Physics of bulk-edge correspondence, Hiroshima Univ. June 27-29, 2012
物理科学特別講義（バルクエッジ対応の物理）

バルクエッジ対応の物理：背景２

background 2

量子液体とトポロジカル秩序

Quantum liquids & topological order

Use of the Bulk-edge correspondence

筑波大学大学院数理物質科学研究科
物理学専攻
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HIROSHIMA UNIVERSITY

筑波大学
University of Tsukuba

Quantum Liquids without Symmetry Breaking

- Quantum Liquids in Low Dimensional Quantum Systems
  - Low Dimensionality, Quantum Fluctuations
  - No Symmetry Breaking
  - No Local Order Parameter
  - Various Phases & Quantum Phase Transitions
- Gapped Quantum Liquids in Condensed Matter
  - Integer & Fractional Quantum Hall States
  - S=1 Spin chain (Haldane Phase)
  - Integer spin chains (S=1,2) with dimerization
  - Generic valence bond solid (VBS) states
  - Dimer models (Rokhsar-Kivelson)
  - Half filled Kondo Lattice

How to Classify the Quantum Liquid Phases

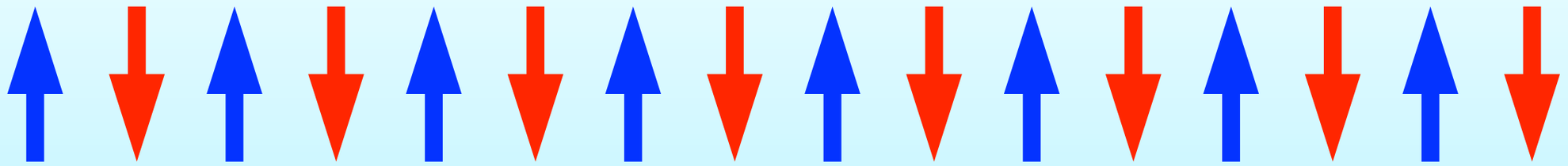
⇨ New way to think about !

Topological/quantum orders

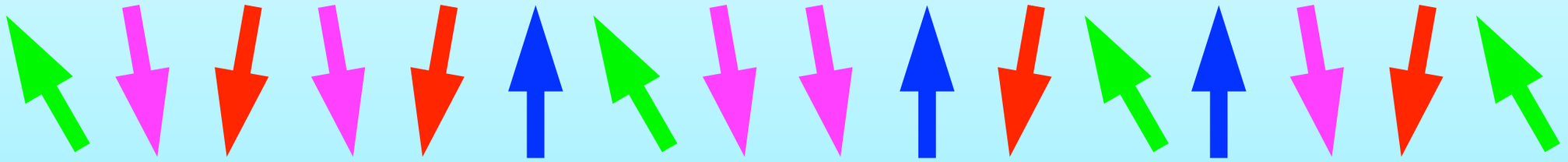
Order & Symmetry

Magnetic Order

Ordered



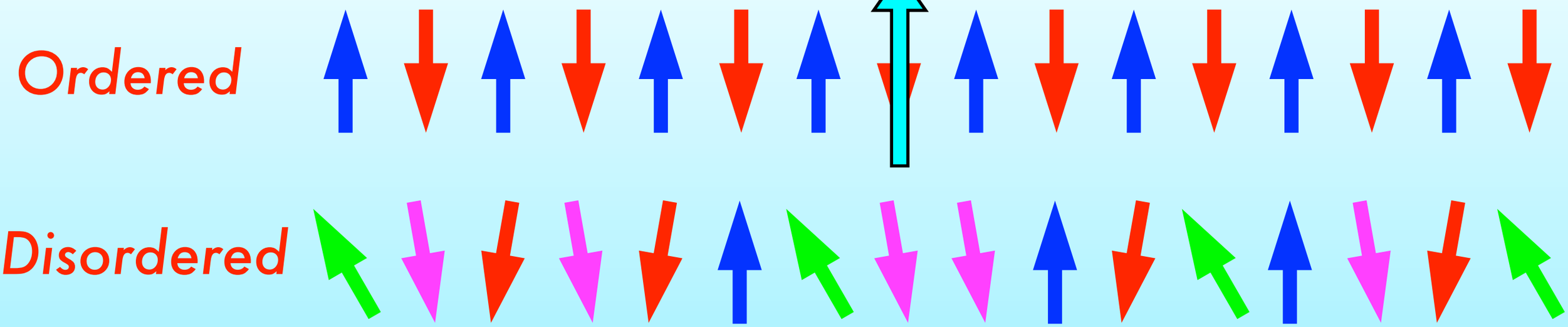
Disordered



Order & Symmetry

Magnetic Order

special direction! (symmetry broken)



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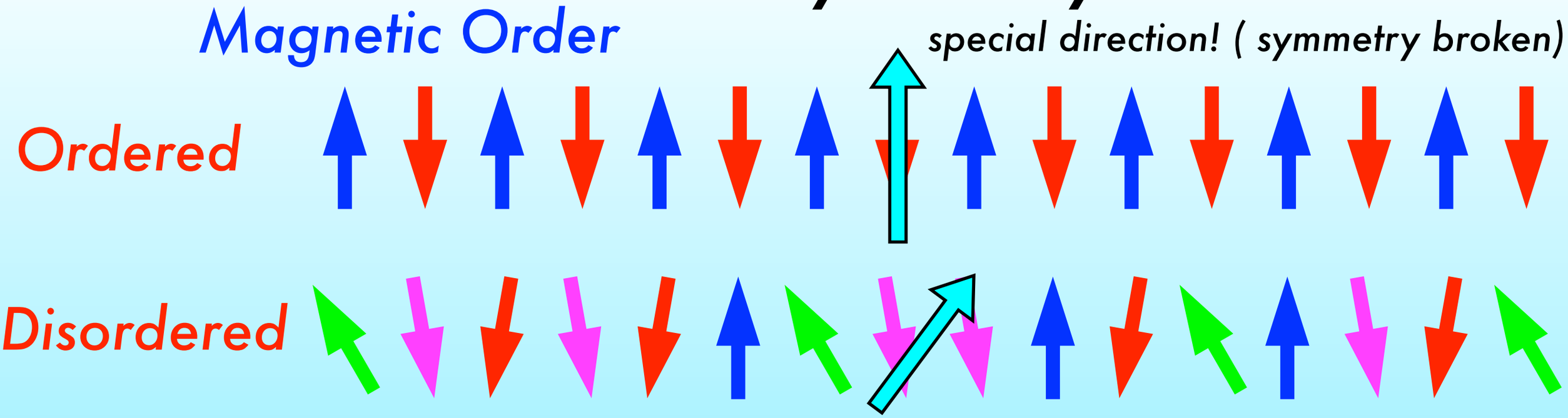

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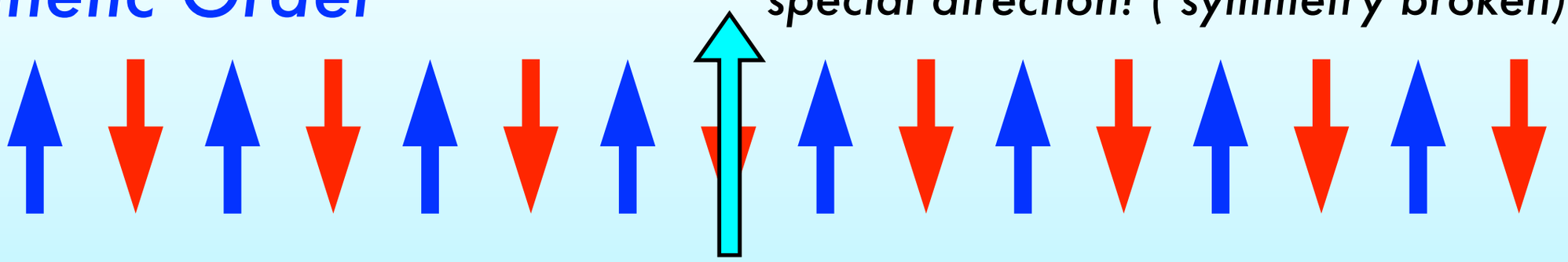


Disordered

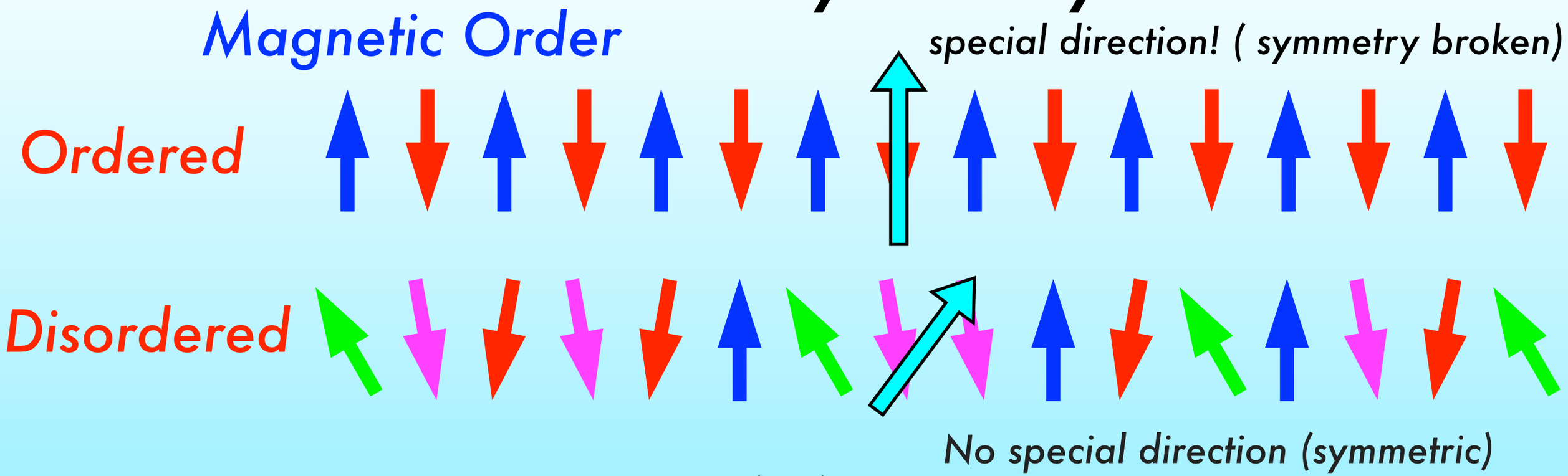
No special direction (symmetric)

Local order parameter : $\vec{m}(\vec{r})$

Local magnetic structure around $\vec{r}$

Ferromagnetic / Antiferromagnetic, ...

Order & Symmetry



Local order parameter : $\vec{m}(\vec{r})$

Local magnetic structure around $\vec{r}$

Ferromagnetic / Antiferromagnetic, ...

- Spontaneous symmetry breaking

$$\langle m(r) \rangle = 0 \quad ?$$

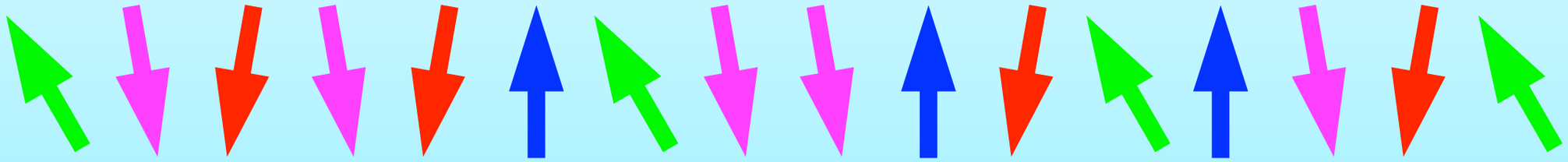
- Long Range Order

$$\langle m(r) m(r') \rangle \to 0 \quad ? \quad (|r - r'| \to \infty)$$

Order & Symmetry

Quantum Disordered ⟺ Gapped

Disordered



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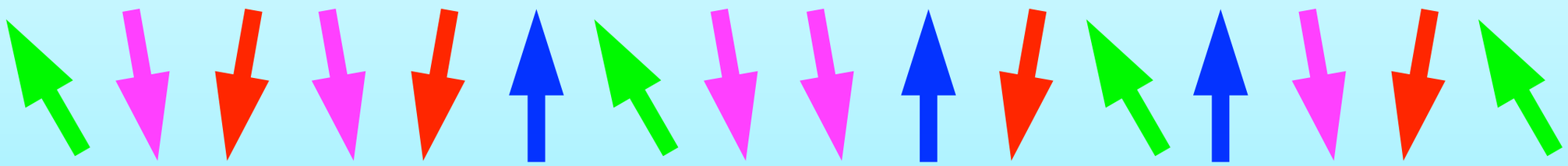

No Observables to Characterize

Order & Symmetry

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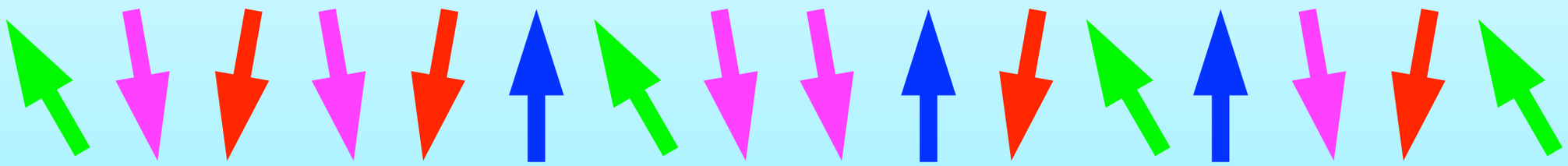

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Quantum Liquids & Spin Liquids

Order & Symmetry

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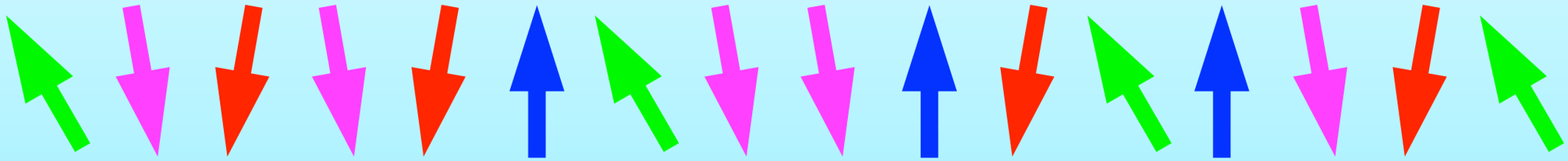

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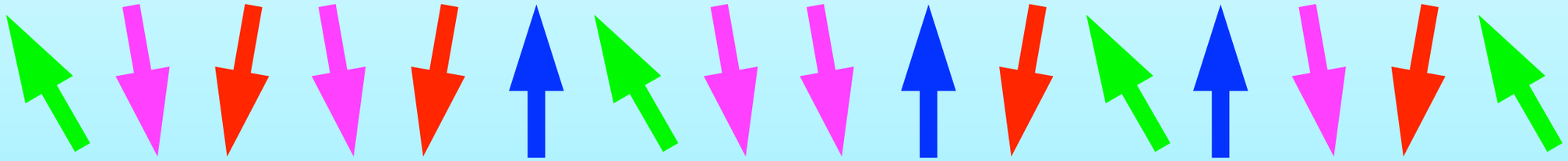

Gapped: Nothing in the gap

Order & Symmetry

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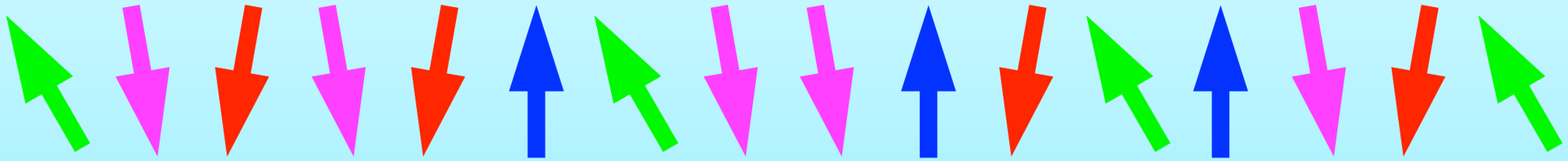

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No low lying excitations

Order & Symmetry

Quantum Liquids & Spin Liquids

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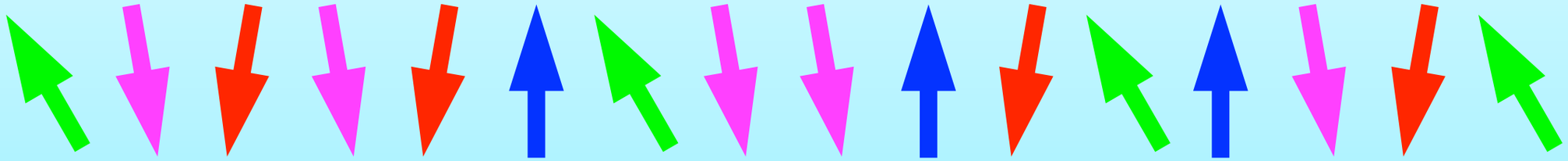

Gapped: Nothing in the gap

No low lying excitations

No Response against small perturbation

Order & Symmetry

Quantum Liquids & Spin Liquids

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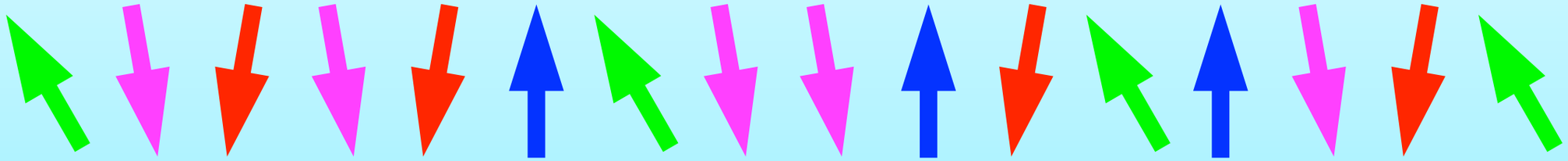
Disordered



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打てども響かず

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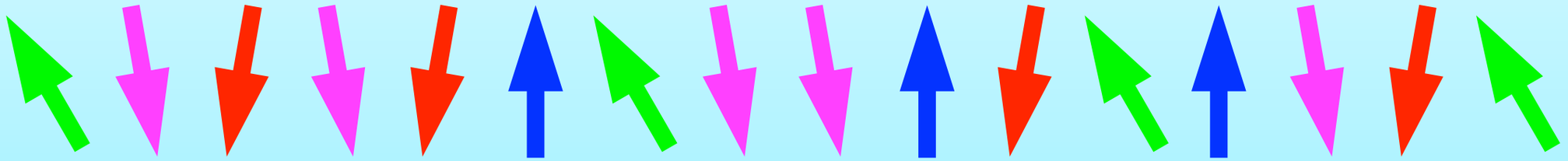

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c.f. gapless modes:

acoustic phonons

zero sounds

spin waves

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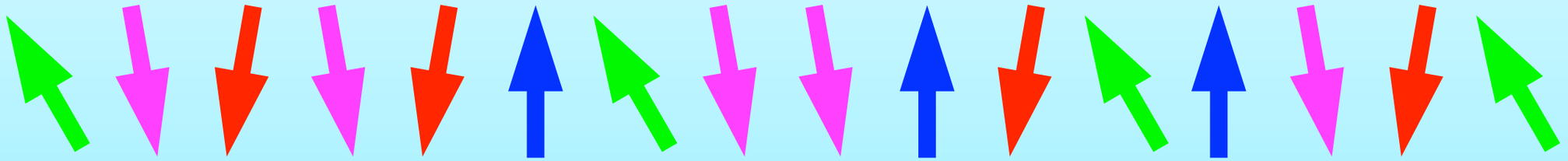

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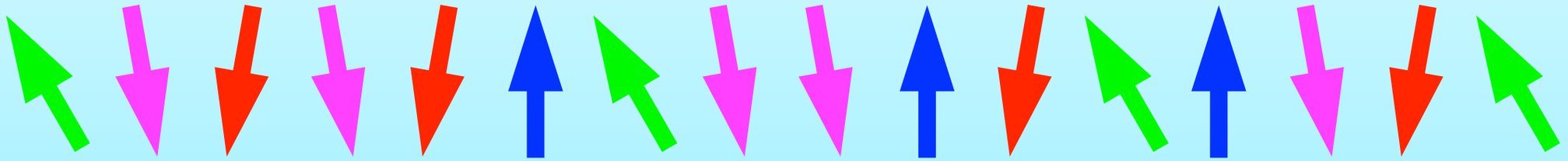
Need Novel Kind of Orders :

Order & Symmetry

Quantum Liquids & Spin Liquids

Quantum Disordered ⇔ *Gapped*

Disordered



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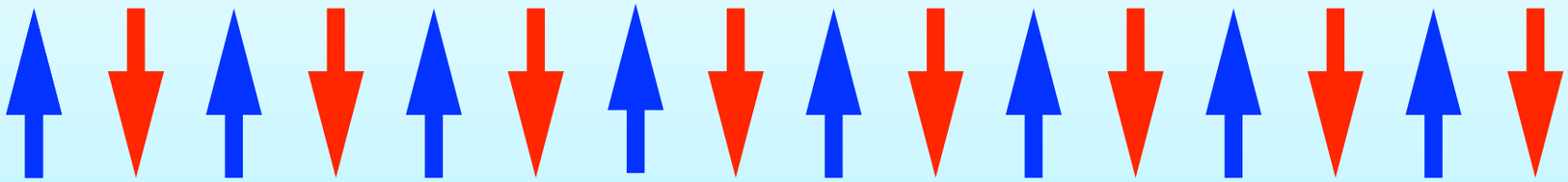
Need Novel Kind of Orders : Topological Order

Excitations and Local Order Parameter

- *Discrete Symmetry Breaking: (ex. Ising order)*

  Local disturbance induces a finite cost



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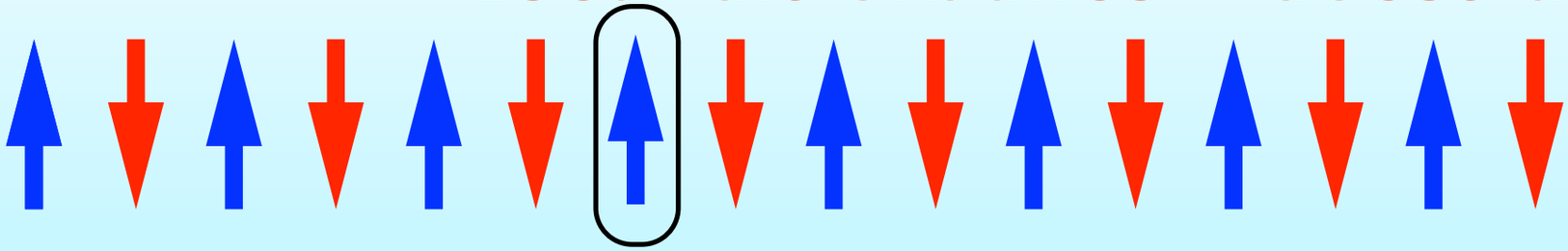

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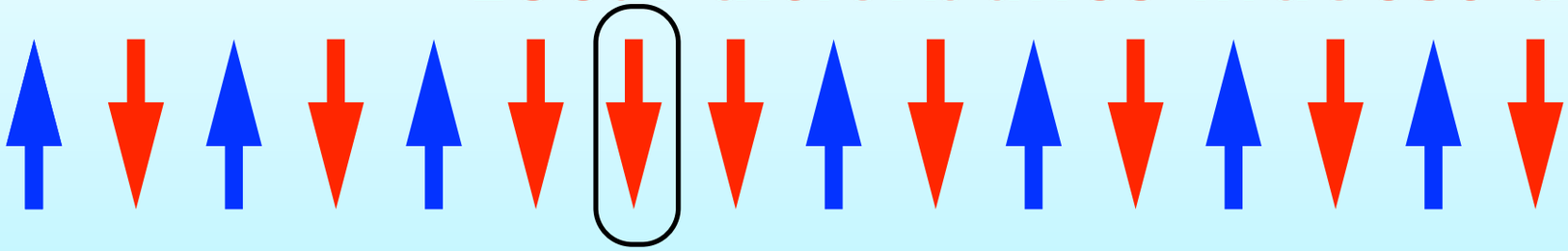

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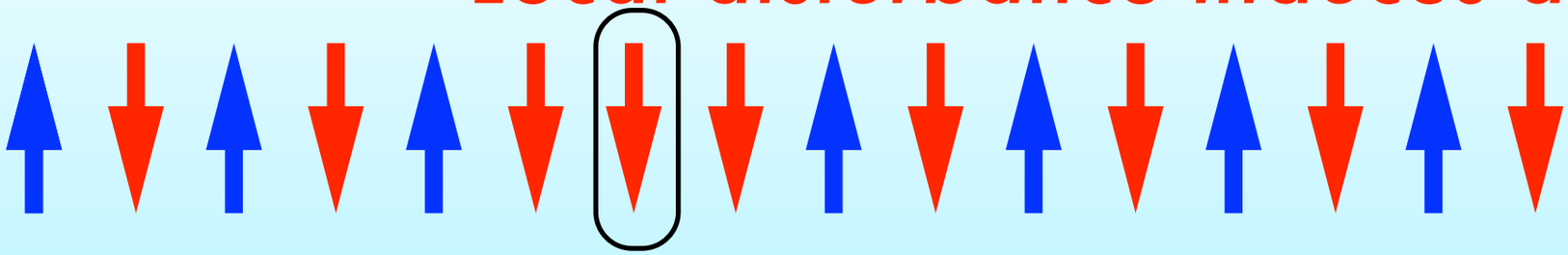

$$\Delta E \sim \mathcal{O}(1)$$

Excitation Gap

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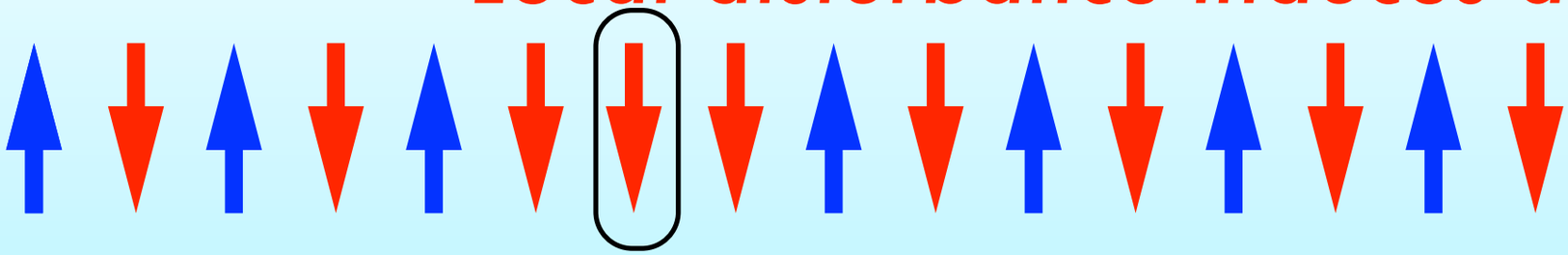

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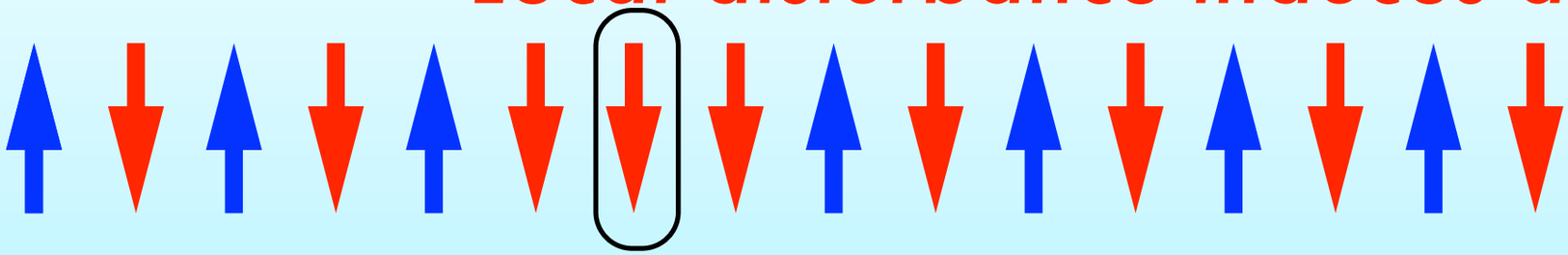

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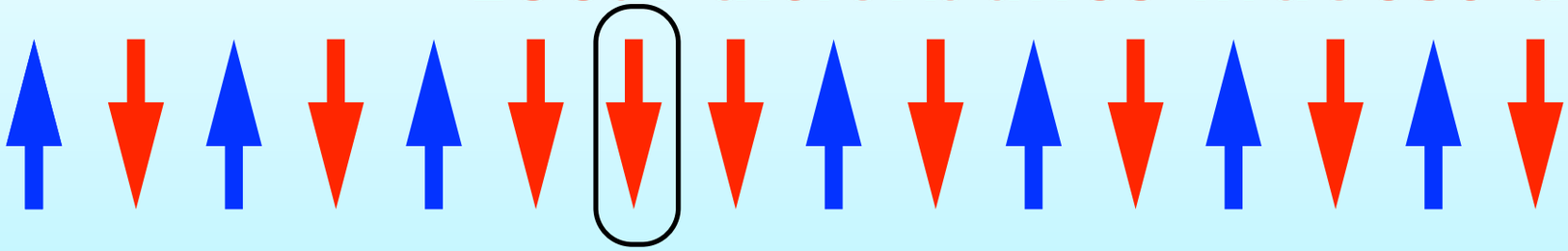

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breaking of continuous symmetry based on the local order parameter

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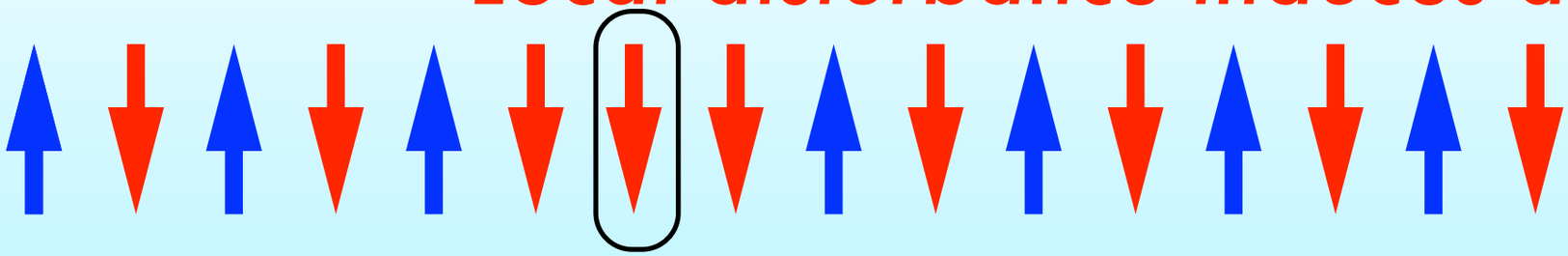

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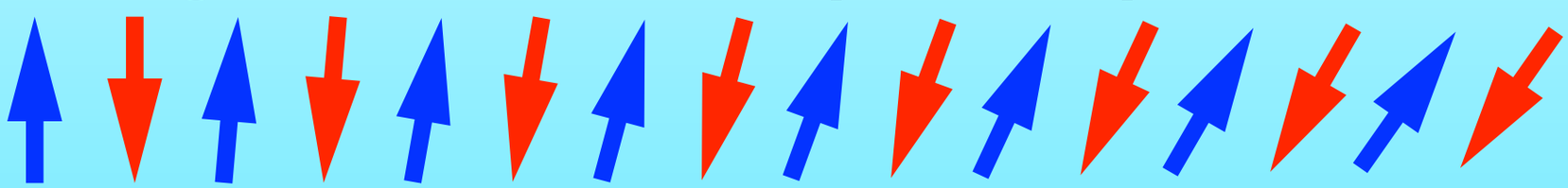

Slowly varying its local order parameter : spin waves

Lieb-Schultz-Mattis

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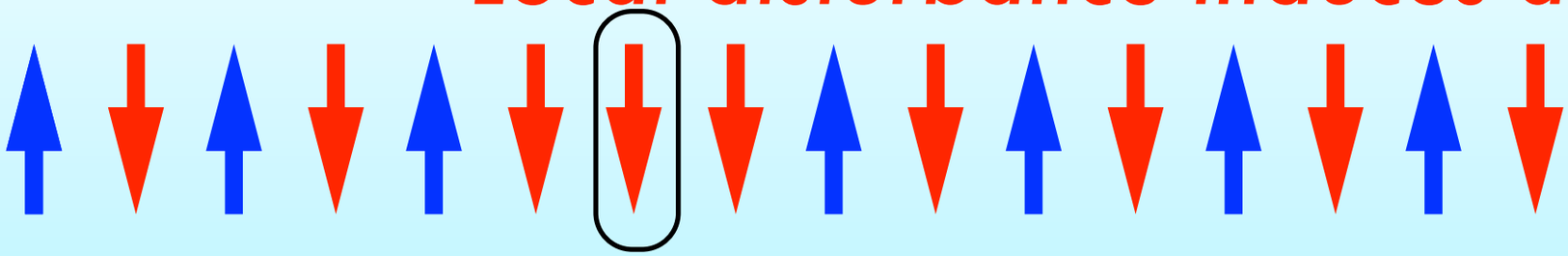

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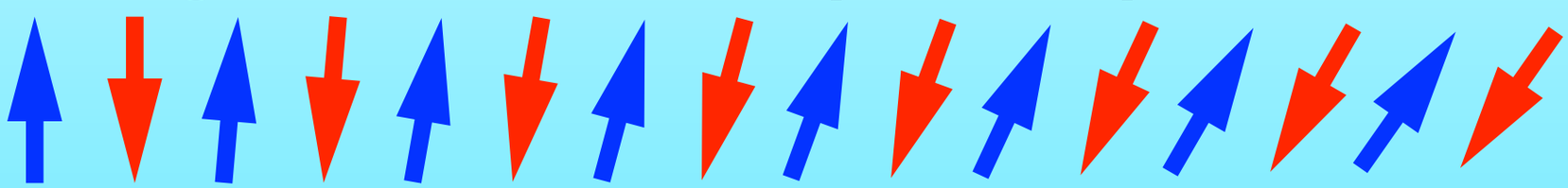

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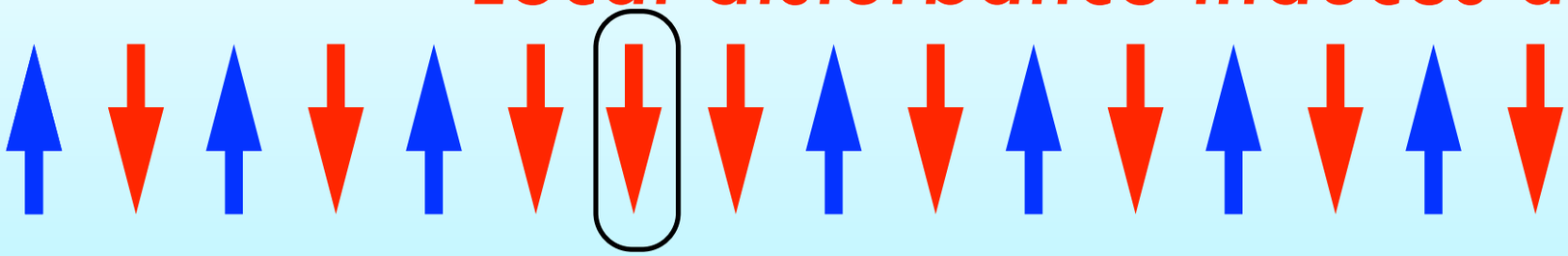

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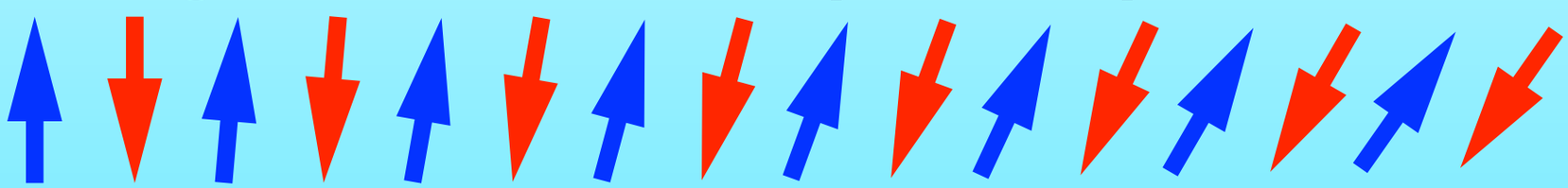

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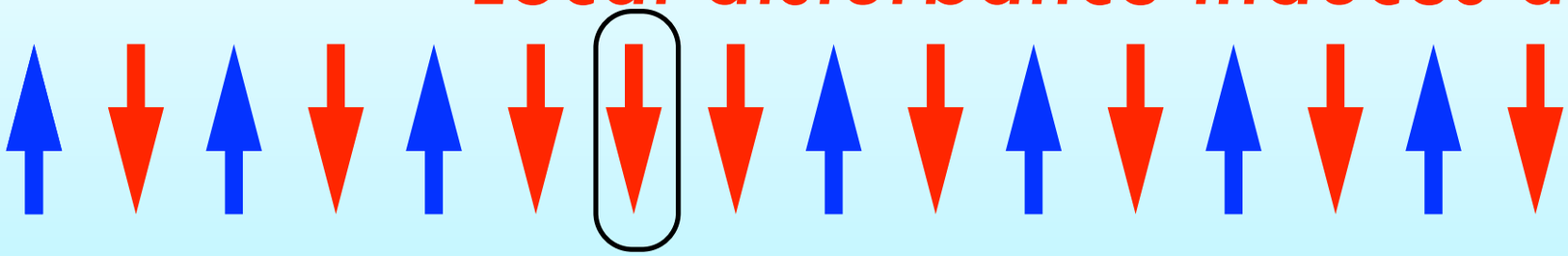

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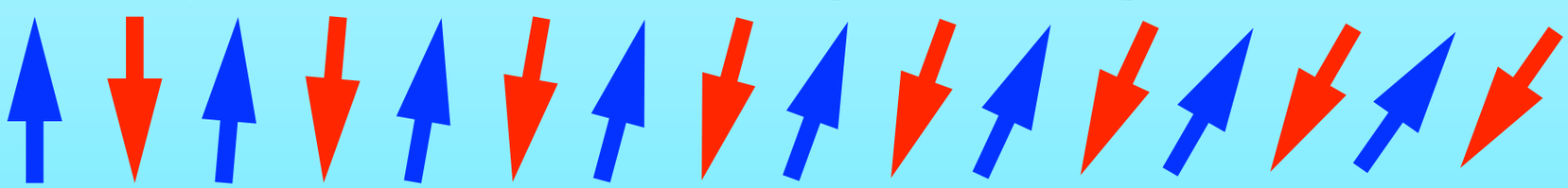

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Topological Origin

quantum/topological order

Quantum Hall Effects & Haldane spin chains

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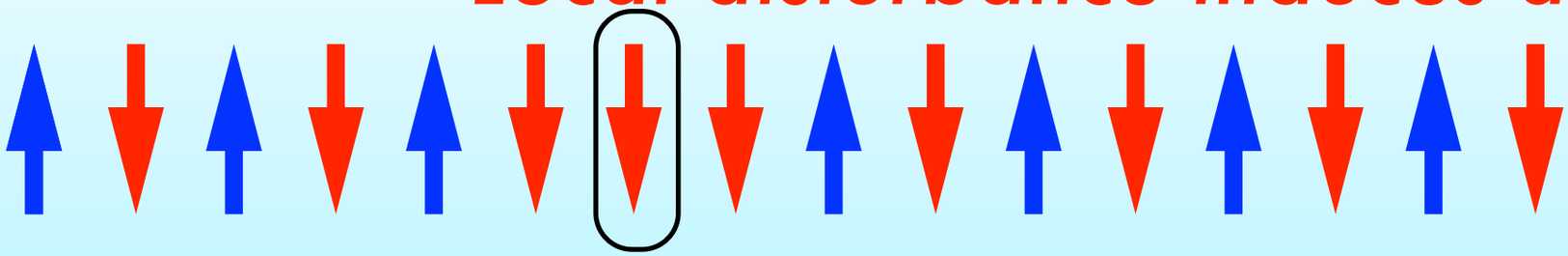

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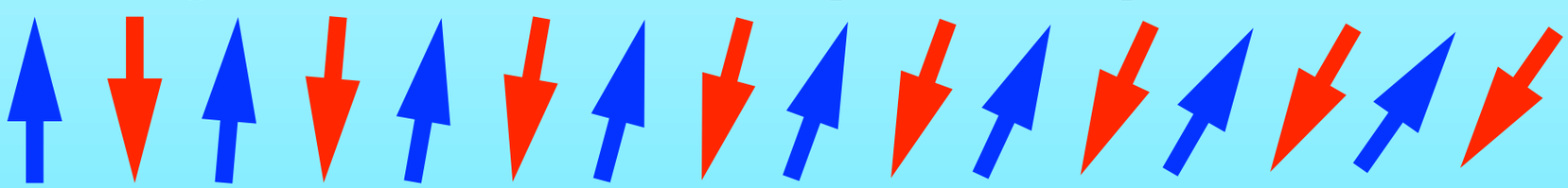

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fermions as boundary states in high dimensions

(domain wall fermions)

Topological Origin

quantum/topological order

Quantum Hall Effects & Haldane spin chains

Quantum Liquids are Featureless !!

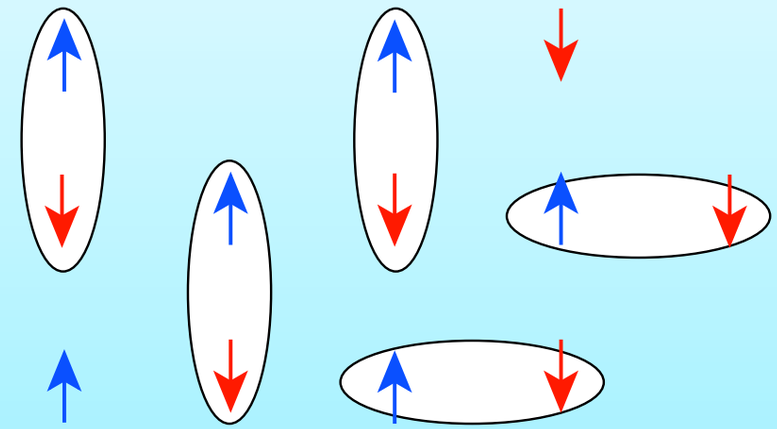
How to characterize the phase

Without Symmetry Breaking

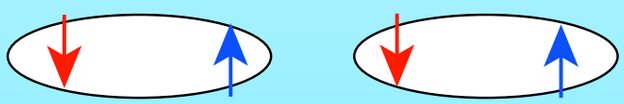
Use Quantum Interference!

Quantum Liquid

☆The RVB state by Anderson



$$|\text{Singlet Pair}_{12}\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle)$$



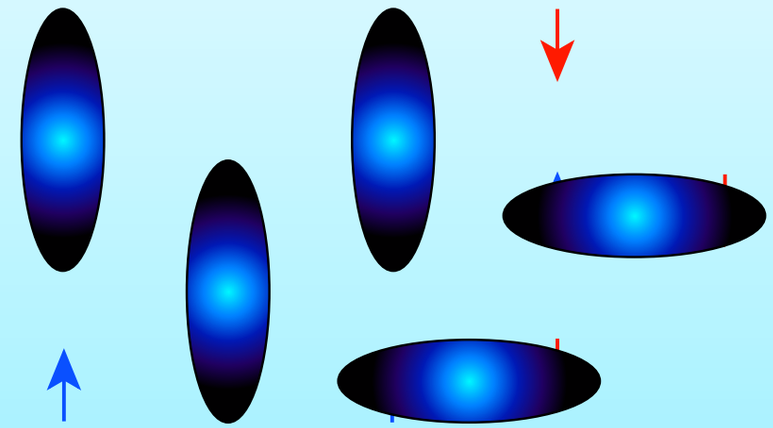
$$|G\rangle = \sum_{J=\text{Dimer Covering}} c_J \otimes_{ij} |\text{Singlet Pair}_{ij}\rangle$$

small magnets

Dimer covering
with non orthogonal
& local states

Quantum Liquid

☆The RVB state by Anderson



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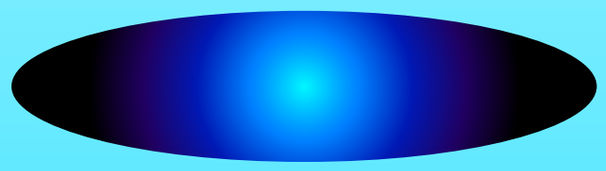
Spins *disappear* as a *Singlet pair*



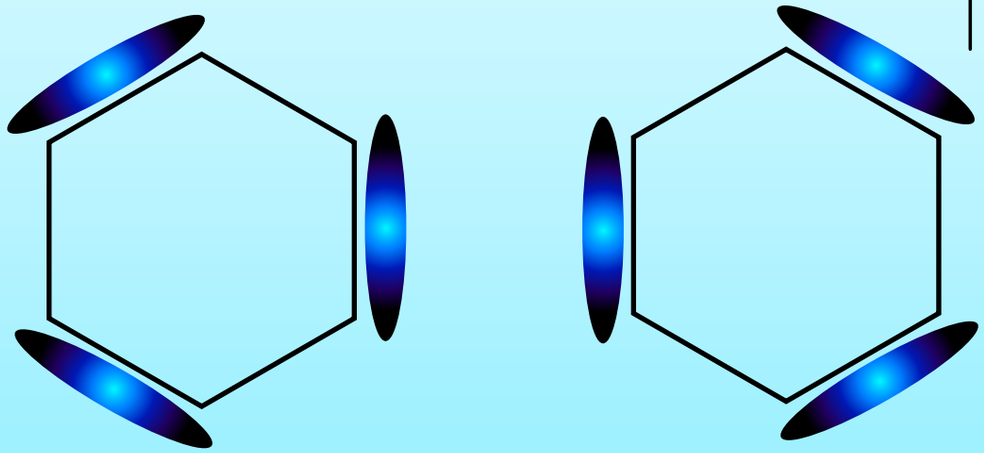
Local Singlet Pairs :

(Fundamental Objects)

Local Quantum Objects for characterization

One more Quantum Liquid

☆The *RVB* state by Pauling

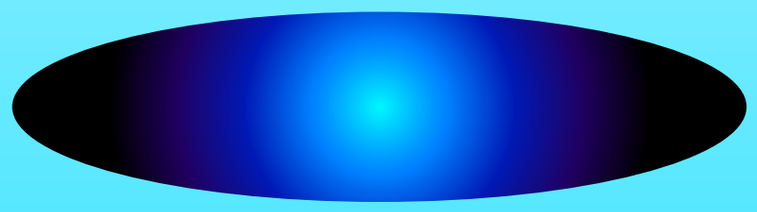


$$|\mathrm{Bond}_{12}\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle) = \frac{1}{\sqrt{2}}(c_1^\dagger + c_2^\dagger)|0\rangle$$

$$|G\rangle = \sum_{J=\text{Dimer Covering}} c_J \otimes_{ij} |\mathrm{Bond}_{ij}\rangle$$

Do Not use the Fermi Sea

Delocalized charge
as a *covalent bond*



Local Covalent Bonds :
(Fundamental Objects)

Local Quantum Objects for characterization

Tools to play with the Quantum Liquids

Quantum Liquids

Featureless !!

- Local Characterization
- Quantized Berry Phases as a Topological Local Order Parameter
- Global Characterization
- Entanglement Entropy as a Novel Tool for the Condensed Matter Physics

Closely related to the edge states

「対称性の破れ」で物質相の記述に本当に十分？

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- 真に量子的な相
  - いかなる対称性の破れもない
  - 古典的な秩序変数が存在しない
  - なおかつ多種多様な物質相が存在する！！

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量子液体相、スピン液体

~~秩序変数~~　　~~対称性の破れ~~

「対称性の破れ」で物質相の記述に本当に十分？

- 真に量子的な相
  - いかなる対称性の破れもない
  - 古典的な秩序変数が存在しない
  - なおかつ多種多様な物質相が存在する！！

量子液体相、スピン液体

~~秩序変数~~ → トポロジカル秩序

~~対称性の破れ~~ → 量子秩序

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量子液体相、スピン液体

~~秩序変数~~

トポロジカル秩序

~~対称性の破れ~~

量子秩序

トポロジカルな量子相転移

量子液体相、スピン液体相とは？？

量子液体相、スピン液体相とは？？

- ☆整数量子ホール相、分数量子ホール相、
- ☆量子スピンホール相：トポロジカル絶縁体
- ☆フラストレートしたスピン系、ダイマー系
- ☆RVB状態、Haldane スピン系、VBS状態
- ☆近藤格子、Kitaev 模型、Levin-Wen模型

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トポロジカルな量子的秩序変数

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トポロジカルな量子的秩序変数

エッジ状態

ベリー位相

チャーン数

エンタングルメント

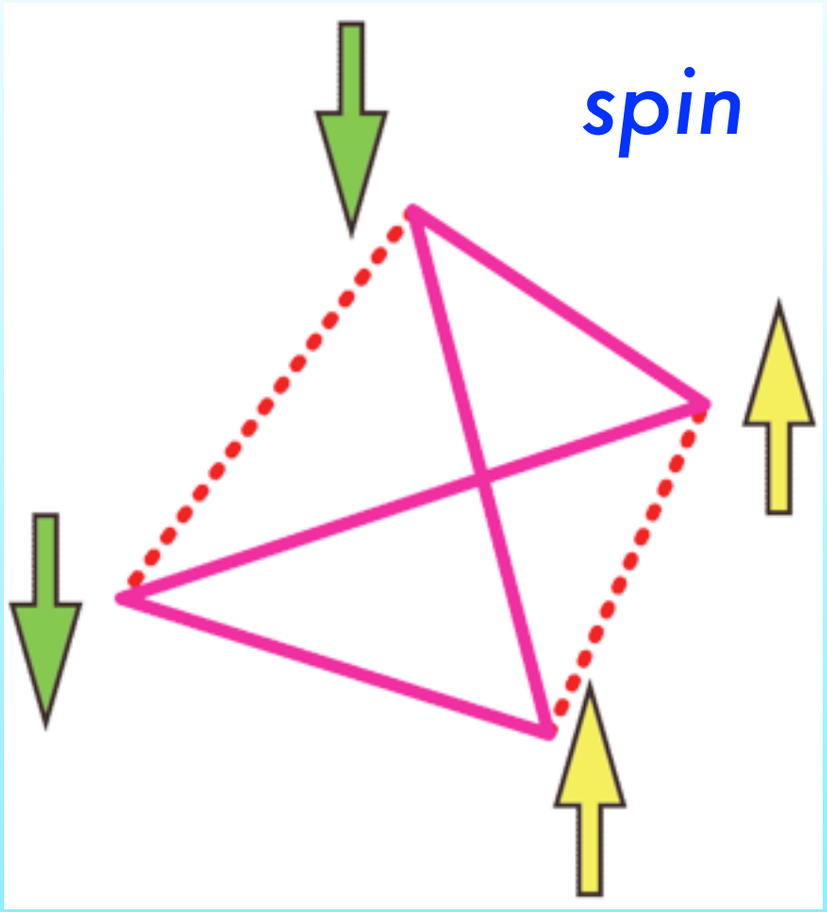
A way to the Quantum liquids

Example: frustration

Relax local degrees of freedom

Frustration : residual entropy

Quantum effects



$$H = \sum H_{\mathrm{loc}}$$

local degeneracy

N: *system size*

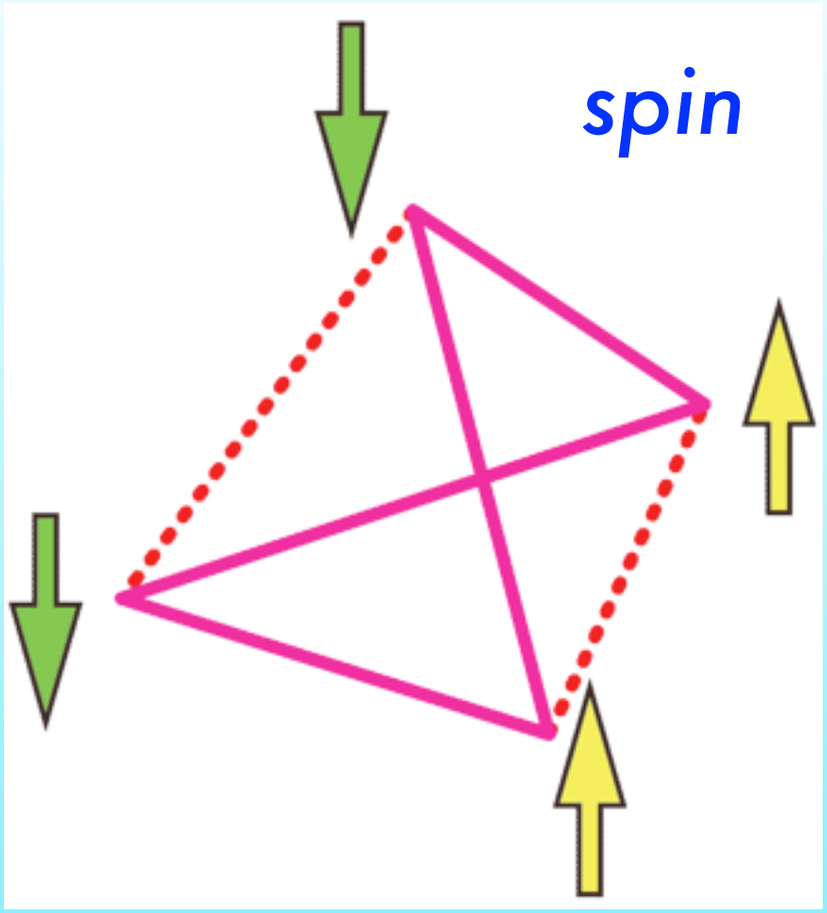
c : *const.*

$$\#\ \mathrm{deg.} \approx e^{cN}$$

 # low energy modes

Frustration : residual entropy

Quantum effects



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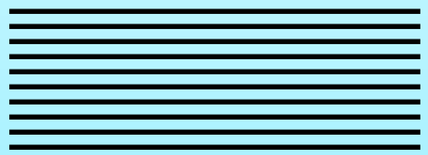

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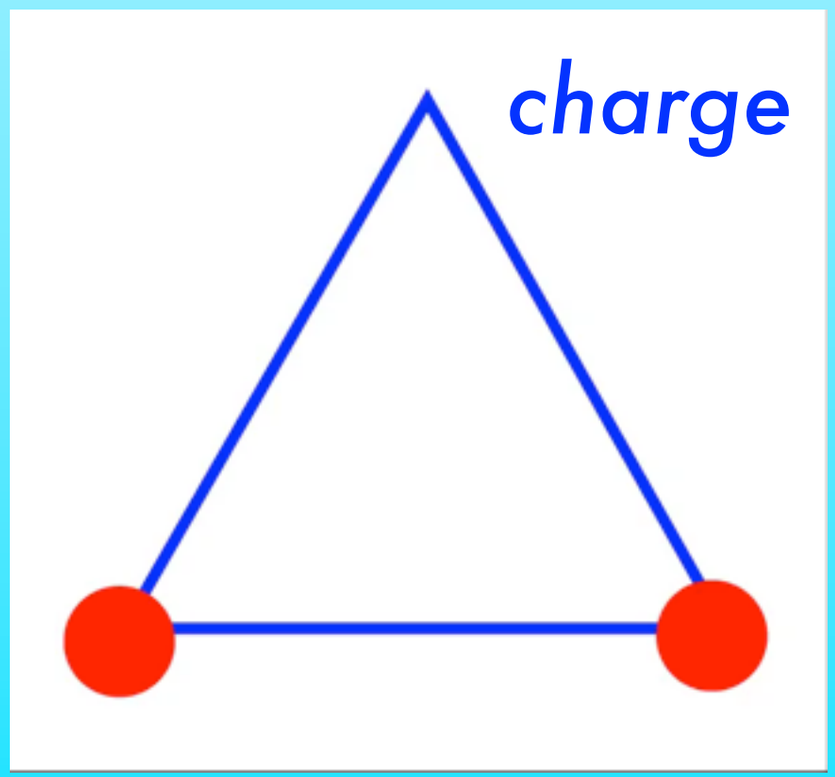
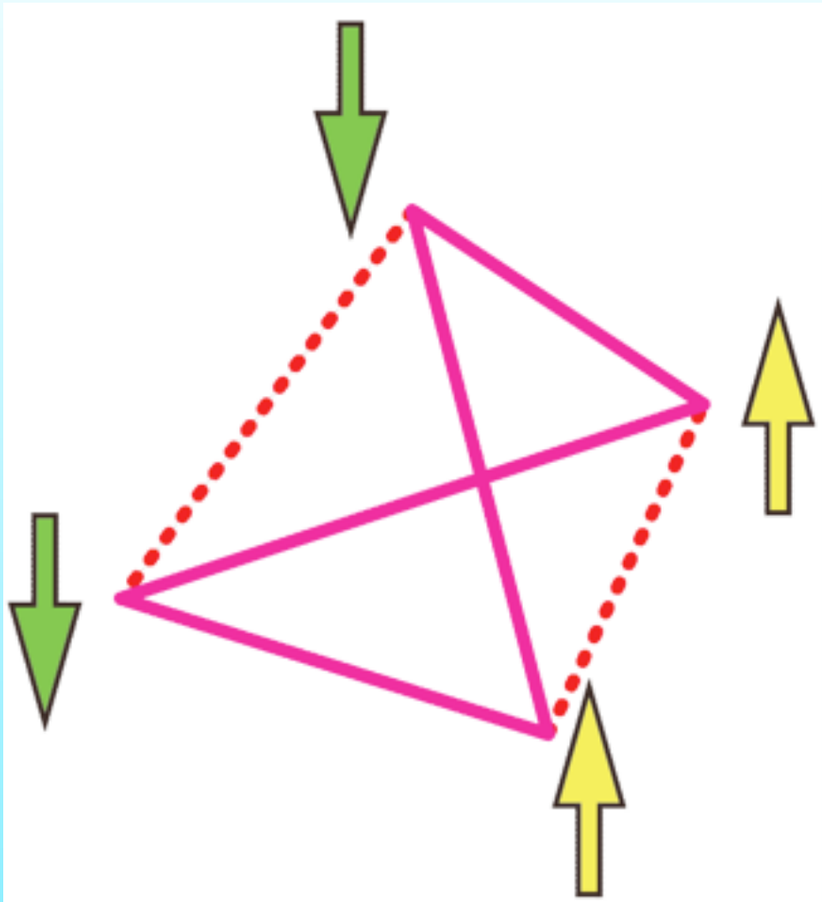


Residual Entropy to form gapped quantum liquid



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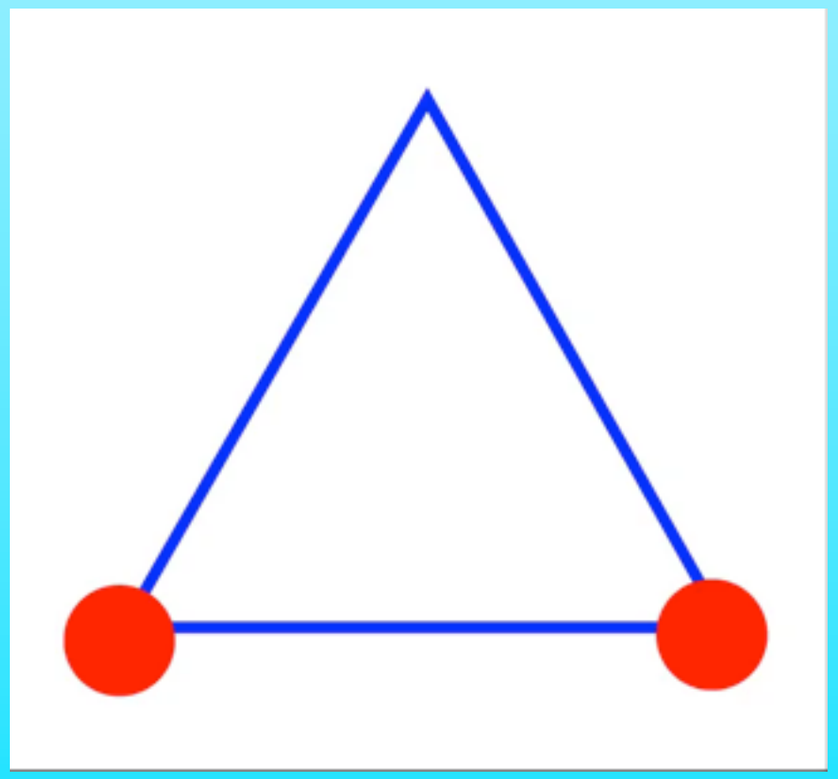
local degeneracy

Quantum effects

Gap opening

Stabilization to relax local entropy



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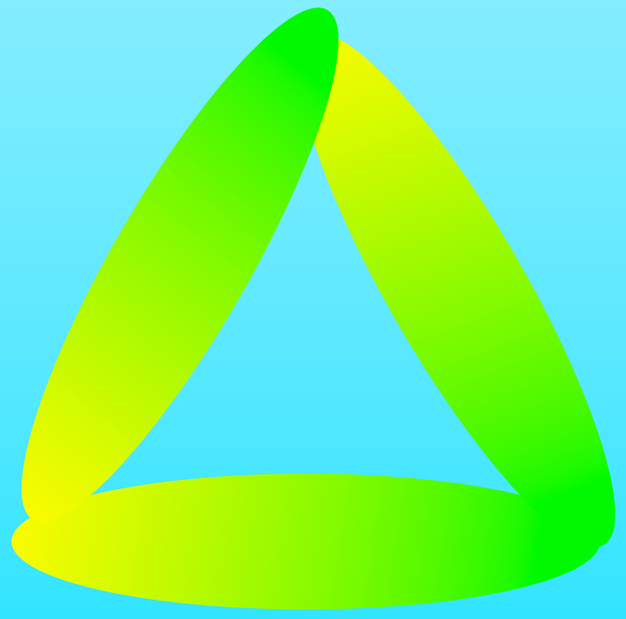


Residual Entropy to form gapped quantum liquid

$$H = \sum H_{\mathrm{loc}}$$

local degeneracy

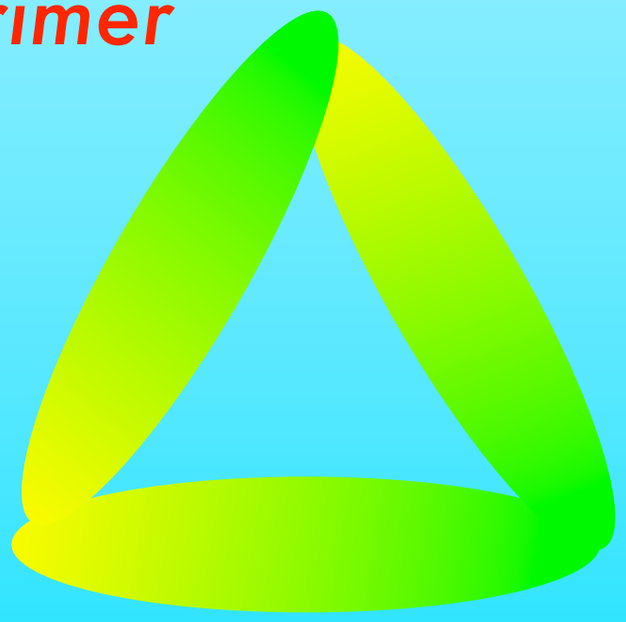
tetramer

Quantum effects

Gap opening

Stabilization to relax local entropy

trimer



Formation of local quantum object

Multimer as a generic dimer

Covalent molecular orbital

c.f. Tamura '06, Matsuda-Motome '06, Katsufuji

Energy band & gap : physicist & chemist ?

Sorry if I'm wrong

physicist

itinerant electrons

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hopping

make energy band

metal

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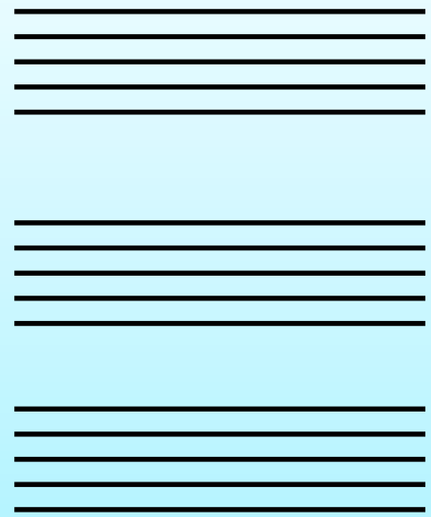

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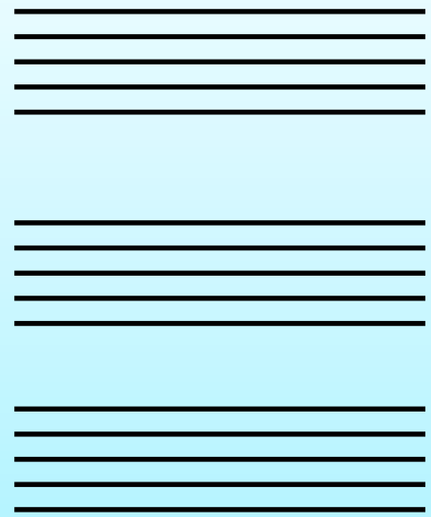
Peierls instability

Opening gap

stabilize



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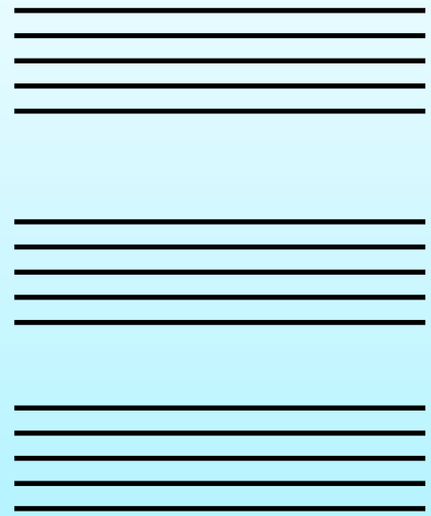

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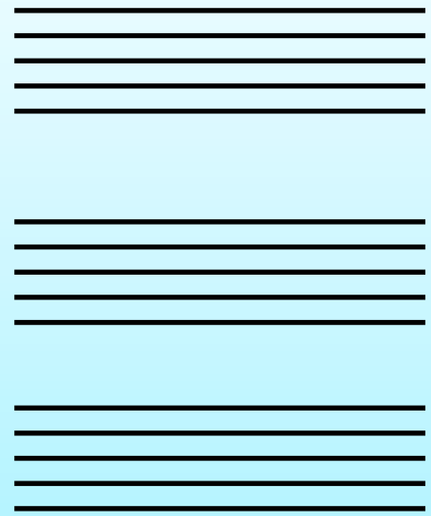

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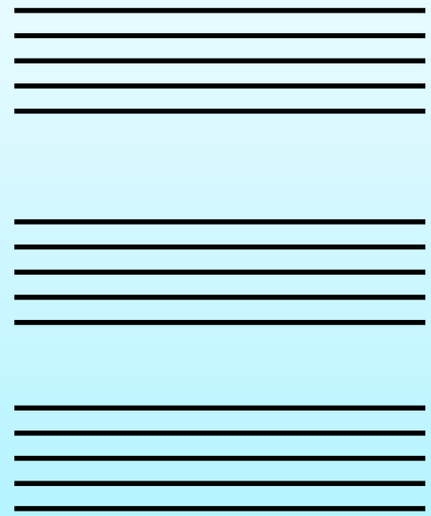

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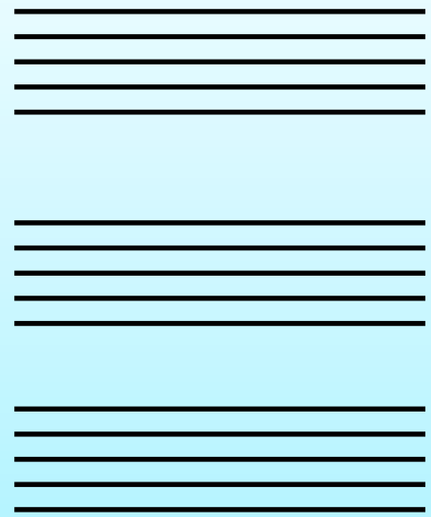

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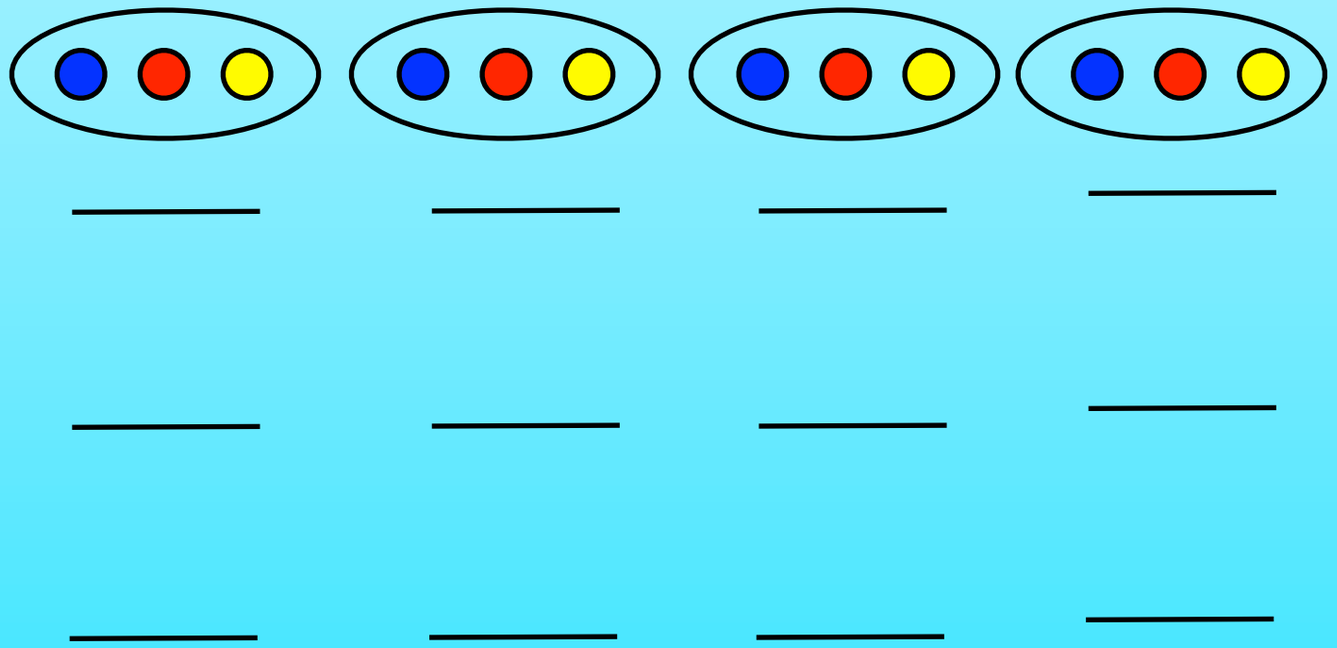

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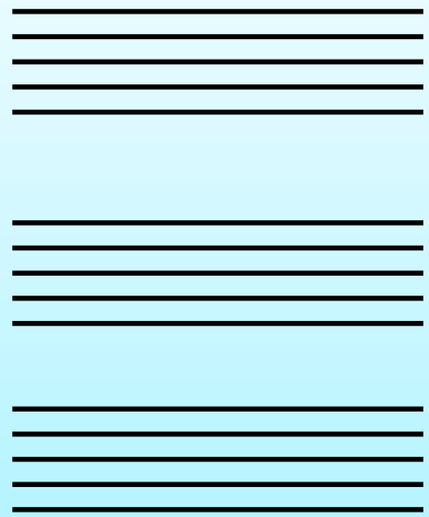

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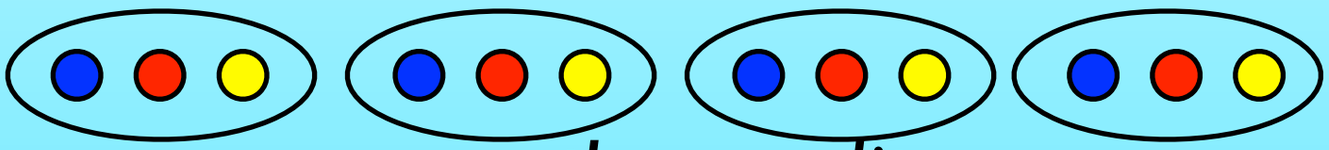
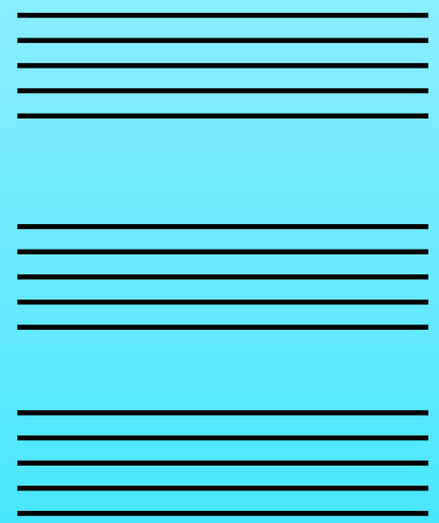


non orthogonality

Dimer & Multimer

make bands of molecules



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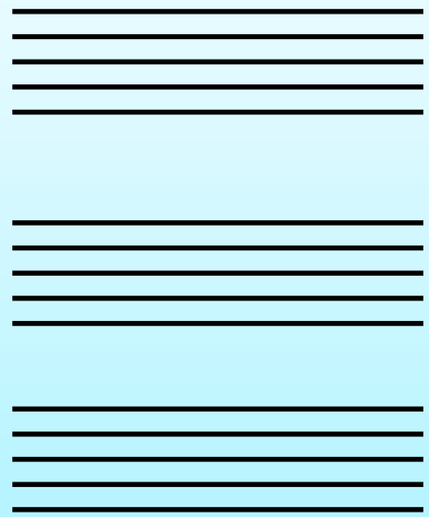

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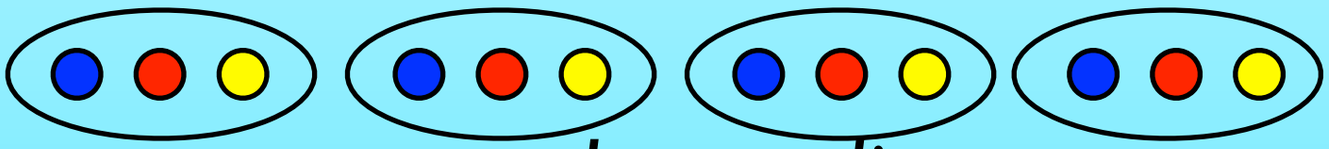


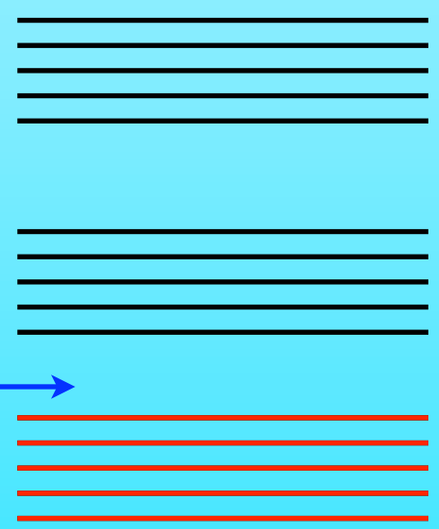
non orthogonality

make bands of molecules

Dimer & Multimer

Adiabatic process

Insulator



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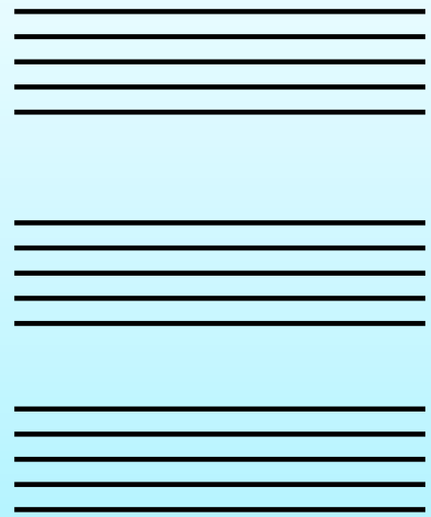

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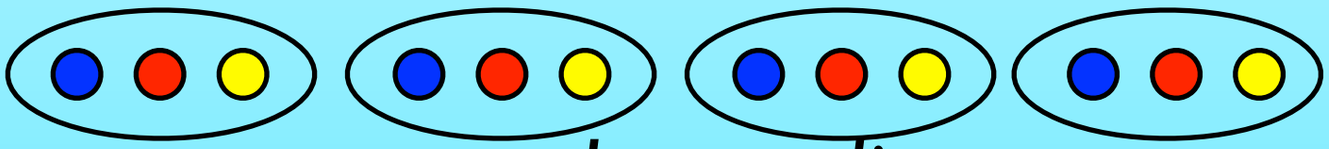


non orthogonality

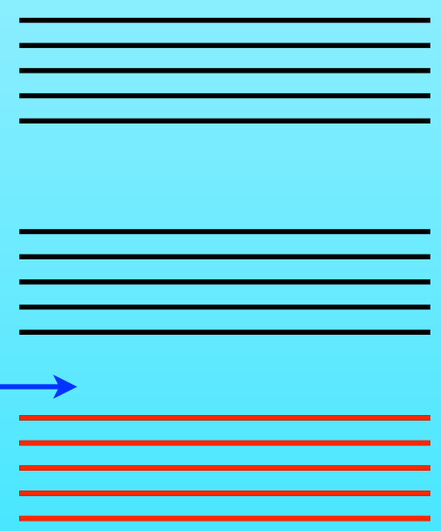
Dimer & Multimer

quantum objects to be respected

make bands of molecules

Adiabatic process

Insulator



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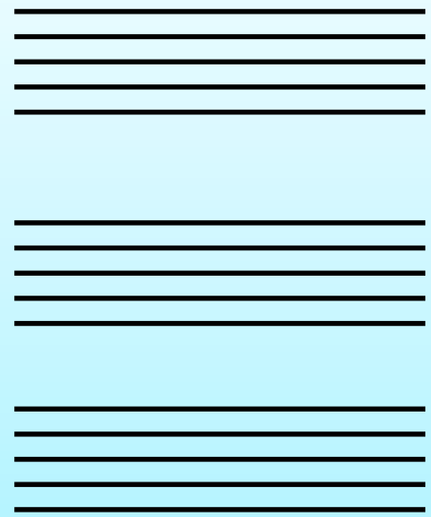
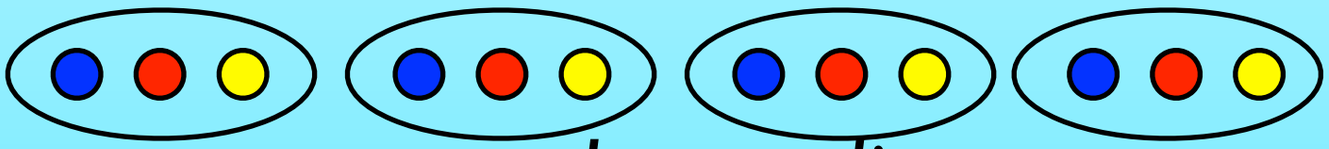


chemist

form molecules first



non orthogonality

short range entanglement

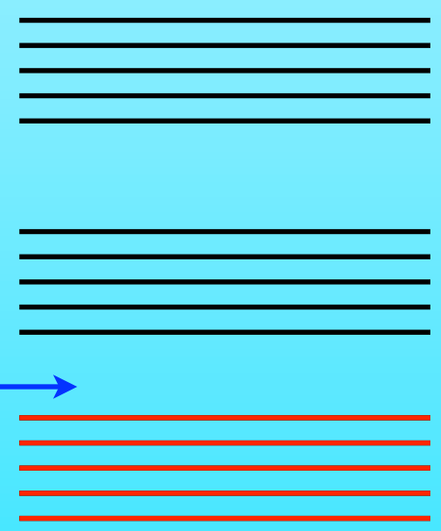
Dimer & Multimer

quantum objects to be respected

make bands of molecules

Adiabatic process

Insulator

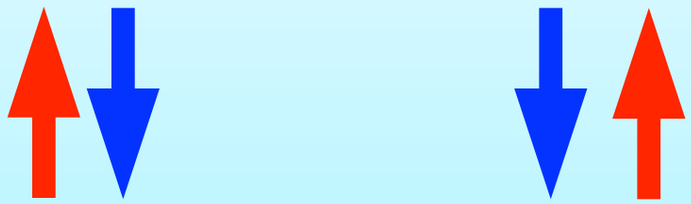


Residual Entropy to form gapped quantum liquid

$$H = \sum H_{\rm loc}$$

local degeneracy

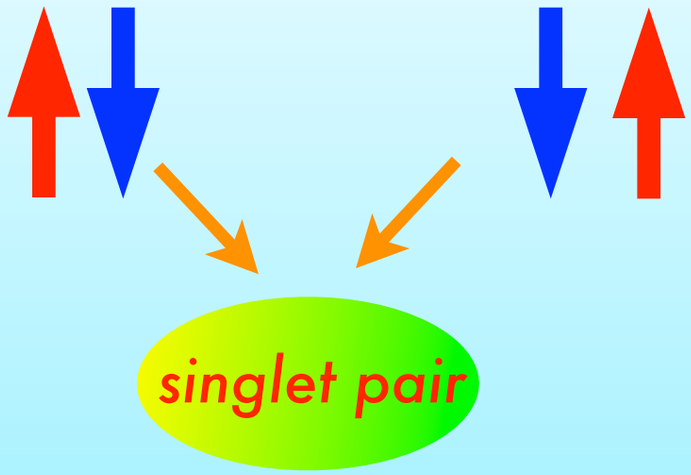
Local anti-ferromagnetism



Local charge ordering

Residual Entropy to form gapped quantum liquid

Singet formation



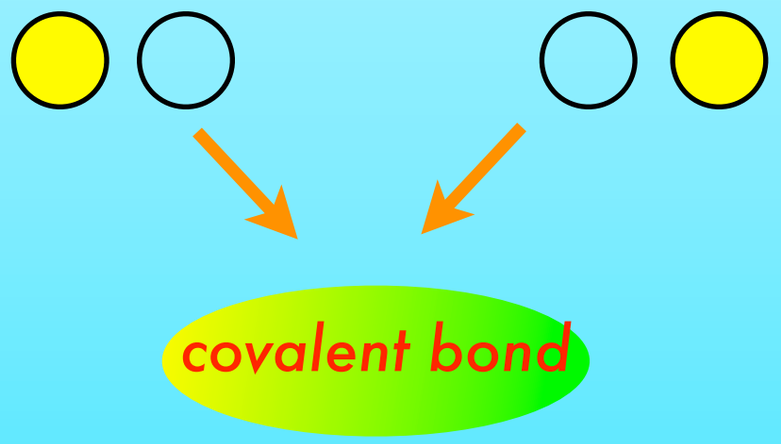
Dimer formation

$$H = \sum H_{\rm loc}$$

local degeneracy

Gap opening
by quantum effect

Stabilization to relax
local entropy



Multimer as a generic dimer

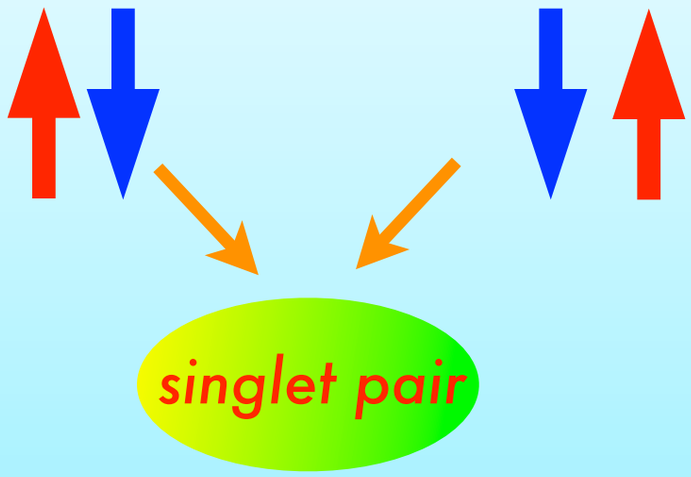
Covalent molecular orbital

Residual Entropy to form gapped quantum liquid

Singet formation

$$H = \sum H_{\rm loc}$$

local degeneracy



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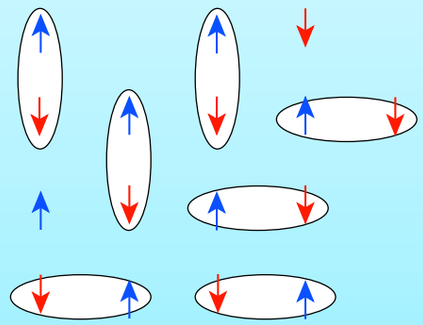
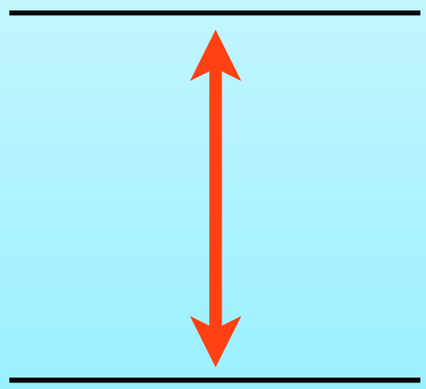


RVB: Anderson



Gap opening
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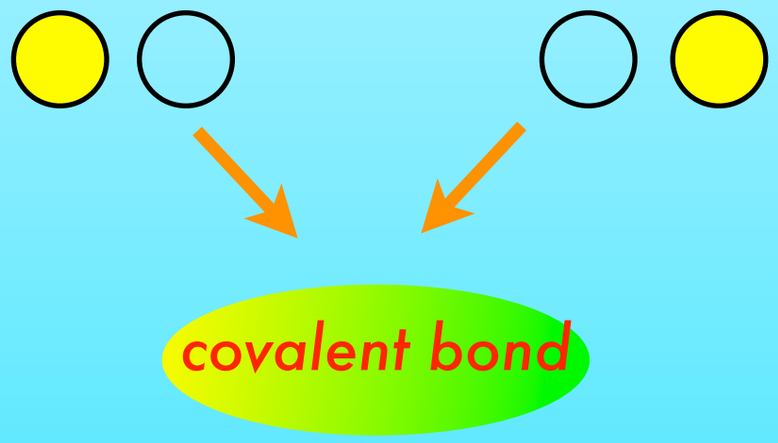

Stabilization to relax
local entropy

Multimer as a generic dimer

Covalent molecular orbital

Residual Entropy to form gapped quantum liquid

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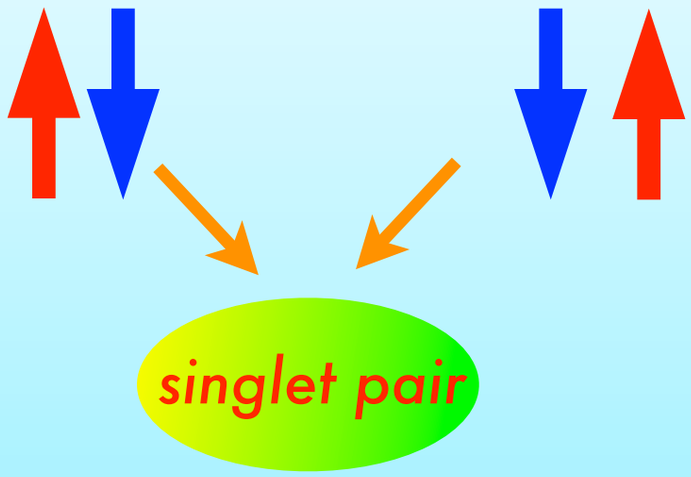
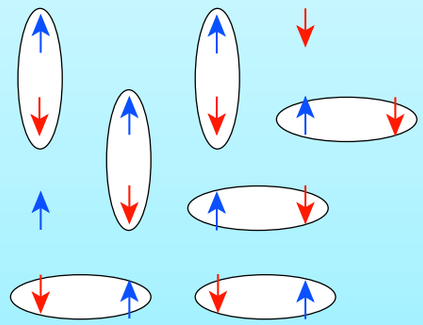


Dimer formation



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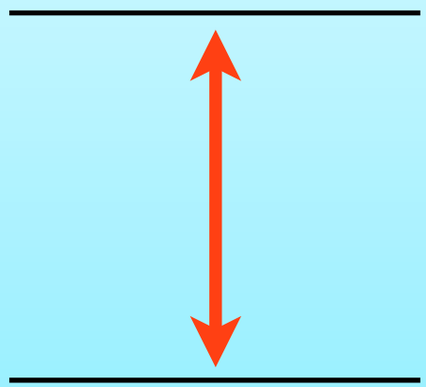
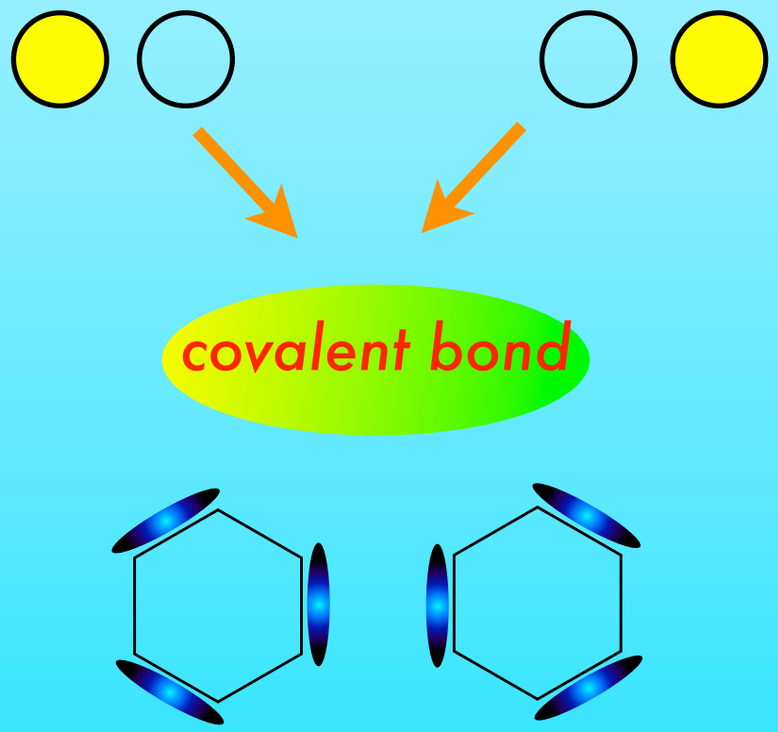


Gap opening
by quantum effect



RVB: Pauling

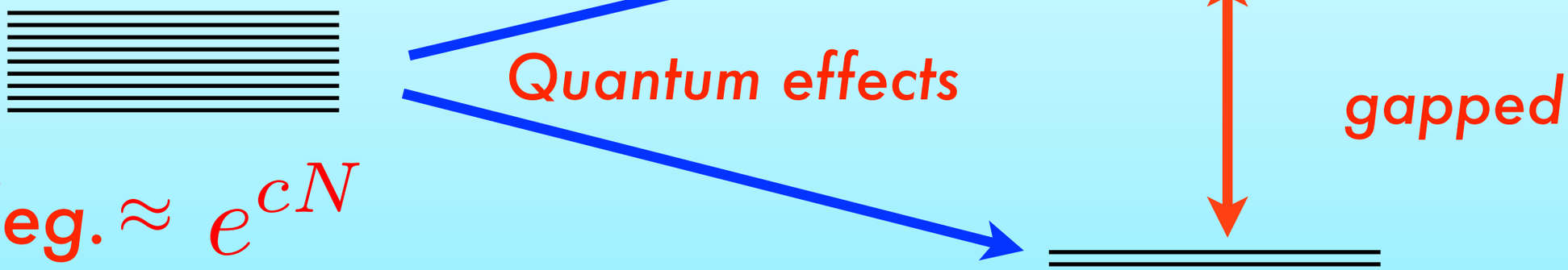
Stabilization to relax
local entropy

Multimer as a generic dimer

Covalent molecular orbital

Quantum effects to relax classical frustration

Classical frustration



$\#\ \text{deg.} \approx e^{cN}$

\# low energy modes

finite low energy states

(topological)

(boundary condition dependent)

Quantum & local ⇔ Gapped

Gapped quantum liquid

"Topological insulators"

(c.f. QHE, QSHE, top.super,

Haldane chain, ladders...)

Gapped quantum liquids

No low lying excitations

No Response against small perturbation

??

Nambu-Goldstone

~~gapless modes:~~

~~acoustic phonons~~

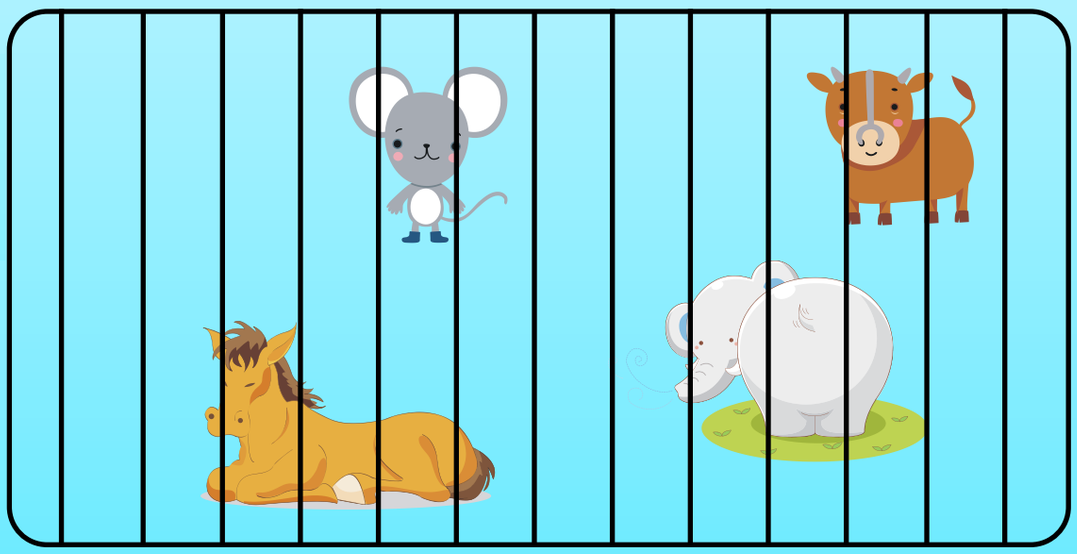
~~zero sounds~~

~~spin waves~~

Absence of fundamental symmetry breaking

Still Lots of variety

Zoo



~~Fermi surface~~

~~massless Dirac fermions~~

with/without doubling

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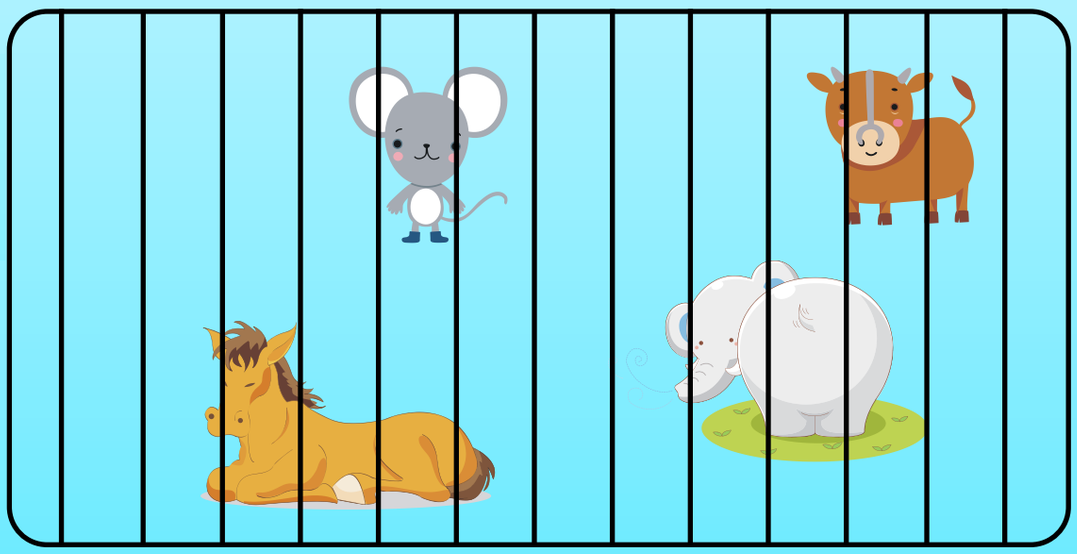

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Something for classification

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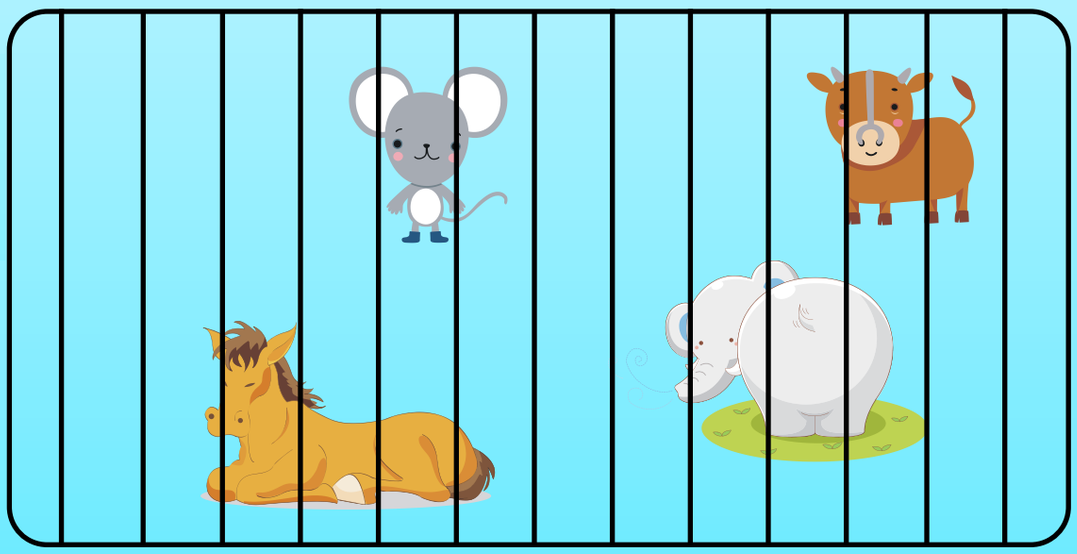

~~Fermi surface~~

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with/without doubling

Something for classification

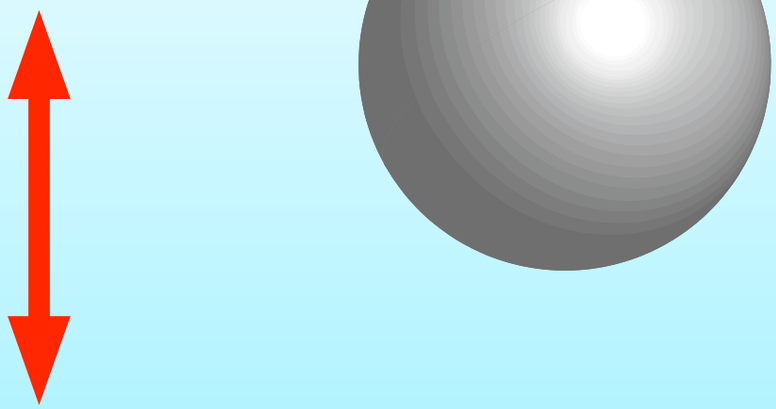
☐ Topological order

☐ Berry connections

☑ Edge states & entanglement

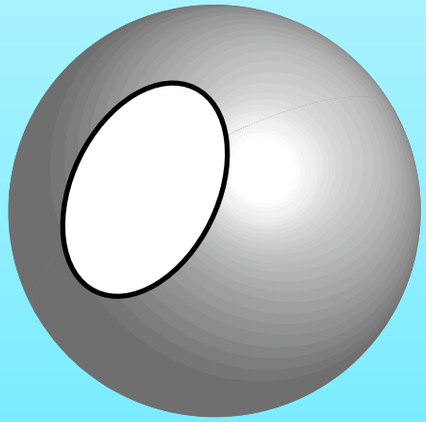
How to understand gapped quantum liquids ?

Bulk state



Bulk without boundary is boring

Edge state



Characteristic *localized states with edges*

Bulk-Edge correspondence *YH'93*

QHE, Spin chains, Graphene, QSHE, Andreev bound st., photonic crystals, cold atoms ...

Quantum Liquids & Entanglement

featureless bulk

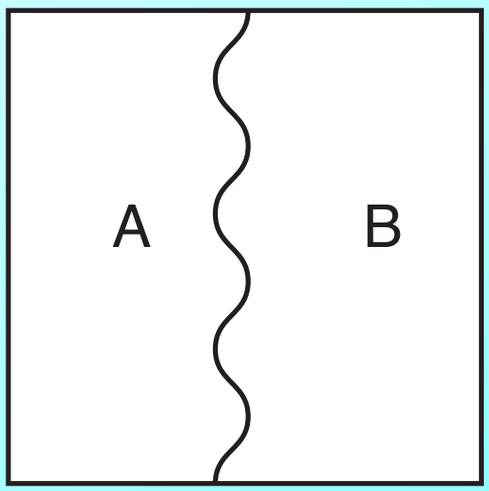
Gapped quantum liquids

Quantum Liquids & Entanglement

featureless bulk

Gapped quantum liquids

Divide into A & B



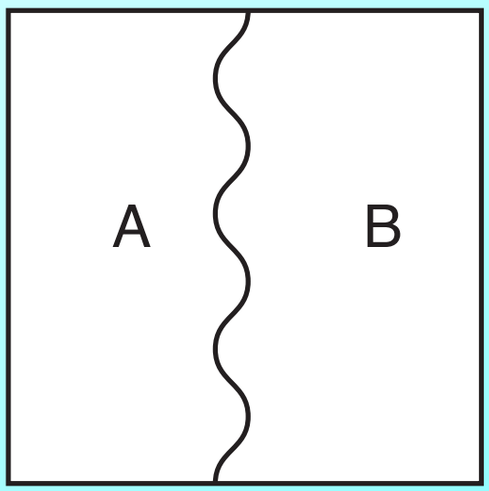
Quantum Liquids & Entanglement

featureless bulk

Gapped quantum liquids

Divide into A & B

How much the states are entangled between A & B?



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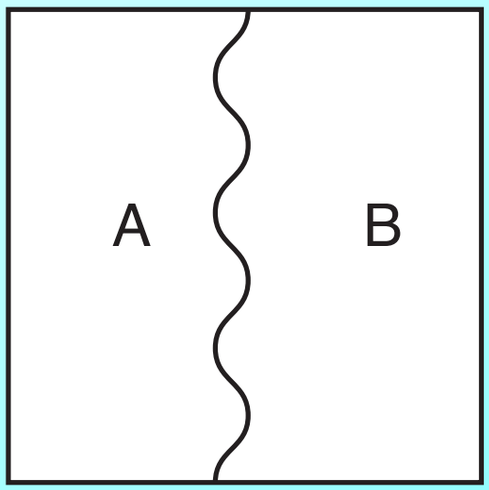

Entanglement entropy

$$\rho_{AB} = |\Psi_{AB}\rangle\langle\Psi_{AB}|$$

$$S_A = -\langle \log \rho_A \rangle \qquad \rho_A = \mathrm{Tr}_B\, \rho_{AB}$$

Characterize the gapped quantum liquids

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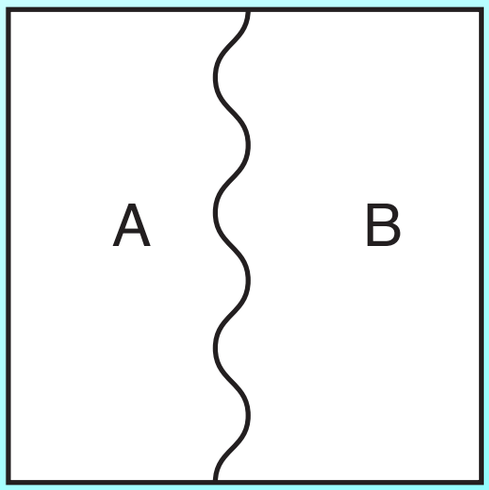

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Edge states near the boundary between A & B

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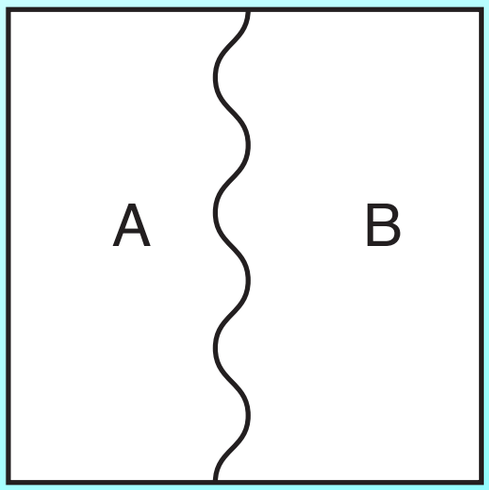

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Characterize the gapped quantum liquids

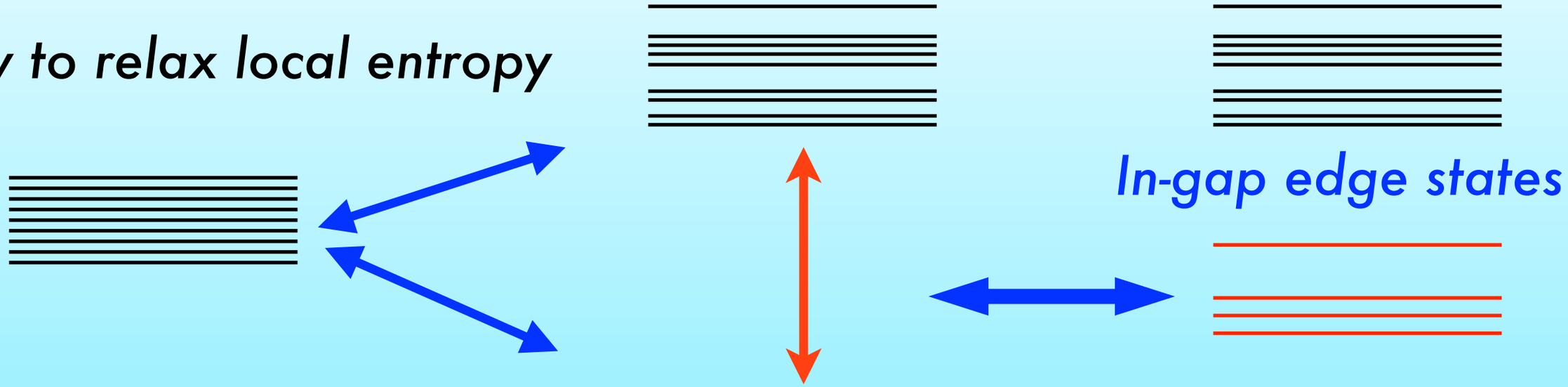
Edge states near the boundary between A & B

Effective edge states reflect quantum entanglement

local deg. freedom, entanglement and edge states

Quantum effects *with boundaries*

How to relax local entropy



 # local degree of freedom

$$\#\ \text{deg.} \approx e^{cN}$$

Entangled & gapped

V_e *: volume (length) of edges*

$$\#\ \text{deg.} \approx e^{V_e/V_0}$$

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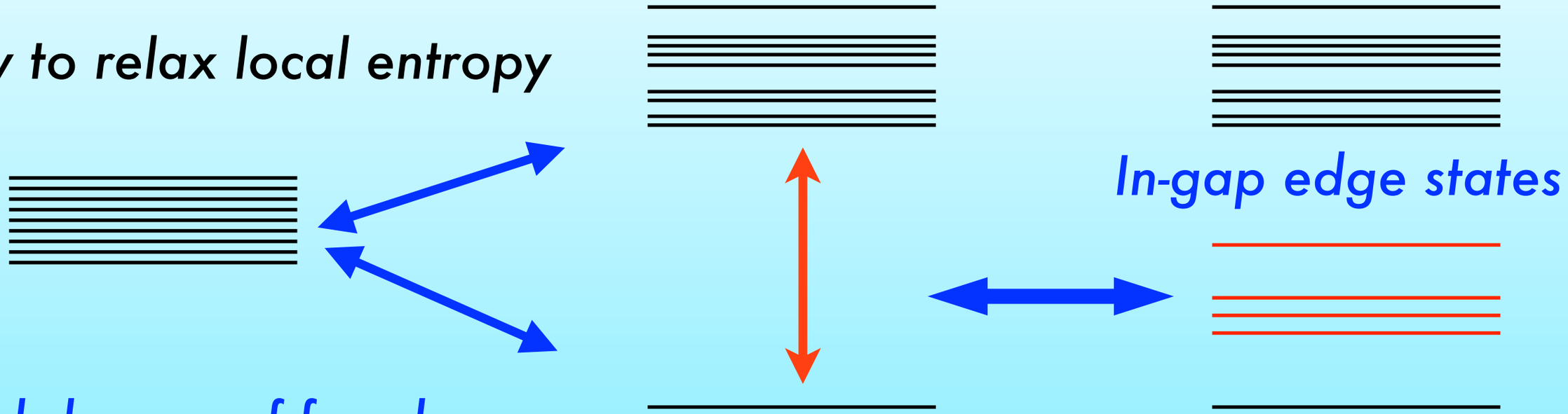

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Quantum entanglement EATS local degrees of freedom and makes the system gapped

local deg. freedom, entanglement and edge states

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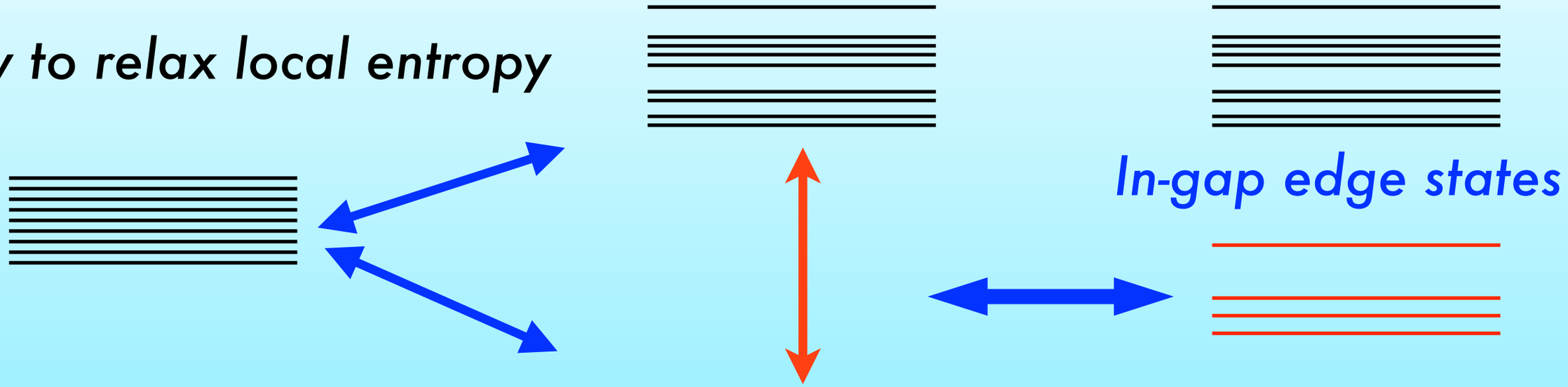

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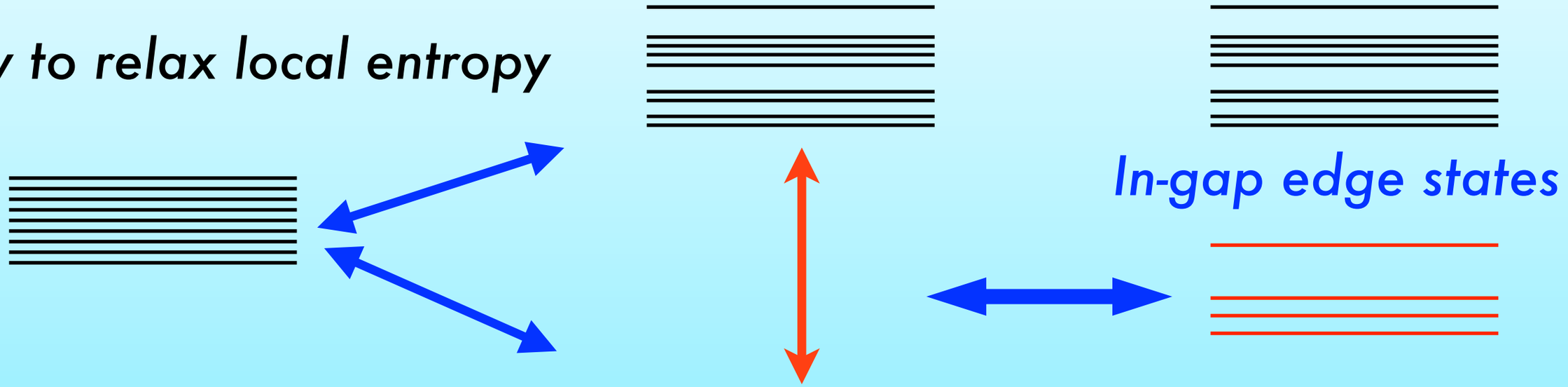
Superconductivity :Anderson mechanism
NG bosons are eaten by gauge field and gapped.

local deg. freedom, entanglement and edge states

Quantum effects

with boundaries

How to relax local entropy



In-gap edge states

local degree of freedom

Entangled & gapped

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Quantum entanglement EATS local degrees of freedom and makes the system gapped

Superconductivity :Anderson mechanism
NG bosons are eaten by gauge field and gapped.

They revive again as edge states near the boundaries

END