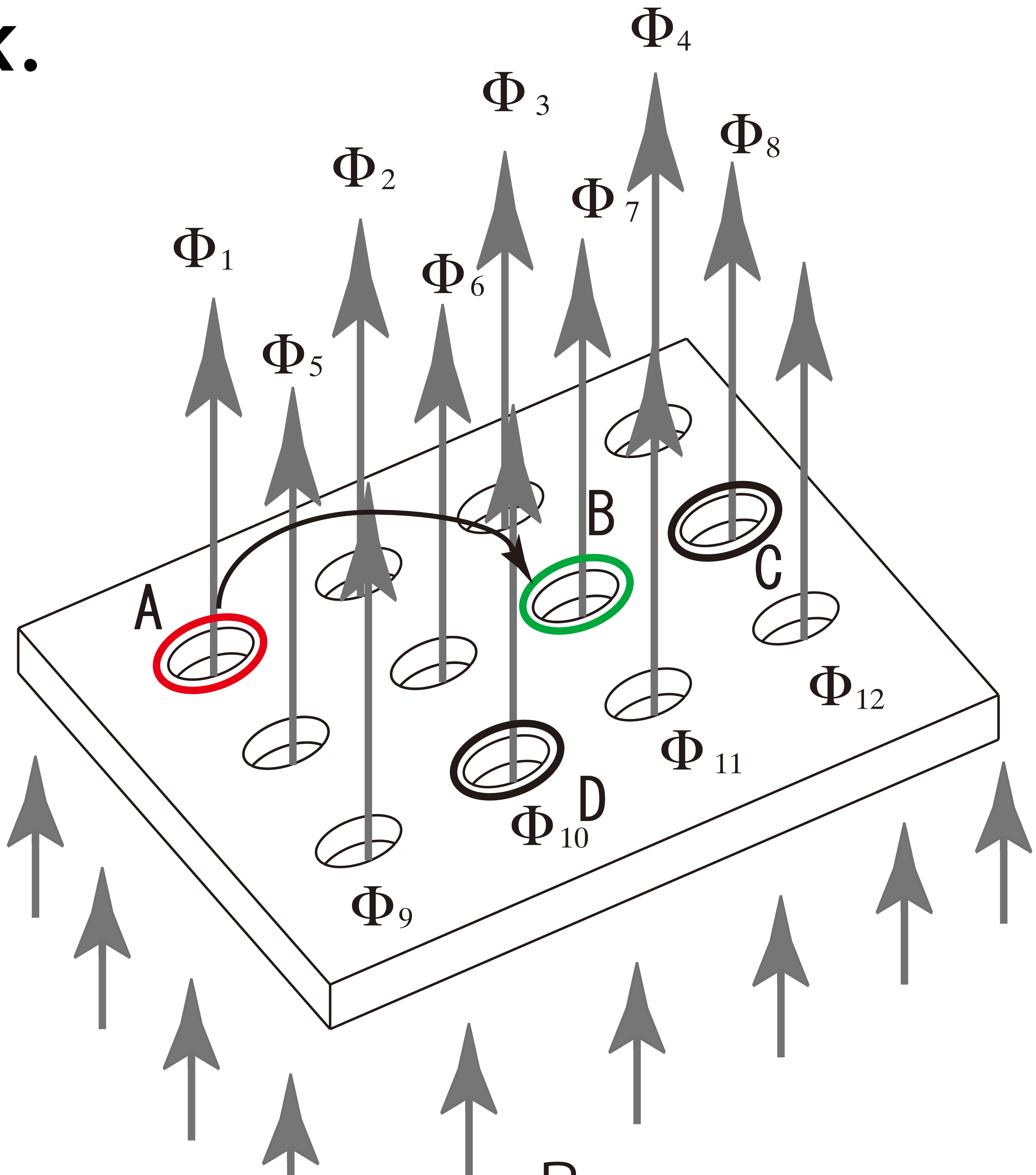
localized particles in the gap

Novel quantum degrees
with topological protection by bulk

Adiabatic transports among edge states

http://jstore.jst.go.jp/seedsDetail.html?seeds_id=3646

ex.



Edge is topological

Right / left to the symmetry

With translation invariance

$$[H,T]=0$$

Bloch theorem $T\psi(\boldsymbol{r})\equiv\psi(\boldsymbol{r}+\boldsymbol{t})=e^{ik}\psi(\boldsymbol{r})$

$$|\psi(\boldsymbol{r})|=|\psi(\boldsymbol{r}+\boldsymbol{t})|=|\psi(\boldsymbol{r}+2\boldsymbol{t})|=\cdots=|\psi(\boldsymbol{r}+10^{10}\boldsymbol{t})|=\cdots$$

Extended over the whole space

Energy bands : energy of the extended states

With boundaries/ impurities

As for extended states, effects of edges can be negligible !

dimension is less ! 0D impurities/1D boundaries in 2D

→ States in the energy gap are localized !

since they can not be extended

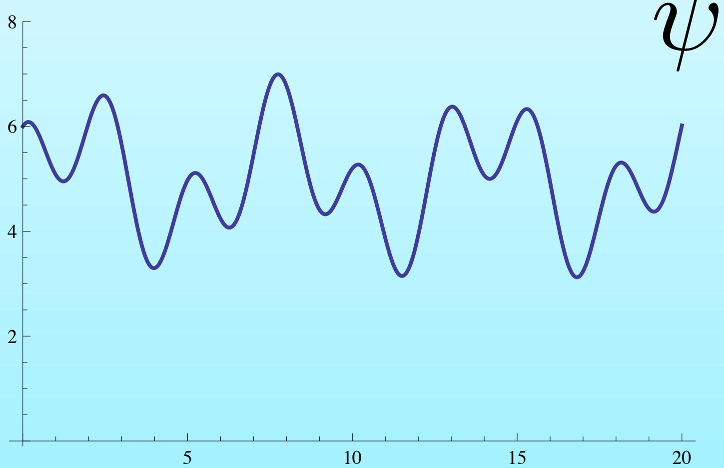
Bound states / Edge states

Edge is topological

Right / left to the symmetry

Extended states

unnormalizable

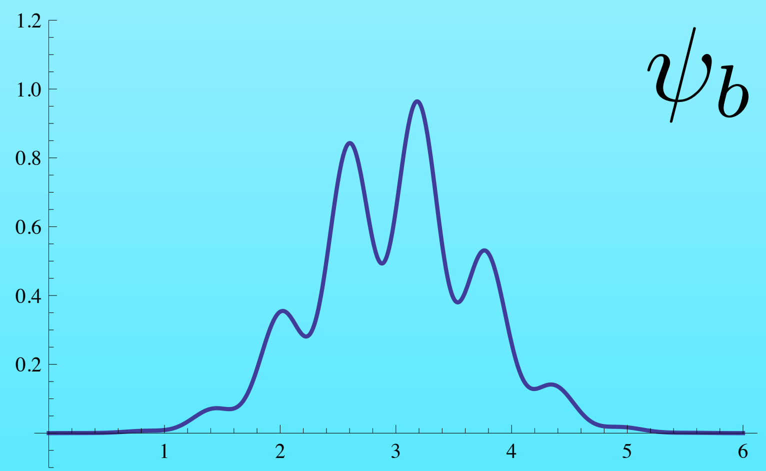


$$\psi_e(r) \sim \frac{1}{\sqrt{V}} e^{ikr} \longrightarrow 0 \quad (V \to \infty)$$

V: Volume

Bound states / Edge states

normalizable



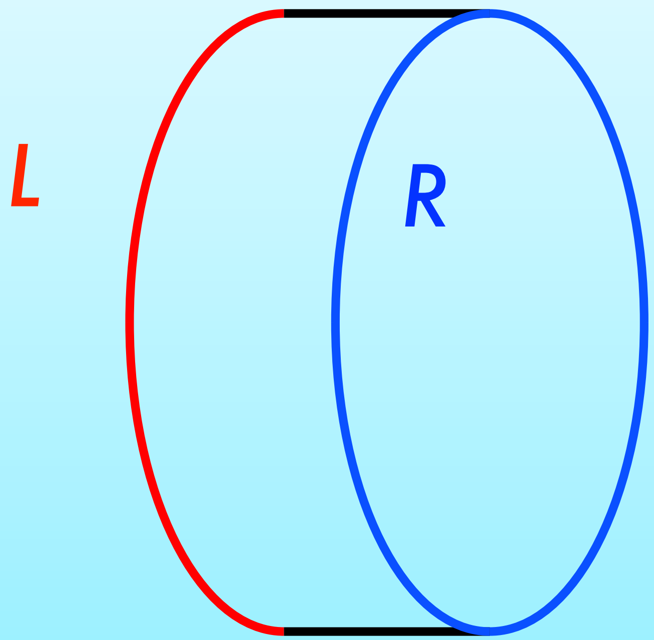
$$\psi_b(r) \sim \frac{1}{\sqrt{a_0^3}} e^{-r/a_0}$$

a_0 :size of the bound state

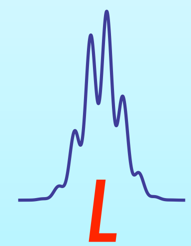
☑ Clear difference only in the infinite system

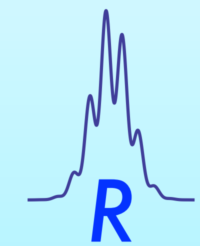
Right / left to the symmetry

On cylinder



Finite system

$|\pm\rangle = |$ 

$\rangle \pm |$ 

$\rangle$

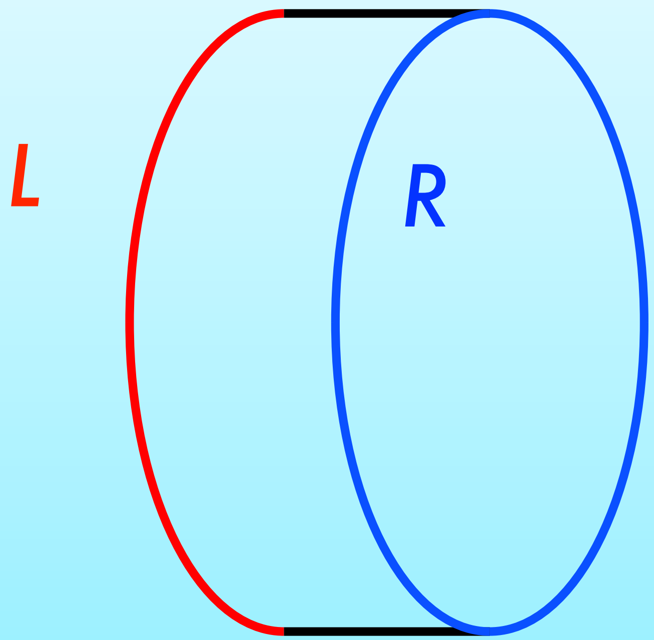
☑ Right / left to the symmetry

only in the infinite system,

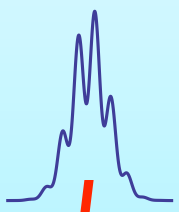
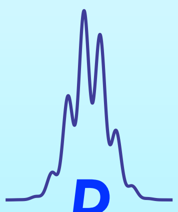
since the Boundaries are far away !

Right / left to the symmetry

On cylinder



Finite system

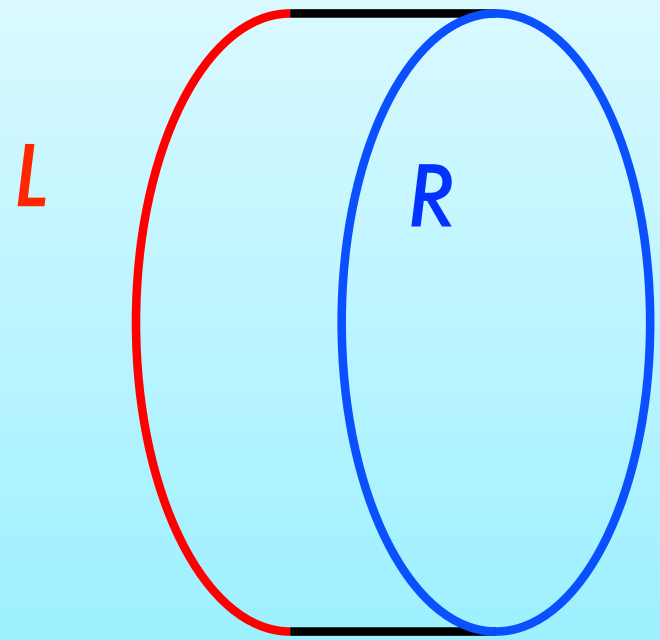
$|\pm\rangle = |$  $\rangle \pm |$  $\rangle$

☑ *Right / left to the symmetry*
only in the infinite system,
since the Boundaries are far away !

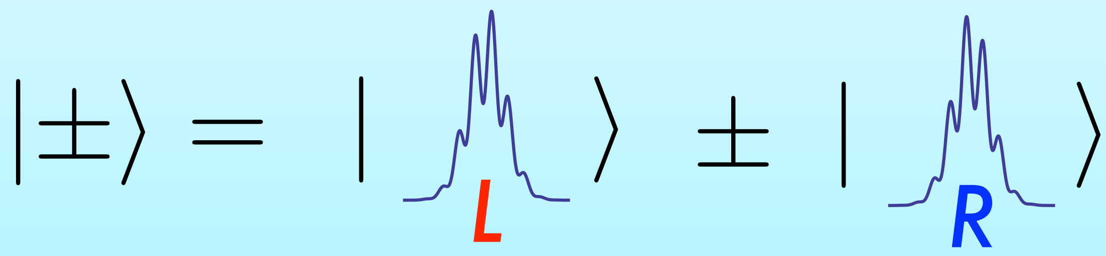
Bulk-edge correspondence : Emergent principle

Right / left to the symmetry

On cylinder



Finite system



☑ **Right / left to the symmetry**

only in the infinite system,

since the Boundaries are far away !

Bulk-edge correspondence : Emergent principle

c.f. More is different

Edge is topological

Why do we care edge states?

Why the Edge States are there??

Accidental ?

NO !

Inevitable reasons

Physical Structures behind:

"Bulk determines the edges"

"Edge determines the bulk"

<u>Bulk-Edge Correspondence</u>

Protected by Topological constraints

Conclusion

Conclusion

Edge states are everywhere
in
condensed matter physics

Conclusion

Edge states are everywhere
in
condensed matter physics

Edge states are useful for
applications
in quantum physics /devices

Thank you

Edge states are everywhere
in
condensed matter physics

Edge states are useful for
applications
in quantum physics /devices